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Implications of a Weaker Form of Complementarity

Jon R. Neill

Abstract

When a non-market good has existence value, the assumption of weak complementarity cannot be used to determine willingness to pay for that good. However, when this assumption is weakened, it is possible to place an upper bound on marginal willingness to pay even when the non-market good has existence value, and thereby, an upper bound on willingness to pay for changes in consumption of non-market goods can be established. Moreover, this upper bound may be relatively easy to compute.

KEYWORDS: marginal willingness to pay, weak complementarity

It is well known that an agent's valuation of a non-market good can be inferred from the agent's observed (market) behavior when the non-market good only has use value. By contrast, if the non-market good has existence value, then an agent's willingness to pay for increases in consumption of the good cannot necessarily be gleaned from the agent's market behavior. This conclusion suggests that when a non-market good has both use and existence value, at best part of its monetary value – its use value – can be extracted from the agent's ordinary demand functions. That is, when a non-market good has existence value as well as use value, it may only be possible to place a bound on an agent's willingness to pay for the non-market good.

Of course, the ability to determine use value from market demand functions depends on how the concept of use value is formalized. Undoubtedly the most widely used definition comes from Maler (1971). Maler argued that changes in an agent's consumption of a quantity of a non-market good may have no effect on his welfare unless he consumes a positive quantity of a particular market good. Implicitly, the non-market good cannot be consumed unless the agent is consuming a positive quantity of the market good. For example, a public television station may have no value to a person unless he owns a television set (an example from Bradford-Hildebrandt). Similarly, water quality in Lake Michigan may have no value to a person unless he swims, fishes, or boats in the lake.

This sort of hedonic relationship has come to be known as weak complementarity. Thus, when a non-market good and a market good are weak complements, if the price of the market good is so high that an agent chooses not to consume it, his marginal willingness to pay for the non-market good is zero. And as Maler demonstrated, this "complementarity" makes it possible to measure an agent's willingness to pay for changes in his consumption of the non-market good using his market demand function for the market good.

In what follows, we consider the implications of a weaker form of complementarity. We find that this notion – a notion which could be no less intuitive than weak complementarity – makes it possible to learn something about willingness to pay for a non-market good from market demand schedules when the non-market good has both use and existence value. More specifically, by weakening Maler's notion of complementarity, it is possible to place an "upper" bound on marginal willingness to pay for a non-market good even though the agent's marginal willingness to pay is always positive. Consequently, since Neill (1999) has shown that a lower bound on marginal willingness to pay can be established when the (ordinary) demand for a market good is affected by consumption of a non-market good, the imposition of this weaker form of complementarity makes it possible to place both upper and lower bounds on marginal willingness to pay for a non-market good.

The problem of determining willingness to pay for non-market goods continues to vex benefit-cost analysts. Although contingent valuation methodology has opened a new, exciting front from which to attack this important problem, some economists remain deeply skeptical of its validity (e.g., Diamond and Hausman, 1994). Thus, it is imperative that the development of revealed preference methodology be continued, despite its limitations. It may very well be impossible to develop a methodology that satisfies everyone, because it appears that unverifiable assumptions are unavoidable in making precise estimates of willingness to pay for non-market goods. Consequently, these two methodologies should be viewed and developed as complements to each other, one a check and balance for the other (Brookshire, Thayer, Schulze, and d'Arge, 1982; Adamowicz and Williams, 1994; Huang, Haab, and Whitehead, 1997; Ebert, 1998; Azevedo, Herriges, and Kling, 2003).

I. Weakening Weak Complementarity

Suppose that z and x denote quantities of some non-market good and a specific market good, respectively. Further suppose that these goods are weak complements. Let x^c and p denote the compensated demand for the market good and its price. As is customary, p^c such that

$$x^c(p^c,z) = 0, x^c(p,z) > 0 \ \forall \ p < p^c$$

will be referred to as the choke price of the market good. Under the assumption of weak complementarity, the agent's marginal willingness to pay for the non-market good is positive when his consumption of the market good is positive, and zero when his consumption is zero. Let $m(p,z)$ denote his marginal willingness to pay for the non-market good, a non-negative, continuously differentiable function for all $p < p^c$, and with a left-side derivative at p^c. Thus, as

$$m(p,z) > 0 \ \forall \ p < p^c$$

and $m(p^c,z) = 0$, it follows that, at least in a neighborhood of p^c, m must be decreasing in p and so $\partial m(p^c,z)/\partial p \leq 0$.

