Inside the Stars, IAU Colloquium 137
ASP Conference Series, Vol. 40, 1993
Werner W. Weiss and Annie Baglin (eds.)

THEORY OF THE OBLIQUE PULSATOR MODEL FOR THE RAPIDLY OSCILLATING AP STARS

HIROMOTO SHIBAHASHI AND MASAO TAKATA
Department of Astronomy, University of Tokyo, Tokyo 113, Japan

ABSTRACT Recently, one of the rapidly oscillating Ap stars, HR 3831, has been found to have an equally split frequency septuplet, though its oscillation seems to be essentially an axisymmetric dipole mode with respect to the magnetic axis which is oblique to the rotation axis (Kurtz et al. 1992; Kurtz 1992). In order to explain this fine structure, we investigate oscillations of rotating magnetic stars by taking account of the perturbations due to the magnetic fields and the rotation. We show that the magnetic field on axisymmetric dipole modes distorts the dipole oscillation pattern to have axisymmetric octapole components so that the frequency pattern becomes a septuplet rather than a triplet, and that the additional effect of the rotation leads the frequency pattern to be asymmetric. The formula allows us to get information about the magnetic fields, the rotation, and the geometrical configuration of the star from the oscillation data.

The rapid oscillations of Ap (roAp) stars are multi-periodic and then they can provide us with much information about the inside of these stars. The Ap stars themselves are unique objects in the sense that the oscillations in these stars are influenced by strong magnetic fields. These fields are thought to play an essential role of chemical peculiarity of these stars. Seismic investigation of these stars is expected to provide us with some information about their magnetic fields, and hence asteroseismology of the roAp stars is worth of doing to understand physics of the Ap phenomena. For more details, see reviews (Kurtz 1990; Matthews 1991; Shibahashi 1991).

The amplitudes of the rapid oscillations of some roAp stars vary definitely synchronously with the magnetic field strength of the star. Hence the oscillations are interpreted as axisymmetric dipole modes with respect to the magnetic axis which is oblique to the rotation axis of the star [the oblique pulsator model of Kurtz (1982)]. According to this model, the change of the aspect angle of the oscillation induces a triplet fine structure in the frequency spectrum; each of the side components is separated exactly by the rotation frequency of the star from the central frequency component. The oblique pulsator model has been theoretically refined. It now explains the asymmetry of the amplitudes of the triplet fine structure, from which information about the magnetic field can be deduced (Dziembowski and Goode 1985, 1986; Kurtz and Shibahashi 1986).

However, Kurtz et al's (1992) recent observations of HR 3831 show that the fine structure so far thought to be a triplet is in reality a part of a septuplet. The discovery of a septuplet fine structure destabilized the fundamentals of the oblique pulsator model, and we have been faced many difficulties. In this situation, Kurtz (1992) tried to decompose the oscillation pattern in terms of spherical harmonics of degrees $l = 0, 1, 2,$ and 3, and succeeded to get some reasonable fits. However, his approach is phenomenological, and needs physical justification. In this paper, we outline our formulation of the nonradial oscillations of rotating magnetic stars to derive a septuplet fine structure in the frequency spectrum (Shibahashi and Takata 1992; in preparation). Our approach is refinement of the oblique pulsator model, and we show the model explains well the observations.

We suppose that the star is rigidly rotating and that the magnetic field is a dipole field. We treat the effects of the rotation and of the magnetic field as perturbations. In doing so, we suppose that the rotation of the star is slow enough so that the effect of the rotation on oscillations is smaller than that of the magnetic field. Hereafter, we consider the oscillation mode which is principally represented by axisymmetric dipole with respect to the magnetic axis. That is, the zero-order eigenfunction is represented by the main component of $(l, m) = (1,0)$.

After some manipulation, the eigenfunction ξ is expressed by

$$\begin{aligned}\xi \simeq\ & (\xi_{n,1,0} + \alpha_1\xi_{n,1,1} + \alpha_{-1}\xi_{n,1,-1} \\ & + \beta_{n-1,3,0}\xi_{n-1,3,0} + \beta_{n-1,3,1}\xi_{n-1,3,1} + \beta_{n-1,3,-1}\xi_{n-1,3,-1}) \\ & \times \exp\{i(\omega^{(0)} + \omega^{(1)})t\}, \end{aligned} \tag{1}$$

where the suffixes of ξ, α, and β represent the radial order n, the degree l, and the azimuthal order m with respect to the magnetic axis, respectively. Here, we set the temporal dependence of eigenfunctions as $\exp(i\omega t)$ in the rotating frame, and the coefficients are

