The second chapter introduces the tensor concept beginning with Cartesian tensors and later general tensors. The invariance of tensor equations and its relevance to the formulation of physical laws are noted. There is also a section on the use of complex variable for transformations which are rotations about an axis, which to me seemed a little out of place.

Chapter Three examines the algebra of tensors, the reduction of Cartesian tensors to principal axes and introduces "pseudo tensors".

Chapter Four is on vector and tensor analysis and includes the integral theorems of Green, Gauss, and Stokes.

The final chapter considers the Christoffel symbols and the covariant derivative with applications to Fluid mechanics and Electromagnetic theory.

Each chapter contains an ample supply of problems both solved and unsolved.

My main criticism of the book is the way in which the authors change from Cartesian tensors to general tensors and vice versa, often without warning. There also seem to be a few errors, possibly misprints, involving the Jacobian of general transformations.

D.R. Westbrook, University of Calgary

Integration, by A.C. Zaanen. John Wiley and Sons, Inc., New York, 1967. xiii + 604 pages. U.S. $16.75.

The chapter titles for this book are: 1) Point Sets, Zorn's Lemma, and Metric Spaces; 2) Measure; 3) Daniell Integral; 4) Stieltjes-Lebesgue Integral; 5) Fubini's Theorem; 6) Banach Space and Hilbert Space; L_p Spaces; 7) The Radon-Nikodym Theorem; 8) Differentiation; 9) New Variables in a Lebesgue Integral; 10) Measures and Functions of the Real Line; 11) Signed Measures and Complex Measures; 12) Conjugate Spaces and Weak Sequential Convergence; 13) Fourier Transformation; 14) Ergodic Theory; 15) Normed Kothe Spaces.

This book is a substantially enlarged version of the author's previous An Introduction to the Theory of Integration (1958). Those familiar with the earlier work will recall that it presented both the measure theoretic and linear functional point of view and showed the natural intimate connections between them. The development in the early chapters of Measure and the Integral is largely unchanged and one may consult Mathematical Reviews 20 (1959) pages 657-8, for comment on it. (The introduction of measurable sets via Carathéodory's condition has been improved.) Suffice it to say that the author succeeds in his "attempt to produce an advanced textbook on integration theory, which makes the student familiar not only with the measure theoretic approach and the linear functional approach to the theory, but also with the fact that the integral of a non-negative function has something to do with the measure... of the ordinate set of the function" and "to make clear... that in the linear functional approach ... measure theory in disguise is used from the very beginning." (Preface.) Throughout the new edition, examples, discussions, and exercises have been liberally added. There are now 448 problems, with hints and solutions to many of them occupying over 100 pages at the end of the book. It is pleasing that although this edition is more than twice the length of the first, one can still cover the core of integration theory (through the σ-finite Radon-Nikodym theorem) in under 250 pages.

https://doi.org/10.1017/S0008439500029921 Published online by Cambridge University Press

The book omits several topics found in some competing works, such as integration on groups, probability theory, martingale theory, measures on Boolean algebras, and directed limits. A particularly attractive feature of the book, however, is the wide variety of topics which ARE presented, and which have special pedagogical value and/or mathematical interest. The following incomplete list will give some idea of the scope of the book: integration by parts; the Gamma function and fractional integrals; the Hahn-Banach theorem; the problem of a measure defined for all bounded subsets of the real line; the Bochner integral; Segal's theorems that localizability of a measure is equivalent to the validity of the (not - necessarily - σ-finite) Radon-Nikodym theorem and also equivalent to the case that L_∞ be the conjugate space of L_1; the Banach-Steinhaus theorem; reflexive spaces; the Poincaré recurrence theorem; and the mean and pointwise ergodic theorems, with Garsia's proof of the maximal ergodic theorem. The final chapter contains basic results in the theory of Köthe and Banach function spaces. This is a fitting conclusion; it draws on a variety of previous material and provides interesting generalizations of earlier results. Results of and references to the work of I. Amemiya; G.G. Gould; I. Halperin; G.G. Lorentz; W.A.J. Luxemburg; T. Ogasawara; D.G. Wertheim; and the author are included. (Consider an extended real valued function ρ defined on the space M^+ of all extended real valued non-negative measurable functions f on a σ-finite measure space X such that

$$0 \le \rho(f) \le \infty,$$

$$\rho(f) = 0 \text{ if and only if } f = 0 \text{ a.e.},$$

$$\rho(af) = a\rho(f) \text{ for all finite constants } a \ge 0,$$

$$\rho(f+g) \le \rho(f) + \rho(g), \text{ and}$$

$$f \le g \text{ a.e. implies } \rho(f) \le \rho(g) \text{ for all } f, g \in M^+.$$

