SESSION VI

PLASMA HEATING BY ALFVÉN WAVES – KINETIC PROPERTIES OF MAGNETOHYDRODYNAMIC DISTURBANCES

Akira Hasegawa
Bell Laboratories, Murray Hill, NJ 07974

Mechanisms of Alfvén wave heating in space-astrophysical plasmas are presented with particular emphasis on the parallel electric field generated in the magnetohydrodynamic perturbations due to the finite Larmor radius effects.

I. INTRODUCTION

Collisionless heating of plasmas by Alfven waves which has been proposed by Grossmann and Tataronis (1973) and Hasegawa and Chen (1974) is now recognized as a major mechanism of plasma heating both in laboratory, in space and in astrophysical plasmas. The original idea of the heating was based on the resonant absorption of the Alfvén wave in a inhomogeneous plasma. The resonant absorption occurs due to the fact that only a local field line at $x = x_o$ in an inhomogeneous plasma can be in resonance with the Alfvén wave with a given frequency ω and parallel wave number $k_{\parallel}$ such that $\omega = k_{\parallel} v_A(x_o)$ where $v_A(x)$ $[= B_o(x)/(\mu_o \rho(x))^{1/2}]$ is the spatially varying Alfvén speed.

It was later recognized by Hasegawa and Chen (1976) that the resonant absorption is a manifestation of the linear mode conversion from the MHD Alfvén wave to the kinetic Alfvén wave and that the physical mechanism of the heating depends on the collisionless absorption of the kinetic Alfvén wave. The kinetic Alfvén wave is the Alfvén wave with perpendicular wavelength comparable to the ion gyroradius, ρ_i.

The important recognition here is the fact that the Alfvén wave accompanies an electric field parallel to the ambient magnetic field if $k_\perp \rho_i \simeq 0(1)$ and the fact this electric field can produce collisionless wave-particle interactions. In space plasmas this electric field has been proposed as a mechanism of accelerating auroral electrons (Hasegawa, 1976; Goertz and Boswell, 1979; Goertz, 1981), as solar (Ionson, 1978) and Stellar (Ionson, 1982) coronal heatings, and acceleration of plasmas in Io torus (Goldstein and Goertz, 1983).

In most cases of plasma heating in space and astrophysical plasmas, the transfer of energy is from the mechanical to the wave and to the particle. Hence the Alfvén wave should first be excited by a mechanical mean. This requires an existence of well defined eigenmodes for an efficient transfer of mechanical energy to the wave energy. Usually the surface Alfvén wave (sometimes called the "kink" mode in laboratory plasmas) is considered for this purpose.

M. R. Kundu and G. D. Holman (eds.), Unstable Current Systems and Plasma Instabilities in Astrophysics, 381–389.

https://doi.org/10.1017/S0074180900075860 Published online by Cambridge University Press

The talks will consist of a brief review of the resonant absorption and resonant mode conversion of the Alfvén wave in an inhomogeneous plasma (II), introduction of the Alfvén surface wave and its modification in the presence of a plasma flow and current (III) and derivation of the local (kinetic) Alfvén wave dispersion relation in a general geometry (IV). Much of the content of the talk can be found in the monograph by Hasegawa and Uberoi (1982).

II. RESONANT ABSORPTION AND RESONANT MODE CONVERSION

In an inhomogeneous plasma with the inhomogeneity in the x direction, the x component of the linearized plasma displacement satisfies the eigen value equation given by (Hasegawa and Chen 1976)

$$\frac{d}{dx}\left[\frac{\epsilon\alpha B_0^2}{\alpha k_\perp^2 B_0^2 - \epsilon}\frac{d\xi_x}{dx}\right] - \epsilon\xi_x = 0 \tag{2.1}$$

where

$$\epsilon(x) = \omega^2 \mu_0\rho - k_\parallel^2 B_0^2 \tag{2.2}$$

and

$$\alpha(x) = 1 + \frac{\omega^2 v_S^2}{v_A(\omega^2 - k_\parallel^2 v_S^2)} . \tag{2.3}$$

