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Regulator Indecomposable Cycles on a Product of Elliptic Curves

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Abstract. We provide a novel proof of the existence of regulator indecomposables in the cycle group $CH^2(X,1)$, where X is a sufficiently general product of two elliptic curves. In particular, the nature of our proof provides an illustration of Beilinson rigidity.

1 Introduction

Let X be a smooth projective algebraic manifold of dimension n and let $CH^k(X,m)$ be the higher Chow group of cycles, introduced in [1]. Our interest is the case $m=1$, where an abridged definition of $CH^k(X,1)$ goes as follows. A class $\gamma \in CH^k(X,1)$ is represented as a formal sum $\gamma = \sum(g_j, Z_j)$ of non-zero rational functions g_j on irreducible subvarieties Z_j of codimension $k-1$ in X such that $\sum \operatorname{div} g_j = 0$. One then quotients out by the image of Tame symbols to arrive at the group $CH^k(X,1)$. The group of decomposable cycles, denoted by $CH^k_{\text{dec}}(X,1)$, is defined to be the image of the intersection product $CH^1(X,1) \otimes CH^{k-1}(X) \to CH^k(X,1)$, where in this situation $CH^1(X,1) = \mathbb{C}^\times$ ([1]).

With this definition, decomposable cycles are represented by those with (non-zero) constant rational functions g_j. The corresponding group of indecomposables is the quotient $CH^k_{\text{ind}}(X,1) := CH^k(X,1)/CH^k_{\text{dec}}(X,1)$. There are a number of results centered around constructing indecomposable higher Chow cycles [3, 5–8], and in some cases countably infinite generation results for group of indecomposables are obtained [3, 7]. One of the methods to detect indecomposable cycles is regulator indecomposability, introduced in [4]. A higher Chow cycle $\zeta = \sum(g_j, Z_j)$ is called *regulator indecomposable* if the current defined by its real regulator

$$r(\zeta)(\omega) = \frac{1}{(2\pi\sqrt{-1})^{d-k+1}} \sum \Big(\int_{Z_j - Z_j^{\text{sing}}} \omega \log|f| \Big)$$

is nonzero for some real d-closed test form ω of Hodge type $(1,1)$, with class in $H^{1,1}(E_1 \times E_2, \mathbb{R})$ orthogonal to $Hg^1(E_1 \times E_2) \otimes \mathbb{R}$. A regulator indecomposable cycle is clearly indecomposable. The proof of [4, Theorem 1] (pertaining to the existence

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of regulator indecomposables in the higher cycle group $CH^2(X, 1)$, where X is a sufficiently general product of two elliptic curves) contains an error that was subsequently fixed in [2] using an entirely different set of techniques. The purpose of this paper is to prove this theorem in the spirit of the original techniques in [4].

2 Notation

Throughout this paper, X is assumed to be a projective algebraic manifold. For a subring $\mathbb{A} \subset \mathbb{R}$, put $\mathbb{A}(k) = \mathbb{A}(2\pi\sqrt{-1})^k$. Our notation is compatible with [4].

3 Constructing a Higher Chow Cycle

For $j = 1, 2$ let $E_j \subset \mathbb{P}^2$ be elliptic curves defined by the Weierstrass equations

$$F_j = y_j^2 - x_j^3 + b_j x_j + c_j \quad \text{and} \quad X = V(\bar{F}_1, \bar{F}_2) \simeq E_1 \times E_2.$$

Clearly X varies with $t = (b_1, c_1, b_2, c_2)$. We consider the family $\mathcal{X} := V(\bar{F}_1, \bar{F}_2) \subset \mathbb{C}^4 \times \mathbb{P}^2 \times \mathbb{P}^2$. *Sufficiently general* X means, $X = X_t$ in a transcendental sense, with t outside a suitable countable union of proper Zariski closed subsets.