As will be seen below, this assumption is equivalent to the following assumption: as z is decreased and compensation takes place, x decreases. In some cases, this would seem to be a very intuitive assumption. For example, it would not surprise anyone to find that as water quality in a lake deteriorates, the representative household makes fewer trips to its beaches, despite receiving compensation for the decrease in water quality.

In any case, a necessary condition for weak complementarity is that $\partial m(p^c,z)/\partial p \leq 0$. That is, the assumption that this partial is negative or zero at p^c is implied by weak complementarity and therefore is a weaker assumption. Moreover,

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it allows the non-market good to have non-use value. In other words, under this assumption an agent's marginal willingness to pay for the non-market good may be positive even though he is not consuming the complementary market good. The primary purpose of this paper is to demonstrate that under this weaker form of complementarity, an upper bound on marginal willingness to pay for a non-market good can be established. The proof of this proposition reveals that by adding a regularity condition, the upper bound is relatively easy to calculate.

II. Bounding Marginal Valuation from Above

We will prove the following claim:

Suppose that $\partial x/\partial z$, $\partial x/\partial y > 0$ and that $\partial m(p^c,z)/\partial p \leq 0$, where y denotes income. Then the agent's marginal willingness to pay for the non-market good can be bounded from above.

Proof: To prove this we use a lemma:

Let x^c denote an agent's compensated demand for a market good. Then

$$-\partial m/\partial p = \partial x^c/\partial z \qquad (1)$$

To prove (1), we recall that for an arbitrary market good i,

$$\partial e/\partial p_i = x_i^c$$

where $e(p,z)$ denotes the agent's twice differentiable expenditure function, the inner product of the n prices of the market goods and his n compensated demand functions. Therefore,

$$\partial^2 e/\partial z\partial p = \partial^2 e/\partial p\partial z = \partial x^c/\partial z$$

Now, as

$$\partial e/\partial z = -m(p,z)$$

it follows that

$$\partial^2 e/\partial z\partial p = -\partial m/\partial p = \partial x^c/\partial z$$

proving the lemma.[1]

The following equation comes from Neill (1988):

$$\partial x^c/\partial z = \partial x/\partial z - m\, \partial x/\partial y \qquad (2)$$

Note that the derivatives on the right-hand side of this equality are the derivatives of the agent's ordinary demand function evaluated with income set equal to $e(p,z)$.

Now, by substituting (1) into (2) and reordering terms we obtain the first order differential equation:

$$\partial m/\partial p = m\, \partial x/\partial y - \partial x/\partial z.$$

[1] This equality also plays an important role in the analysis by Bullock and Minot (2006).

This differential equation has a well-known solution which can be written:

$$m(p) = m(p^c)\exp[\int_{p^c}^{p} a(s)ds] + \int_{p^c}^{p} \exp[\int_{u}^{p} a(s)ds]\, b(u)du \quad (3)$$

where $p < p^c$ and $a(s) = \partial x(s)/\partial y$, $b(u) = -\partial x(s)/\partial z$.[2]

Now, because $\partial m(p^c,z)/\partial p \leq 0$, (1) and (2) imply that:

$$m(p^c) \leq (\partial x(p^c)/\partial z)/(\partial x(p^c)/\partial y).$$

Thus, because the term by which $m(p^c)$ is multiplied in (3) is positive, we conclude that:

$$m(p) \leq [(\partial x(p^c)/\partial z)/(\partial x(p^c)/\partial y)]\exp[\int_{p^c}^{p} a(s)ds] + \int_{p^c}^{p} \exp[\int_{u}^{p} a(s)ds]\, b(u)du \quad (4)$$

proving the claim.

Note that all the terms on the right-hand side of (4) can be calculated because the quantity of the non-market good is fixed at z, and therefore these terms are determined by $e(p,z)$, which can be calculated from the ordinary demand functions. Of course, this is not a simple calculation.[3] However, this claim leads to something of a corollary, that simplifies the problem of calculating an upper bound on marginal willingness to pay somewhat.