Here $B_0(x)$ is the ambient magnetic field, $v_A(x)$ is the Alfven speed, $v_S(x)$ is the MHD sound speed and $\rho(x)$ is the mass density of the plasma. The equation (2.1) shows that for a given frequency $\omega = \omega_0$, the equation becomes singular at $x = x_0$ where $\epsilon(x_0) = 0$. Near $x = x_0$, the solutions of Eq. (2.1) becomes logarithmic,

$$\xi_x = C \ln(x - x_0 + i\delta) . \tag{2.4}$$

The energy absorption rate dW/dt is obtained from the jump in the power flow in the x direction in the cross sectional area $S = L_y L_z$,

$$p = \frac{L_y L_z}{2} \mathrm{Re}(\mathbf{E}\times\mathbf{H}^* + vp^*)$$

$$= -\frac{L_y L_z \omega_0}{2} \mathrm{Im}\, \xi_x^* \frac{\epsilon}{k_\perp^2\mu_0}\frac{\partial\xi_x}{\partial x}$$

Thus

$$\frac{dW}{dt} = -\frac{L_y L_z \omega_0}{2}\frac{|C|^2}{k_\perp^2\mu_0}\left|\frac{d\epsilon r}{dx}\right|_{x=x_0} \mathrm{Im}[\ln(x - x_0 + i\delta)]_{x_0^-}^{x_0^+}$$

$$= \frac{\omega_0}{2}\pi L_y L_z \frac{|C|^2}{k_\perp^2\mu_0}\left|\frac{d\epsilon_r}{dx}\right|_{x=x_0} \tag{2.5}$$

The singularity in Eq. (2.1) originates from the "fluid" approximations of a MHD plasma, hence it can be eliminated by taking into account the plasma "kinetic" property. In fact if the Vlasov equation is used, the singularity of Eq. (2.1) disappears due to the appearance of terms with higher spatial derivatives (Hasegawa and Chen, 1976). Then the absorbed energy can be identified to be mode converted to the kinetic Alfvén wave whose dispersion relations in a uniform plasma (local dispersion relation) is given by

$$\omega^2 = k_{\parallel}^2 v_A (1 + k_{\perp}^2 \bar{\rho}^2) \tag{2.6}$$

where

$$\bar{\rho}^2 = (\frac{3}{4} + \frac{T_e}{T_i})\, \rho_i^2 \tag{2.7}$$

with

$$\rho_i^2 = \frac{T_i}{m_i} \frac{1}{\omega_{ci}^2}\,. \tag{2.8}$$

The kinetic Alfvén wave has the electric field in the direction parallel to the ambient magnetic field whose magnitude is

$$E_z \equiv E_{\parallel} \simeq E_{\perp} \frac{k_{\parallel}}{k_{\perp}} \tag{2.9}$$

This electric field produces the wave-particle interaction. For example, the linear response of the resonant electrons becomes (Hasegawa and Mima, 1978).

$$f_k = \pi\delta(k_z v_Z - \omega) \left[\frac{e}{m} \frac{\partial f_o}{\partial v_Z} - \frac{k_y v_Z}{\omega B_o} \frac{\partial f_o}{\partial x} \right] E_{zk}\,. \tag{2.10}$$

The first term produces the velocity space diffusion (momentum and energy transfer) while the second term produces the coordinate space diffusion. The heating rate for example is readily obtained by taking the quadratic velocity moment,

$$\begin{aligned} n_o \frac{dT_e}{dt} &= \mathrm{Re}\, \frac{1}{2} (J_{zk} E_{zk}{}^*) \\ &= n_o T_e \omega \left(\frac{\pi}{8} \right)^{1/2} \frac{m_i}{m_e} \sum_k \frac{|B_{xk}|^2}{B_o^2} \frac{\lambda_S}{(1 + \lambda_S)^{3/2}} \\ &\times F \left(\frac{v_A}{v_{Te}} \right) \end{aligned} \tag{2.11}$$

where

$$F(x) = x^3 \exp(-x^2/2)$$

and

$$\begin{aligned} \lambda_S &= (k_{\perp}\rho_S^2) \\ &= \frac{T_i}{T_e} (k_{\perp}\rho_i)^2 \end{aligned} \tag{2.12}$$

https://doi.org/10.1017/S0074180900075860 Published online by Cambridge University Press

Note that the heating rate vanishes in the MHD limit, $k_{\perp}\rho_i = 0$.

