Radial and Nonradial Pulsations as Probes of Stellar Physics
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Seismic Inversions for White Dwarf Stars

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Abstract. Inversion methods have been used successfully for the Sun. The stars with the next richest set of observed frequencies are the white dwarfs. We consider here the viability of numerical inversions for these stars. We find that, while the number of presently observed modes in the white dwarf GD 358 is too small for structural inversions, such inversions *would* be possible if the frequencies of all modes with $1 \leq \ell \leq 3$ were observed. This is possible for space observations by e.g. Eddington.

1. Introduction

By means of a genetic algorithm, Metcalfe et al. (these proceedings) have placed constraints on the interior Carbon/Oxygen profile of the pulsating white dwarf GD 358. This in turn places constraints on the $^{12}C(\alpha,\gamma)^{16}O$ nuclear reaction rate. Given the importance of this result, we wish to see what can be learned by the complementary approach of structural inversions. Previous treatments of inversions for white dwarf stars include asymptotic inversions for N^2 (Shibahashi et al., 1988) and rotational inversions (Kawaler et al., 1999).

2. Numerical models and inversions

Our equilibrium model was computed with the evolutionary code described in Montgomery & Winget (1999), and has the following parameters: $T_{\rm eff} = 22,600$ K, $M_\star = 0.65\ M_\odot$, $M_{\rm He} = 3.0 \times 10^{-6}\ M_\star$. We adopt the "optimally localized averaging method", in which we determine the differences in the inversion variables (e.g., c^2 and ρ) between the model and the actual star. Following Gough (1996), we numerically obtained kernels for $v = (r/g)N^2$ and Γ_1 from the standard ones for sound speed and density. These kernels describe how the frequencies of individual modes change in response to small changes in the Brunt-Väisälä frequency and the adiabatic index:

$$\frac{\delta\sigma_i}{\sigma_i} = \int K^{(i)}_{v,\Gamma_1} \frac{\delta v}{v}\,{\rm d}x + \int K^{(i)}_{\Gamma_1,v} \frac{\delta\Gamma_1}{\Gamma_1}\,{\rm d}x, \tag{1}$$

where $\delta\sigma_i$ is the difference between the observed and model frequencies, the Ks are the kernels for v and Γ_1, and δv and $\delta\Gamma_1$ are the differences between those quantities in the model and the actual star—the quantities we wish to measure. The subscript i stands for the quantum numbers (k,ℓ) and indicates that the frequencies and kernels are for a particular mode, and $x = \ln(r/p)$

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is our chosen radial variable. Using these kernels, we performed an inversion involving a known perturbation to v using a synthetic set of periods, consisting of all periods between 100 and 1000 s with ℓ less than some value, as a test of the accuracy of the inversion method. The errors of these periods were taken to correspond to frequency errors of 0.1 μHz.

3. Results and discussion

In Fig. 1 we show the results involving the large set of modes (116) with $1 \leq \ell \leq 3$. We find that including modes up to $\ell = 3$ (116 modes) gives well-localized kernels (Fig. 1a) which can be used to perform inversions (Fig. 1b). In addition we found that using modes up to only $\ell = 2$ (61 modes) gives averaging kernels which are localizable only in the envelope, and that using only the observed modes in GD 358 (11 modes, all $\ell = 1$) is insufficient.

The reserve ESA mission Eddington will be capable of observing pulsating white dwarf stars. Given the resulting increase in sensitivity which this will provide, the number of oscillation modes detected in white dwarfs may increase dramatically. This would allow inversion techniques to be applied to these stars.

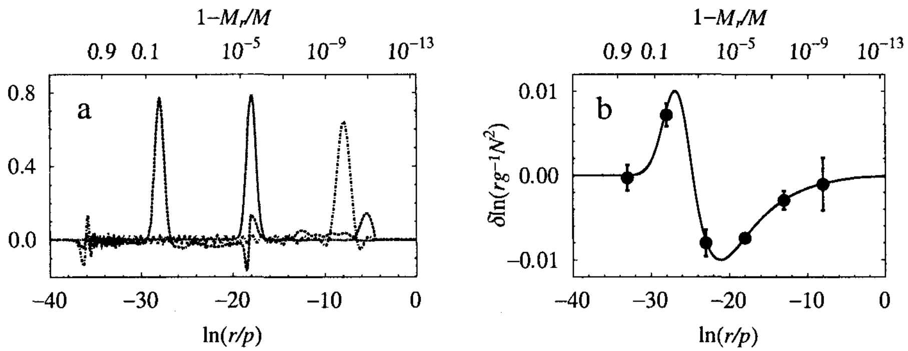

Figure 1. a) Localized averaging kernels for all modes with $1 \leq \ell \leq 3$ and periods between 100 and 1000 seconds. b) A test inversion for the assumed perturbation to $v = (r/g)N^2$: the points show the inversion results and the solid curve is the actual value of $\delta v/v$.

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