Let D be the curve of intersection of X with the hypersurface given by $V(s_1t_1+s_2t_2)$, where $[s_0, s_1, s_2]$ and $[t_0, t_1, t_2]$ are homogeneous coordinates of $\mathbb{P}^2 \supset E_1$ and $\mathbb{P}^2 \supset E_2$ respectively, as in [4], with $x_1 = \frac{s_1}{s_0}$, $x_2 = \frac{t_1}{t_0}$, $y_1 = \frac{s_2}{s_0}$, $y_2 = \frac{t_2}{t_0}$. Under the Segre embedding $s\colon \mathbb{P}^2 \times \mathbb{P}^2 \hookrightarrow \mathbb{P}^8$, given by

$$s\colon [s_0, s_1, s_2; t_0, t_1, t_2] \mapsto [s_0t_0, s_1t_0, s_2t_0, s_0t_1, s_1t_1, s_2t_1, s_0t_2, s_1t_2, s_2t_2],$$

D corresponds to a $\mathbb{P}^7 \subset \mathbb{P}^8$ intersecting with X. By [4, Lemma 2.2], D is smooth and irreducible for general t.

In [4] the function $f = x_1 - \sqrt{-1}$ and the form $\omega := (\frac{dx_1}{y_1} \wedge \frac{d\bar{x}_2}{\bar{y}_2} + \frac{d\bar{x}_1}{\bar{y}_1} \wedge \frac{dx_2}{y_2})$ in affine coordinates are considered, and it is claimed that

$$\int_D \omega \log |f| \neq 0.$$

For general X, $w \in (Hg^1(X) \oplus \mathbb{R})^\perp$ (see [4, Lemma 2.5]), where $Hg^1(X)$ denotes the group of Hodge cycles of codimension 1 on X. This claim is proved by means of two deformation arguments; first, deforming D_t from generic point $t = (b_1, c_1, b_2, c_2)$ to $t = (b_1, 0, b_2, 0)$ and then considering the limit case as $(b_1, b_2) \mapsto (0, 0)$. However, there is an error in the second deformation argument. We discuss this error briefly below.

When $t = (b_1, 0, b_2, 0)$, we have $X = E_1 \times E_2$ where E_j is given by the equation $y_j^2 = x_j^3 + b_j x_j$ and $D_t = X \cap V(x_1x_2 + y_1y_2 = 0)$. Notice that on D_t we have

$$x_1^2x_2^2 = y_1^2y_2^2 = x_1x_2(x_1^2 + b_1)(x_2^2 + b_2),$$

and we can decompose

$$D_t = \big(E_1 \times [1, 0, 0]\big) + \big([1, 0, 0] \times E_2\big) + \grave{D}_t,$$

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where $x_1x_2 = (x_1^2 + b_1)(x_2^2 + b_2)$ on $\grave{D}_t$. We can cancel a factor of x_1x_2, corresponding to the curve $(E_1 \times [1, 0, 0]) + ([1, 0, 0] \times E_2)$, since the pull back of the real 2-form ω to this component is zero. Hence we have

$$\int_{D_t} \omega \log |f| = \int_{\grave{D}_t} \omega \log |f|,$$

and we are left with the family $\sum := \bigcup_{t \in U} \grave{D}_t$ for some neighbourhood U of t.

In the second degeneration argument, $(b_1, b_2) \mapsto (0, 0)$, we have $X = E_1 \times E_2$, where the elliptic curves E_j themselves degenerate to $y_j^2 = x_j^3$, and we can decompose $\grave{D}_t$ into three pieces $\acute{\acute{D}}$, $(E_1 \times [1, 0, 0])$ and $([1, 0, 0] \times E_2)$ where $\acute{\acute{D}} = D \cap V(x_1x_2 - 1)$. Moreover, we have $x_1x_2 = x_1^2x_2^2$ on $\acute{\acute{D}}$, but this time we cannot cancel the factor x_1x_2, since the real 2-form ω acquires singularities and contributions to the real regulator from different parts cancel each other.