III. ALFVEN SURFACE WAVES

When there exists a sharp discontinuity in the plasma density and the magnetic field in the direction perpendicular to the magnetic field, the surface Alfvén wave appears in addition to the bulk Alfvén waves. Using ϵ defined in Eq. (2.2), the dispersion relation of the surface Alfvén wave becomes,

$$\epsilon_{I} + \epsilon_{II} = 0 \tag{3.1}$$

where ϵ_I and ϵ_{II} are values of ϵ at the two sides of the discontinuity. If one side is vacuum, Eq. (3.1) gives

$$\omega = \omega_S = \sqrt{2}k_{\parallel} v_A . \tag{3.2}$$

Since ω_S lies between the Alfvén frequencies in the vacuum (∞) and the plasma ($k_{\parallel}v_A$) regions, there exists a location $x = x_o$ between the vacuum and bulk region where "local" Alfvén frequency $\omega = k_{\parallel}v_A(x_o)$ becomes equal to ω_S. From the result of Section II, we note that the surface wave is resonantly mode converted to the kinetic Alfvén wave at $x \equiv x_o$ and the wave energy is absorbed through the wave-particle interactions.

The surface wave dispersion relation is modified when there exists a flow or a current in the plasma. In the presence of a flow $\mathbf{v} = \mathbf{v}_o$ in region I, it is given by (Chandrasekhar, 1961)

$$\frac{1}{n_{0I}[(\omega - \mathbf{k}\cdot\mathbf{v}_0)^2 - (\mathbf{k}\cdot\mathbf{v}_A)_I^2]} + \frac{1}{n_{0II}[\omega^2 - (\mathbf{k}\cdot\mathbf{v}_A)_{II}^2]} = 0 . \tag{3.3}$$

Here n_0 is the plasma number density and the subscripts I and II show quantities in the region I and II respectively and the direction of the vector Alfvén velocity is that of the ambient magnetic field. When $\mathbf{v}_o$ exceeds the threshold given by

$$(\mathbf{k}\cdot\mathbf{v}_0)^2 > \left(\frac{1}{n_{0I}} + \frac{1}{n_{0II}}\right)[n_{0I}(\mathbf{k}\cdot\mathbf{v}_A)_I^2 + n_{0II}(\mathbf{k}\cdot\mathbf{v}_A)_{II}^2] . \tag{3.4}$$

the surface wave becomes unstable (Kelvin-Helmholtz instability). the mechanical energy of the flow can be converted to the heat through the excitation of the Alfvén surface wave and subsequent mode conversion to the kinetic Alfvén wave (Osawa, et al., 1976).

In the presence of a current, the dispersion relation becomes geometry dependent. For example, in a cylindrical plasma with radius a surrounded by vacuum, a current which flows on the plasma surface modifies the dispersion relation (Kadomtsev 1966) for the surface wave with a structure $f(r)\, e^{i(n\theta+kz-\omega t)}$,

$$\mu_o\rho_o\omega^2 = k^2B_{0z}^2 + \left(kB_{0z}^e + \frac{n}{a}B_{0\theta}^e\right)^2 - \frac{n(B_{0\theta}^e)^2}{a^2} . \tag{3.5}$$

Here the subscript zero indicates the ambient quantities, z and θ are axial and azimuthal components and the superscript e indicates the value external to the plasma. In the absence of the current, $B_{0\theta}^e = 0$ and the dispersion relation gives that of the surface Alfvén wave, $\omega = \sqrt{2}\,k_{\parallel}v_A$. In the presence of the current, the kink instability sets in for n = 1 mode when

$$\frac{B^c_{0\theta}}{B^c_{0z}} > \frac{2\pi a}{L}, \tag{3.6}$$

where $L = 2\pi/k$ is the length of the plasma. For $n \neq 1$, the plasma is stable but the surface wave frequency becomes n-dependent. In particular a mode with frequency *lower* than the bulk Alfvén frequency appears. This mode does not suffer the resonant absorption hence remains to be an undamped high-Q mode. When the current is distributed in the cylinder, the dispersion relation does not have a simple form as shown in Eq. (3.5) and the eigen frequency should be obtained numerically (Appert et al. 1982).