Suppose that $\partial x/\partial z$, $\partial x/\partial y > 0$ and that $\partial m(p,z)/\partial p \leq 0$ for all p. Then

$m(p) \leq (\partial x(p)/\partial z)/(\partial x(p)/\partial y)$

Proof: As

$$\partial m(p,z)/\partial p = -\partial x^c/\partial z = -\partial x/\partial z + m\, \partial x/\partial y,$$

it follows that

$$-\partial x/\partial z + m\, \partial x/\partial y \leq 0,$$

and therefore, because $\partial x/\partial z$ and $\partial x/\partial y$ are positive, that

$$m(p) \leq (\partial x(p)/\partial z)/(\partial x(p)/\partial y).$$

III. Comparing Upper and Lower Bounds

Note that the second term on the right-hand side of (4) is the lower bound that Neill (1999) identified. Thus, by imposing the assumption of weaker complementarity on

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[2] See any advanced undergraduate or graduate text on differential equations for the derivation of this solution. For example, Corduneanu (1977).

[3] See Vartia (1983) and the most recent contribution to this literature, Bullock and Minot (2006).

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a good whose demand is positively affected by income and consumption of the non-market good, both an upper and lower bound on the agent's marginal willingness to pay for that good can be established, and the difference between the bounds is:

$$[(\partial x(p^c)/\partial z)/(\partial x(p^c)/\partial y)]\exp[\int_{p^c}^{p} a(s)\mathrm{d}s]$$

Consider the conclusion reached by Neill (1999):
if $\partial x/\partial z$ is positive over the interval $[p_0,p_1]$, then $\{\int_{[p0,p1]} \lambda^c\ \partial x/\partial z\ \mathrm{d}p\}/\lambda(p_0,z,y)$ is a lower bound on $m(p_0,z)$ where λ^c is the Lagrangian multiplier from the agent's utility maximization problem when there is compensation for price changes; $\lambda(p_0,z,y)$ is the value of the Lagrangian initially, when price is p_0, the quantity of the non-market good consumed is z, and income is y; and $\partial x/\partial z$ is evaluated with income set at $e(p,z \mid p_0,z,y)$.

What we want to do is make first approximations of this lower bound and the upper bound identified above, in order to get some idea of the range in which marginal willingness to pay could be placed.

From the Slutsky equations:[4]

$$\partial\lambda/\partial p = -\lambda\ \partial x/\partial y$$

and thus

$$\lambda = \lambda_0$$

$$\exp\{-\int_{[p0,p1]} \partial x/\partial y\ \mathrm{d}p\}.$$

Initially let $\gamma = \partial x/\partial y$, $\delta = \partial x/\partial z$, and suppose that over the relevant range, the effects of price and income on these derivatives are negligible. If so, the upper bound identified above is approximately $(y/z)(\varepsilon_z/\varepsilon_y)$.[5] In contrast, the approximation of Neill's lower bound is:

$$(y/z)(\varepsilon_z/\varepsilon_y)(1 - \exp\{-\gamma(p^c - p_0)\}.$$

However, the difference between price initially and the choke price can also be approximated, by solving a first order Taylor's expansion of the compensated demand function. Let p^* be the solution to:

$$0 = x(p_0,z,y) + (\partial x^c/\partial p)(p^* - p_0).$$

Thus, p^* is an estimate of the choke price, p^c. Let $\eta = \partial x^c/\partial p$. Then

$$\delta/\gamma(1 - \exp\{-\gamma(p^c - p_0)\} = \delta/\gamma(1 - \exp\{\gamma x/\eta\})$$

$$= (y/z)(\varepsilon_z/\varepsilon_y)(1 - \exp\{s\ \varepsilon_y/\varepsilon_p{}^c\}),$$

4 For the derivation of this expression, see Samuelson (1983), pp. 101–103.
5 These elasticities would be evaluated with the quantity demanded set at the initial level.

where s is the budget share of x, and ε_p^c is the elasticity of compensated demand ($\varepsilon_p^c = \varepsilon_p + s\ \varepsilon_y$). Thus, under the assumption that marginal willingness to pay falls as the price of this good rises, the percentage difference between the upper bound and this approximation of the lower bound is determined by $s\ \varepsilon_y/\varepsilon_p^c$. Specifically, as this value increases in absolute value, the upper and lower bounds converge.