We will keep track of this deformation and show that the contributions to real regulator from the parts $\acute{\acute{D}}$ and $(E_1 \times [1, 0, 0])$ cancel each other by direct calculation of integrands in the limit case. To see this, and for notational simplicity, let us take $b_1 = b_2 = \epsilon$. On $\grave{D}$, we have $x_1x_2 = (x_1^2 + \epsilon)(x_2^2 + \epsilon)$ and x_1 is a local coordinate on a Zariski open subset of each irreducible component of $\grave{D}$ (provided we discard the component $[1, 0, 0] \times E_2$ when $b_1 = b_2 = 0$, which we can do, as this amounts to the observation that $\log |f| = \log |x_1 - \sqrt{-1}| = 0$ there). We now apply some first order approximations. For small values of $|\epsilon|$, we have $x_1x_2 \approx x_1^2x_2^2$, and if $x_1x_2 \neq 0$, then $x_1x_2 = 1$, and $x_2 \approx x_1^{-1}$ is a solution. On the other hand, regarding $E_1 \times [1, 0, 0]$, we look at small values of $|x_2|$ and we get $x_1x_2 \approx \epsilon(x_1^2 + \epsilon) \approx \epsilon x_1^2$, and $x_2 \approx \epsilon x_1$ is a solution. Clearly, the former one limits to $\acute{\acute{D}}$ and the latter to $E_1 \times [1, 0, 0]$. To reiterate, we can discard the other component $[1, 0, 0] \times E_2$. So we will compute the limiting integral of $\log |x_1 - \sqrt{-1}|\omega$ for these two approximate solutions.

Consider

$$\omega = \left(\frac{dx_1}{\sqrt{x_1^3 + \epsilon x_1}} \right) \wedge \overline{\left(\frac{dx_2}{\sqrt{x_2^3 + \epsilon x_2}} \right)} + \overline{\left(\frac{dx_1}{\sqrt{x_1^3 + \epsilon x_1}} \right)} \wedge \left(\frac{dx_2}{\sqrt{x_2^3 + \epsilon x_2}} \right). \tag{3.1}$$

For $x_2 = x_1^{-1}$, $dx_2 = -x_1^{-2}dx_1$. Plugging this in above equation,

$$\omega = \left(\frac{dx_1}{(x_1^3 + \epsilon x_1)^{\frac{1}{2}}} \right) \wedge \overline{\left(\frac{-x_1^{-2}dx_1}{(x_1^{-3} + \epsilon x_1^{-1})^{\frac{1}{2}}} \right)} + \overline{\left(\frac{dx_1}{(x_1^3 + \epsilon x_1)^{\frac{1}{2}}} \right)} \wedge \left(\frac{-x_1^{-2}dx_1}{(x_1^{-3} + \epsilon x_1^{-1})^{\frac{1}{2}}} \right).$$

Arranging the terms, we get

$$\begin{aligned} \omega &= -\frac{dx_1}{x_1^{\frac{1}{2}}(x_1^2 + \epsilon)^{\frac{1}{2}}} \wedge \frac{d\overline{x_1}}{\overline{x_1}^{\frac{1}{2}}\overline{(1 + \epsilon x_1^2)}^{\frac{1}{2}}} - \frac{d\overline{x_1}}{\overline{x_1}^{\frac{1}{2}}\overline{(x_1^2 + \epsilon)}^{\frac{1}{2}}} \wedge \frac{dx_1}{x_1^{\frac{1}{2}}(1 + \epsilon x_1^2)^{\frac{1}{2}}} \\ &= \left(\frac{-1}{x_1^{\frac{1}{2}}(x_1^2 + \epsilon)^{\frac{1}{2}}\overline{x_1}^{\frac{1}{2}}\overline{(1 + \epsilon x_1^2)}^{\frac{1}{2}}} + \frac{1}{\overline{x_1}^{\frac{1}{2}}\overline{(x_1^2 + \epsilon)}^{\frac{1}{2}}x_1^{\frac{1}{2}}(1 + \epsilon x_1^2)^{\frac{1}{2}}} \right) dx_1 \wedge d\overline{x_1} \\ &= \left(\frac{x_1^{\frac{1}{2}}(x_1^2 + \epsilon)^{\frac{1}{2}}\overline{x_1}^{\frac{1}{2}}\overline{(1 + \epsilon x_1^2)}^{\frac{1}{2}} - \overline{x_1}^{\frac{1}{2}}\overline{(1 + \epsilon x_1^2)}^{\frac{1}{2}}x_1^{\frac{1}{2}}(1 + \epsilon x_1^2)^{\frac{1}{2}}}{|x_1||1 + \epsilon x_1^2||x_1^2 + \epsilon||x_1|} \right) dx_1 \wedge d\overline{x_1}. \end{aligned}$$