IV. FINITE LARMOR RADIUS MHD EQUATIONS

In an inhomogeneous plasma, the kinetic Alfvén wave couples with drift waves, ballooning modes and other electrostatic modes through the presence of the parallel electric field. The dispersion relation including these effects as well as the wave-particle interactions can be obtained by the use of the gyrokinetic equation (Hasegawa, 1979; Freeman and Chen, 1982). However, the gyrokinetic equation is difficult to use for a nonlinear problem in particular due to the basic two components (electron-ion) property. Recently Hasegawa and Wakatani (1983) derived a new set of *fluid* equations which is capable of treating MHD problems with the finite ion gyroradius correction. The appropriate equations are listed in the following. The first equation is $\nabla \cdot \mathbf{J} = 0$ (which is equivalent to the equation for vorticity) with $J_\perp$ given by guiding center currents. In a low beta plasma,

$$\begin{aligned} \hat{\mathbf{b}} \cdot \nabla (J_{zi} + J_{zc}) &= \frac{m_i n_o}{B_0^2} \frac{d}{dt} \nabla_\perp^2 \phi \\ &+ \frac{p_{io}}{B_o^2 \omega_{ci}} (\nabla \nabla_\perp^2 \phi \times \hat{z}) \cdot \nabla \ln (p_{io} + p_i) \\ &+ \sum_{j=i,e} \Big[\nabla p_{\perp j} \cdot (\nabla B_o \times \hat{z})/B_o^2 \\ &+ \nabla p_{\| j} \cdot (\mathbf{B}_o \times \mathbf{R})/B_o^2 R^2 \Big] \\ &\equiv - \nabla_\perp \cdot \mathbf{J}_\perp , \end{aligned} \tag{4.1}$$

where $\mathbf{R}/R^2 = -(\hat{\mathbf{b}}_o \cdot \nabla)\hat{\mathbf{b}}_o$ is the curvature of the unperturbed magnetic field, p_j and p_{jo} are perturbed and unperturbed pressure of the jth species. The second term on the right hand side originates from the difference of $\mathbf{E} \times \mathbf{B}$ drift between electrons and ions, due to the fact that the ion sees the electrostatic field which is reduced by $\rho_i^2 \nabla^2$. $\hat{\mathbf{b}} \cdot \nabla$ and d/dt are given by,

$$\begin{aligned} \nabla_\| &= \hat{\mathbf{b}}(\hat{\mathbf{b}} \cdot \nabla) \\ &= \hat{z}\left(\frac{\partial}{\partial z} + \frac{B_{oy}}{B_o} \partial/\partial y + \frac{\mathbf{B}_\perp}{B_o} \cdot \nabla_\perp\right) , \end{aligned} \tag{4.2}$$

where $\hat{\mathbf{b}}$ is the unit vector in the direction of the total magnetic field, B_{oy} represents the shear field,

$$\mathbf{B}_\perp(\mathbf{x},t) = \nabla \perp A_z \times \hat{z} \tag{4.3}$$

is the perturbed magnetic field,

$$\frac{d}{dt} = \frac{\partial}{\partial t} + v_E \cdot \nabla$$

$$= \frac{\partial}{\partial t} - \frac{\nabla \phi \times \hat{z}}{B_o} \cdot \nabla \, , \tag{4.4}$$

and ϕ is the scalar potential for the electric field. Maxwell equations become

$$\nabla^2 A_Z = - \mu_o J_z \, ,$$

Parallel component of the generalized Ohm's law which may be obtained from the first moment of the electron drift kinetic equation is given by

$$\eta J_{ze} = - \frac{\partial A_z}{\partial t} - \hat{\mathbf{b}} \cdot \nabla \phi + \frac{1}{en} \hat{\mathbf{b}} \cdot \nabla (p_{\parallel_e} + p_{\perp e}) \tag{4.6}$$

Here η is the resistivity, and J_{ze} is the z component of the electron current. If $k_\parallel$ is small enough such that the parallel ion current J_{zi} is ignorable, appropriate equations of state which relate p_i and p_e to the other field variables complete the equations. One possible choice of the equations of state is incompressive ions,

$$\frac{dp_{\parallel i}}{dt} = \frac{dp_{\perp i}}{dt} \equiv \frac{dp_i}{dt} = 0$$

and isotropic, isothermal electrons, (4.7)

$$\frac{dp_{\parallel e}}{dt} = \frac{dp_{\perp e}}{dt} = \frac{T_e}{e} \hat{\mathbf{b}} \cdot \nabla J_{ze} \, .$$