$$\alpha_{\pm 1} \propto \frac{\text{Coriolis Force } (\propto \Omega)}{\text{Lorentz Force } (\propto B^2)}, \tag{2}$$

$$\beta_{n-1,3,0} \propto \frac{\text{Lorentz Force } (\propto B^2)}{\text{Gas Press.} + \text{Grav. Force}}, \tag{3}$$

$$\beta_{n-1,3,\pm 1} \propto \frac{\text{Coriolis Force } (\propto \Omega)}{\text{Gas Press.} + \text{Grav. Force}}. \tag{4}$$

It should be noted that the previous theoretical works on the oblique pulsator model (e.g. Dziembowski and Goode 1985, 1986; Kurtz and Shibahashi 1986) discarded the first-order components represented by the coefficients β and the octapole components and took account of only the zero-order components expressed by the coefficients α and the dipole components. It is these octapole components that are responsible for the observed septuplet fine structure of the amplitude spectrum.

The spherical harmonic $Y_l^m(\theta_B, \phi_B)$ expressed by the spherical coordinates with respect to the magnetic axis (θ_B, ϕ_B) is written in terms of $(2l+1)$ spherical harmonics of the same degree l with respect to the spherical coordinates (θ_L, ϕ_L) measured from the line-of-sight. Hence the dipole components ($\xi_{n,1,0}$ and $\xi_{n,1,\pm 1}$) of the eigenfunction ξ are observed as a triplet fine structure of the frequency spectrum $(\omega-\Omega,\omega,\omega+\Omega)$ and the octapole components ($\xi_{n-1,3,0}$ and $\xi_{n-1,3,\pm1}$) are observed as a septuplet $(\omega-3\Omega,...,\omega+3\Omega)$ overlapping with the triplet. That is, the observable luminosity variation is formally expressed as

$$L_{\rm obs}=\sum_{m''=-1}^{1} A_{m''}^{(l=1)}\exp[i(\omega-m''\Omega)t]+\sum_{m''=-3}^{3} A_{m''}^{(l=3)}\exp[i(\omega-m''\Omega)t]. \quad (5)$$

The fine structure is asymmetric with respect to its central frequency because of contamination due to the non-axisymmetric components $\xi_{n,1,\pm1}$ and $\xi_{n-1,3,\pm1}$. These features are qualitatively consistent with Kurtz et al's (1992) recent observational results, and provide theoretical justification of Kurtz's (1992) decomposition of the amplitude spectrum of HR 3831 by a series of spherical harmonics.

After some manipulation, we derive the theoretically expected frequency pattern and relations, which are dependent of only the geometrical configuration and the ratio of the Coriolis force and the Lorentz force, among the fine structure amplitudes $A_{m''}^{(l)}$. The degree of asymmetry of the fine structure is independent on the geometrical configuration but depends on the ratio of the Coriolis force and the Lorentz force. The relative contributions of the dipole and the octapole components depend on not only the geometrical configuration and the magnetic field strength but also the limb darkening effects. These relations among $A_{m''}^{(l)}$ allow us to get information about the magnetic fields, the rotation, and the geometrical configuration of the star from the oscillation data, and hence they are promising asteroseismological tools. We are now going to apply the present formulation to Kurtz et al.'s (1992) observational data of HR 3831.

ACKNOWLEDGMENTS

One of the authors (H. S.) acknowledges the Mitsubishi Foundation, which financed the travel expenses.

REFERENCES

Dziembowski, W., and Goode, P. R. 1985, *ApJ*, **296**, L27.

Dziembowski, W., and Goode, P. R. 1986, in *Seismology of the Sun and Distant Stars*, ed. D. O. Gough (Reidel, Dordrecht), p. 441.

Kurtz, D. W. 1982, *MNRAS*, **200**, 807.

Kurtz, D. W. 1990, *ARA&A*, **28**, 607.

Kurtz, D. W. 1992, *MNRAS*, in press.

Kurtz, D. W., Kanaan, A., and Martinez, P. 1992, *MNRAS*, in press.

Kurtz, D. W., and Shibahashi, H. 1986, *MNRAS*, **223**, 557.

Matthews, J. M. 1991, *PASP*, **103**, 5.

Shibahashi, H. 1991, in *Challenges to Theories of the Structure of Moderate-Mass Stars*, ed. D. O. Gough and J. Toomre (Springer, Berlin), p. 393.