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Taking the limit as $\epsilon \to 0$, we have

$$\omega = \left(\frac{x_1^{\frac{3}{2}} \overline{x_1}^{\frac{1}{2}} - x_1^{\frac{1}{2}} \overline{x_1}^{\frac{3}{2}}}{|x_1|^4} \right) dx_1 \wedge d\overline{x_1} = \left(\frac{x_1 - \overline{x_1}}{|x_1|^3} \right) dx_1 \wedge d\overline{x_1} \text{ on } \tilde{\tilde{D}}.$$

As $\epsilon \to 0$, $x_2 = x_1^{-1}$ has limit $\tilde{\tilde{D}}$ and

$$\log |f| \omega \to \log |x_1 - \sqrt{-1}| \left(\frac{x_1 - \overline{x_1}}{|x_1|^3} \right) dx_1 \wedge d\overline{x_1}.$$

Let us consider the latter approximation $x_2 = \epsilon x_1$. When $x_2 = \epsilon x_1$, we have $dx_2 = \epsilon dx_1$. Plugging these relations into equation (3.1), we get;

$$\begin{aligned}
\omega &= \left(\frac{dx_1}{(x_1^3 + \epsilon x_1)^{\frac{1}{2}}} \right) \wedge \overline{\left(\frac{\epsilon dx_1}{(\epsilon^3 x_1^3 + \epsilon^2 x_1)^{\frac{1}{2}}} \right)} + \overline{\left(\frac{dx_1}{(x_1^3 + \epsilon x_1)^{\frac{1}{2}}} \right)} \wedge \left(\frac{\epsilon dx_1}{(\epsilon^3 x_1^3 + \epsilon^2 x_1)^{\frac{1}{2}}} \right) \\
&= \left(\frac{dx_1}{(x_1^3 + \epsilon x_1)^{\frac{1}{2}}} \right) \wedge \frac{\overline{\epsilon} d\overline{x_1}}{\overline{(\epsilon^3 x_1^3 + \epsilon^2 x_1)^{\frac{1}{2}}}} + \frac{d\overline{x_1}}{\overline{(x_1^3 + \epsilon x_1)^{\frac{1}{2}}}} \wedge \left(\frac{\epsilon dx_1}{(\epsilon^3 x_1^3 + \epsilon^2 x_1)^{\frac{1}{2}}} \right) \\
&= \left(\frac{\overline{\epsilon}}{(x_1^3 + \epsilon x_1)^{\frac{1}{2}} \overline{(\epsilon^3 x_1^3 + \epsilon^2 x_1)^{\frac{1}{2}}}} - \frac{\epsilon}{\overline{(x_1^3 + \epsilon x_1)^{\frac{1}{2}}} (\epsilon^3 x_1^3 + \epsilon^2 x_1)^{\frac{1}{2}}} \right) dx_1 \wedge d\overline{x_1}.
\end{aligned}$$

Taking the limit as $\epsilon \to 0$, we get

$$\omega = \left(\frac{1}{x_1^{\frac{3}{2}} \overline{x_1}^{\frac{1}{2}}} - \frac{1}{\overline{x_1}^{\frac{3}{2}} x_1^{\frac{1}{2}}} \right) dx_1 \wedge d\overline{x_1} = \left(\frac{\overline{x_1} - x_1}{|x_1|^3} \right) dx_1 \wedge d\overline{x_1} \text{ on } E_1 \times [1, 0, 0].$$

In the limit as $\epsilon \to 0$, $x_2 = \epsilon x_1$ has limit $E_1 \times [1, 0, 0]$ and

$$\log |f| \omega \to \log |x_1 - \sqrt{-1}| \left(\frac{\overline{x_1} - x_1}{|x_1|^3} \right) dx_1 \wedge d\overline{x_1}.$$

(As a reminder, when $b_1 = b_2 = 0$, $E_1 = E_2$ are (singular) rational curves.) In the limit, the contributions of these parts to the real regulator cancel one another.