The local dispersion relation for a case $J_o = \eta = 0$ may be obtained from Eqs. (4.1), (4.2), (4.3), (4.6), and (4.7). For a simple curved field line,

$$\nabla p \cdot (R \times B_o)/R^2 B_o^2 \simeq \nabla p \cdot z \times \nabla B_o / B_o^2 = - \frac{\partial p}{\partial y} \frac{1}{RB_o} \tag{4.8}$$

where x axis is taken in the direction of the radius of curvature and that of the pressure gradient. If we define the electron and the ion drift wave frequency,

$$\omega_e^* = \frac{k_y T_e}{eB_o} \frac{\partial}{\partial x} \ln p_{eo} \tag{4.9}$$

and

$$\omega_i^* = \frac{k_y T_i}{eB_o} \frac{\partial}{\partial x} \ln p_{io} \tag{4.10}$$

the local dispersion relation becomes,

$$(\omega + \omega_e^*)\,[\omega^2 - \omega_i^*\,\omega - k_z^2 v_A^2 - \frac{2k_y^2}{m_i R k_\perp^2}\,(T_i \frac{\partial}{\partial x}\,\ln p_{io} + T_e \frac{\partial}{\partial x}\,\ln p_{eo})] = \frac{k_z^2 v_A^2}{\omega}\,(\omega - \omega_i^*)\,[k_\perp^2 \rho_s^2 \omega + \omega_i^*(R \frac{\partial}{\partial x}\,\ln p_{io})^{-1}] \quad (4.11)$$

Equation (4.11) shows that the kinetic Alfvén wave dispersion relation is modified significantly when $k_y \rho_i \simeq 0(1)$ through the couplings to the drift waves. In addition the curvature of the field line can produce an instability of the field line to balloon in the region where $2p_o/\partial x < 0$ (ballooning instability).

V. CONCLUDING REMARKS

Magnetohydrodynamic perturbations can couple to kinetic properties of a plasma through the finite Larmor radius effects. This originates from the generation of electric fields in the direction of the ambient magnetic field and subsequent wave-particle interactions. In case of the Alfvén wave, this interaction heats the bulk region of the electron phase space while accelerates ions to the Alfvén speed if $m_e/m_i < \beta < 1$. Through the mode conversion in an inhomogeneous plasma and through the wave steeping (ultraviolet catastrophy) MHD perturbations with $k_\perp \rho_i \simeq 0(1)$ can easily be excited. Thus the process discussed here is quite universal and is expected to play an important role in the exchange of energy between MHD modes and particles.

REFERENCES

Appert, K., Balet, B., Gurber, R., Troyon, F., Tsunematsu, T. and Vaclavik, J., 1982, Plasma Phys. *24*, 1147.

Chandrasekhar, S., 1961, *Hydrodynamic and Hydromagnetic Stability*, Clarendon Press, Oxford, Chapt. 13.

Freeman, E. A. and Chen, L., 1982, Phys. Fluids *25*, 502.

Goldstein, M. L. and Goertz, C. K., 1983, *Physics of the Jovian Magnetosphere*, Ed. by A. J. Dessler, Cambridge Univ. Press. NY, 317.

Goertz, C. K., and Boswell, R. W., 1979, J. Geophys. Res. *84*, 7239.

Goertz, C. K., 1981 AGU Monography *25*, *Amoral Arc Formation*, 451.

Grossmann, W. and Tataronis, J., 1973, Z. Phys. *261*, 217.

Hasegawa, A. and Chen, L., 1974, Phys. Rev. Lett., *32*, 454.

Hasegawa, A. and Chen, L., 1976, Phys. Fluids *19*, 1924.

Hasegawa, A., 1976, J. Geophys. Res. *81*, 5083.

Hasegawa, A. and Uberoi, G., 1982. *The Alfvén Wave*, Tech. Inf. Center, US Dept. of Energy, Oak Ridge, Tenn.

Hasegawa, A. and Mima, K., 1978, J. Geophys. Res. *83*, 1117.

Hasegawa, A., 1975, *Plasma Instabilities and Nonlinear Effects*, Springer Verlag, NY, 128.