Obviously the percentage difference in these approximations of the bounds on marginal willingness to pay may be very large. However, when we look at the derivation of the lower bound, it becomes apparent that we may be able to increase it, possibly substantially. Suppose there are n market goods, and that the demands for goods 1 through $k < n$ are such that:

$$p_1\ \partial x_1/\partial z + p_2\ \partial x_2/\partial z + \ldots + p_k\ \partial x_k/\partial z > 0.$$

Now suppose the prices of these k goods are increased by the same percentage. That is, the price of good i is changed from p_i to $t_0\ pi$, $t_0 > 1$, $i = 1, 2,\ldots, k$. Then it is a straightforward exercise to show that:

$$m(p_1,p_2,\ldots,p_k) \geq$$
$$\{\textstyle\int_{[1,t01]} \lambda^c\ (p_1\ \partial x_1/\partial z + p_2\ \partial x_2/\partial z + \ldots + p_k\ \partial x_k/\partial z)\mathrm{d}t\}/\ \lambda(p_1,p_2,\ldots,p_k, z,y).$$

Moreover,

$$\lambda^c\ (t) = \lambda(p_1,p_2,\ldots,p_k, z,y)\ \exp\{\ -(p_1\ \partial x_1/\partial y + p_2\ \partial x_2/\partial y + \ldots + p_k\ \partial x_k/\partial y)\}.$$

Once again, by ignoring the effect of the change in prices on the partials, $\partial x_i/\partial z$, $\partial x_i/\partial y$, we can approximate this bound. What we find is that, to a first approximation, marginal willingness to pay must be greater than

$$(y/z)(\sum_1^k s_i\ \varepsilon_z^{\ i}\ /\sum_1^k s_i\ \varepsilon_y^{\ i})(\ 1 - \exp\{-\textstyle\int_{[1,t0]} \sum_1^k s_i\ \varepsilon_y^{\ i}\})$$

In this case, t_0 would be that number greater than one such that $x_i \geq 0$, $i = 1, 2,\ldots, k$. Although there is certainly no guarantee that the lower bound on marginal willingness to pay can be substantially raised by using the effect of the non-market good on spending on a number of goods, it is a possibility.

IV. Conclusion

One of the contributions of this analysis should be obvious. Namely, it provides an assumption making it possible to place an upper bound on willingness to pay for non-market goods, an assumption which relates to, but is in fact weaker than, an assumption that has interested economists for over 30 years. Yet there is a second contribution here. When the assumption of weak complementarity is applied to Eq. (3), we can obtain a measure of willingness to pay for an increase in the consumption of a non-market good through a procedure that is different from the procedure implied by Maler's analysis. Therefore, if weak complementarity is in fact present, the procedure based on Eq. (3) must produce the same value that precipitates from Maler's procedure. Consequently, this equality is a necessary condition for the presence of weak complementarity.[6]

The discovery that under weak complementarity it is possible to determine willingness to pay for non-market goods using market demand schedules was an epochal breakthrough in public economics. However, the impossibility of verifying if such complementarity exists has been a glaring problem. However, the necessary conditions for weak complementarity that emerge from Bullock and Minot (2006) and our analysis may mitigate that problem considerably and, thereby, give Maler's approach greater credibility.

By contrast, it certainly may be that agents have a positive marginal willingness to pay for wilderness preservation, to improve water quality in the Great Lakes, etc., even though they may never visit those places. In other words, non-market goods may have existence value, and thus if we were forced to reject the hypothesis that weak complementarity is present, where would that leave us? We think that the claim proved here provides the cost-benefit analyst with a viable alternative. But if nothing else, these results are at least a modest extension of indirect methods for determining willingness to pay for non-market goods, results which hold out the hope of further advances in this important methodology.

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[6] Note that the procedure developed by Bullock and Minot is also different from that developed by Maler. If there is weak complementarity, then the Bullock-Minot procedure would likewise have to produce the same value as Maler's. Thus, their procedure provided the first empirical test of the assumption of weak complementarity.

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