In order to solve this problem, we consider the function $f = x_1^2 x_2 - \sqrt{-1}$ and the same form ω. Note that for the solution $x_2 = \epsilon x_1$, which limits to the component $E_1 \times [1, 0, 0]$, $\log |x_1^2 x_2 - \sqrt{-1}| = \log |\epsilon x_1^3 - \sqrt{-1}|$, goes to zero as $\epsilon \to 0$, so in the limit, $\log |f| \omega$ vanishes. However for the second solution $x_2 = x_1^{-1}$, we have $\log |x_1^2 x_2 - \sqrt{-1}| = \log |x_1 - \sqrt{-1}|$. In the limit we get the component $\tilde{\tilde{D}}$ and recover the function $\log |x_1 - \sqrt{-1}|$ introduced in [4], which contributes to the real regulator nontrivially.

Since the function $f = x_1^2 x_2 - \sqrt{-1}$ is not linear as in [4], it requires a more complicated and different argument to complete the tuple (f, D) to a higher Chow cycle.

Let $E_{j,\mathrm{tor}}$ denote the set of torsion points on E_j. We define $D_{\mathrm{tor}} := \{E_{1,\mathrm{tor}} \times E_2\} \cap D$. For sufficiently general X, D is a smooth irreducible curve. Moreover, $E_{1,\mathrm{tor}}$ is dense in E_1 and $D \subset X = E_1 \times E_2$ projects onto first factor, so D_{tor} is dense in D. In projective coordinates the function f is given by

$$f = x_1^2 x_2 + \sqrt{-1} = \frac{s_1^2 t_1 t_0 + s_0^2 t_0^2 \sqrt{-1}}{s_0^2 t_0^2}.$$

Under the Segre embedding f is a quotient of two quadrics

$$Q_{1,0} = s_1^2 t_1 t_0 + s_0^2 t_0^2 \sqrt{-1} = (s_1 t_1)(s_1 t_0) + (s_0 t_0)^2 \sqrt{-1}$$

and

$$Q_{2,0} = s_0^2 t_0^2 = (s_0 t_0)^2.$$

Counted with multiplicities, the divisor of f along D is given by

$$\operatorname{div}(f)_D = V(Q_{1,0}) \cap D - V(Q_{2,0}) \cap D.$$

Note that for a quadric $Q \in \mathbb{P}^8$, deg $(Q \cap D) = 36$. Consider the family of quadrics lying in a $\mathbb{P}^7 \subset \mathbb{P}^8$ cutting out $D \subset E_1 \times E_2$ under the Segre embedding. This family is a projective space of dimension 35. Hence the family of quadrics passing through 35 general points of D is zero dimensional. If we set $Q \cap D = \{p_1 + \cdots + p_{36}\}$, and assume that $\{p_1 \cdots p_{35}\} \in D_{\text{tor}}$, then $p_{36} \in D_{\text{tor}}$.

Let $q_1^i \cdots q_{36}^i \in \operatorname{div}_D(Q_{i,0})$. Since D_{tor} is dense in D, for any given collection of analytic neighborhoods $\{U_i\}$ around q_i for $i = 1 \cdots 36$, we can find 36 points $p_1^i, \ldots, p_{36}^i \in D_{\text{tor}}$, lying in a quadric intersected with D, such that $p_j^i \in U_i$. By the above argument these points define quadratic functions $Q_{i,n}$ for $i = 1, 2$ and $\widetilde{f}_n = Q_{1,n}/Q_{2,n}$ such that $p_1^i, \ldots, p_{36}^i \in \operatorname{div}_D(\widetilde{f}_n) \subset D_{\text{tor}}$; moreover, using the fact that if $h_1, h_2 \in \mathbb{C}^\times$ with $\operatorname{div}(h_1) = \operatorname{div}(h_2)$ then $h_1 = c \cdot h_2$ for some $c \in \mathbb{C}^\times$, we can arrange for $\lim_{n \to \infty} \widetilde{f}_n = f$.

Let Δ_j be a small open polydisk in the space of quadratic polynomials in $\mathbb{C}[z_0, \ldots, z_7]$ centered at 0 for $j = 1, 2$. Then for $t \in \Delta := \Delta_1 \times \Delta_2$, one has a corresponding function $f_t = Q_{1,t}/Q_{2,t}$ with $f_0 = Q_{1,0}/Q_{2,0} = f$.

Note that the set

$$\bigcup_{t \in \Delta} |\operatorname{div}(f_t)|$$

has real codimension ≥ 2 in $\Delta \times D$. Considering ϵ tubular neighborhoods in $\Delta \times D$ about this set and applying standard estimates as $\epsilon \mapsto 0^+$, we conclude that the integral $\int_D \log |f_t| \omega$ varies continuously with $t \in \Delta$.