Hasegawa, A., 1979, Phys. Fluids *22*, 1988.

Hasegawa, A., and Wakatani, M., 1983, Phys. Fluids to be published.

Ionson, J. A., 1978, Ap. J., *226*, 650.

Ionson, J. A., 1982, Ap. J., *254*, 318.

Kadomtsev, B. B., 1966, *Review of Plasma Physics*, Ed. by M. A. Leontovich, Consultant Bureau, NY, 174.

Osawa, Y., Nozaki, K. and Hasegawa, A., 1976, Phys. Fluids *19*, 1139.

DISCUSSION

Davila: You indicate that the normal and inverse cascade processes result in a concentration of turbulent power at scales on the order of the ion gyroradius. This, however, is also where wave damping by the thermal protons is maximized. It would seem that one would expect turbulent wave power to be a minimum at scales on the order of the ion gyroradius.

Hasegawa: The spectral cascade is purely a consequence of a "fluid" picture of the magnetohydrodynamic turbulence. When the kinetic picture is taken into account one should consider the detailed balance. In an inhomogeneous plasma many different types of micro-instability emit waves in this regime leading to a quasi-equilibrium state of a high level of turbulence at $k_\perp \rho_i \gtrsim O(1)$, a situation somewhat similar to the consequence of the fluctuation dissipation theory. In fact recent measurements of density and magnetic field fluctuations in Tokamak type plasmas have revealed that super-thermal fluctuations are concenterated at $k_\perp \sim \rho_i^{-1}$.

Lotko: Does the inverse cascade depend on the dimensionality of the turbulence and is there coupling to the compressional Alfvén wave?

Migliuolo: Is the inverse cascade a correct picture for high-β plasmas, where the shear Alfvén wave couples to the compressional Alfvén wave?

Hasegawa: "Two dimensionality" is essential in the inverse cascade. This requires a low β situation in which the magnetic field controls the plasma dynamics.

Vasyliunas: Your kinetic MHD theory holds for $(k_\parallel/k_\perp)^2 \leq \rho_i^2/R^2$, but in astrophysical and space applications one expects a minimum value $(k_\parallel/k_\perp)^2 \gtrsim (v/v_A)^2$ where v is, e.g., the speed of magnetospheric disturbance sources relative to the earth's field, and this ratio can be much larger than $(\rho_i/R)^2$. What happens to the theory in that case?

Hasegawa: Generation of perturbation with a very large value of $k_\perp(\rho_i^{-1})$ originates through 1. singularity of MHD perturbations in an inhomogeneous plasma, 2. through the so called ultraviolet catastrophy; the continuous generation of shorter wavelengths due to nonlinear mode couplings, and 3. micro-instabilities such as drift wave modes.

Norman: It would be very nice to apply the physics to jets, but here β may be at least of order unity. Do you have any ideas of how this may be done?

Hasegawa: In a high β case ($\beta \simeq 1$), the present expansion scheme breaks down. This could mean that the microturbulence may not be important in a high β case except for specific cases (such as the drift mirror instability).

Vlahos: In your talk you mention only the Landau resonance for obliquely propagating Alfvén waves. Are the higher order cyclotron interactions $\omega - k_{\parallel}v_{\parallel} - n\Omega_e$ important?

Hasegawa: The Landau resonance is important for "heating" because it affects the bulk of the distribution of plasma particles. The cyclotron resonance is relevant to the high energy tail, such as for cosmic rays.

Sturrock: What role do these processes play in heating the solar corona?

Hasegawa: Microscopic perturbations produce direct wave-particle interactions through the parallel electric field that they accompany. This indicates that there is a coupling between MHD perturbations and kinetic effects in plasmas. A number of papers seem to have been published on coronal heating through a process described in the talk.

Ionson: In solar loops the dissipational scale length is $\sim 10^5$ cm compared to an ion gyroradius of 10^2 cm. Therefore, since the resonance layer thickness is 10^3 times larger than ρ_{ci} (i.e. $k_\perp \rho_{ci} \sim 10^{-3}$) it appears that finite Larmor radius corrections are not necessary.

Hasegawa: You are right. Finite Larmor radius corrections and related kinetic effects are important for relatively collision free plasmas such that the ion Larmor radius is smaller than the dissipation scale length.