Denoting the extended real valued functions f on X by M, extend ρ to M by $\rho(f) = \rho(|f|)$, and denote by L_ρ those $f \in M$ such that $\rho(f) < \infty$. (Identify any functions in M which differ only on a null set.) It is shown that the functions in L_ρ are finite valued a.e., form a linear space, and ρ is a norm on L_ρ. Any such normed linear space is a normed Köthe space. If, in addition, $0 \le f_n \uparrow f$ a.e. implies $\rho(f_n) \uparrow \rho(f)$ (for all f_n, $f \in M^+$), then L_ρ is complete and is called a Banach function space. The obvious examples are the L_p spaces.) Among the questions considered are the Riesz-Fischer and Fatou properties of the norm, conjugate spaces, and reflexivity.

Many interesting topics are developed in the exercises, which are grouped and titled according to content. Included are, for instance, finitely additive measures (following the work of B.C. Strydom); infinite product integrals; a change of variable formula in integration on R^k for non-invertible mappings; and the determination of a function by the values of its integral along lines in R^k. Moreover, there are several series of exercises where one has to evaluate classical integrals over subsets of R^1. Thus the student should actually be able to integrate if he uses this text on integration.

The book is well written and should be suitable for mature undergraduates and beginning graduate students. The 15 chapters consist of 73 shorter sections in all, and at the beginning of each chapter a fine print paragraph sketches its contents. The author avoids referring in the text to results proved in previous exercises until the final chapter. The references at the end of the book, list numerous journal articles as well as books. The only difficulties the reviewer experienced were the author's occasional assumptions made at the beginning of a section, valid for the section, but which might be overlooked by one who simply looks up a theorem in the section. Very few typographical errors were found, of which the following may be worth mention:

p. 293, last line: the second of the three equality signs should read "$\leq$" .

p. 429, line 6 down: "2ε" may be replaced by "ε" .

p. 429, line 14 down: "ε" should be replaced by "2ε" .

The reviewer recommends the book both as a lucid text and reference.

Norton Starr, Amherst College

Eléments de mathématique, by N. Bourbaki. Fasc. XXXII. Théories spectrales. Chapitre 1 : Algèbras normées. Chapitre 2 : Groupes localement compact commutatifs. Actualités Scientifiques et industrielles 1332, Hermann, Paris, 1967. 45 F.

This is a very concise and clear treatment of Banach algebras and Fourier Analysis on locally compact commutative groups. The Bourbaki exposition is clean and neat; only topics of major importance appear in the text, and these are given a detailed and elegant treatment. Many topics of interest are relegated to the exercises.

The first two sections of Chapter 1 are of a preliminary nature. Then the Gelfand Theory of commutative Banach algebras is presented. This is followed by an excellent section entitled "Calcul fonctionnel holomorphe" which is the study of morphisms of algebras of germs of analytic functions into a Banach algebra. The problem of harmonic synthesis in regular commutative Banach algebras is then discussed. A section on normed $*$-algebras including C^*-algebras and the C^*-algebra of a locally compact group follows. The chapter ends with a section on algebras of continuous functions on a compact space. There are close to 100 exercises at the end of the chapter.

Chapter 2 begins by developing the standard results of Fourier Analysis : Plancherel's theorem, the inversion formula, the Pontriagin duality theorem, etc. . The next section presents the structure theory of locally compact commutative groups. The final section is a discussion of harmonic synthesis in L^1, L^∞ and L^2 . Some more recent results concerning harmonic synthesis appear in the exercises.

There are some misprints in the text, some of which are in references to previous results. The most surprising "misprint" however, is the following ".... l'operateur qui transforme la fonction f(t) en la fonction tf(t) " The unbeliever may find this on lines 6 and 7 on the page immediately following page 86 and preceeding page 88. The page (appropriately) is not numbered.

Roger Rigelhof, McGill University