We may assume that

$$\int_D \log |f_t| \omega \neq 0, \qquad \forall t \in \Delta.$$

Since Δ parameterizes all quadratic quotients in a neighborhood of $(0, 0) \in \Delta$, then for large enough n we will have $\widetilde{f}_n = f_t$ for some $t \in \Delta$. Therefore

$$\left| \int_D \log |f| \omega - \int_D \log |\widetilde{f}_n| \omega \right| < \epsilon,$$

for any small $\epsilon > 0$ and large enough n dependent on ϵ.

The divisor of $\widetilde{f}$ along D can be written as

$$\mathrm{div}_D(\widetilde{f}) = \sum_j n_j(p_j \times q_j) \in D_{\mathrm{tor}}, \quad \text{where } n_j \in \mathbb{Z} \text{ and } \sum_j n_j = 0.$$

Let e_1 denote the identity element on E_1. By our construction, the p_j's are torsion points, so $m_j p_j \sim_{\mathrm{rat}} m_j e_1$ for some m_j (*i.e.*, there exist rational functions $h_j \in \mathbb{C}(E_1)^\times$ such that $\mathrm{div}_{E_1}(h_j) = m_j e_1 - m_j p_j$). Then for $m = \prod_j m_j$, we have $mp_j \sim_{\mathrm{rat}} me_1$ for all j. So we likewise have rational functions $h_j \in \mathbb{C}(E_1 \times q_j)^\times$ such that $\mathrm{div}_{E_1\times q_j}(h_j) = m(e_1 \times q_j) - m(p_j \times q_j)$. Consider the precycle $(\widetilde{f}^m, D) + \{h_j^{n_j}, E_1 \times q_j\}_j$:

$$\begin{aligned}
&\mathrm{div}_D(\widetilde{f}^m) + \sum_j \mathrm{div}_{E_1\times q_j}(h_j^{n_j}) \\
&\quad = \sum_j mn_j(p_j \times q_j) + \sum_j \big(mn_j(e_1 \times q_j) - mn_j(p_j \times q_j)\big) \\
&\quad = \sum_j mn_j(e_1 \times q_j) := \xi.
\end{aligned}$$

The remaining term ξ is the divisors of the functions $\widetilde{f}$ and $\{(h_j)\}_j$, hence it is rationally equivalent to zero on $E_1 \times E_2$. The projection of ξ to the second factor, $\mathrm{Pr}_{2,*}(\xi)$, is rationally equivalent to zero on E_2. So there exists a rational function g defined on $e_1 \times E_2$ such that $\mathrm{div}_{e_1\times E_2}(g) = -\sum_j mn_j(e_1 \times q_j)$. Let

$$\gamma = (\widetilde{f}^m, D) + \big\{ (h_j^{n_j}, E_1 \times q_j) \big\}_j + (g, e_1 \times E_2).$$

Then

$$\mathrm{div}_D(\widetilde{f}^m) + \sum_j \mathrm{div}_{E_1\times q_j}(h_j^{n_j}) + \mathrm{div}_{e_1\times E_2}(g) = 0.$$

Hence $\gamma \in CH^2(X, 1; \mathbb{Q})$ is a higher Chow cycle.

Note that the curves $E_1 \times q_j$ and $p_j \times E_2$ cannot support the real 2-form ω. Therefore the contributions of the terms $\{(h_j^{n_j}, E_1 \times q_j)\}_j + (g, e_1 \times E_2)$ to the real regulator are zero ($\int_{E_1\times q_j} \log|h_j|\omega = 0 = \int_{e_1\times E_2} \log|g|\omega$), so

$$r(\gamma)(\omega) = \int_D \omega \log|\widetilde{f}^m| \neq 0.$$

That is, $\gamma \in CH^2(X, 1; \mathbb{Q})$ is regulator indecomposable, so it is indecomposable, and hence we have the following theorem.

Theorem 3.1 $CH^2_{\mathrm{ind}}(E_1 \times E_2, 1; \mathbb{Q})$ *is nontrivial for sufficiently general product* $E_1 \times E_2$ *of elliptic curves* E_1 *and* E_2.

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