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Prognostic opportunity of the shifted correlation between Wolf numbers and their time derivatives

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Abstract. We correlate the annual Wolf numbers W and their time derivatives W' by shifting time fragments of W and W' relative to each other. The most significant (up to 0.874) correlation is with 3 years shifts for fragments covering 14 years. For longer and shorter periods, the correlation coefficients 0.771–0.855 with 2–3 years shift. The most significant 9 years shift corresponds to -0.852/-0.824 anti-correlation coefficient for 14/11 years period. The other periods are less significant. To evaluate predictive estimates, we use the times series fragments of W shifted back into the past. A forecast can be made using the leading graphs based upon the derived calibration factor. Test calculations show that the most effective is the calibration factor calculated for changing the phase of the cycle. The best linear pairwise correlation coefficient of the approximation is 0.94.

Keywords. Solar activity cycles, sunspots, Wolf numbers, physical foundations of the solar dynamo, forecast

1. Introduction

Despite numerous attempts, forecasting solar-stellar magnetic activity has remained an outstanding challenge (Pesnell (2012), Obridko & Nagovitsyn (2017), Nandy (2021)). In a detailed review, Nandy (2021) shows that forecasts for Solar Cycle XXV based on different techniques still diverge widely, with a majority of the forecasts indicating that Solar Cycle XXV would be stronger than Solar Cycle XXIV. The mean of the various forecasts of the maximum amplitude in solar cycle XXV is 136.2 ± 41.6. Some of the most frequently used forecasting methods are statistical methods, including the study of the correlation properties of Wolf numbers W (Vitinsky (1973), Bondar *et al.* (1995), Ishkov & Shibaev (2006), Pishkalo (2008), Abdel–Rahman & Marzouk (2018), Petrovay (2020), McIntosh *et al.* (2020)). Therefore, correlations, and in fact autocorrelations, are primarily sought between original or transformed fragments of W series, separated by time intervals. In (Starchenko & Yakovleva (2022)), the basic correlation dependencies between the series of Wolf numbers shifted in time relative to each other and their derivatives (W and W') were determined. Annual means of Wolf numbers were used from 1700 to 2022 in version v2 http://sidc.oma.be/silso/datafiles. Time derivatives $dW/dt = W'$ were obtained at the same middle of the year as W, by taking the average of the derivative on the left and right. This technique allows us to smooth some errors originated from the observational and contractual definition of W. Both the Wolf numbers and their derivatives in the considered interval was modified by extracting the corresponding average values. We use such deviations for more clearly demonstration of variability,

Table 1. Correlation coefficients between temporal rows of W and W' of different lengths. K is the resulting correlation coefficient, I is the length of the time series fragments in years, Z is the step (in years) by which the time fragments are shifted (the sign "−" means an anti-correlation).

Z	K(I=11)	K(I=14)	K(I=22)	K(I=33)	K(I=44)	K(I=66)	K(I=88)	K(I=176)	K(I=322)
0	0.015	-0.076	0.023	0.012	0.011	0.004	0.002	0.006	0.000
1	0.584	0.521	0.531	0.551	0.553	0.554	0.548	0.542	0.531
2	0.771	0.783	0.793	**0.807**	**0.812**	**0.814**	**0.812**	**0.793**	**0.761**
3	**0.855**	**0.874**	**0.836**	**0.808**	0.805	0.790	0.792	0.769	0.714
4	0.672	0.737	0.639	0.545	0.533	0.518	0.528	0.515	0.463
5	0.314	0.401	0.281	0.123	0.116	0.117	0.131	0.143	0.115
6	0.120	0.077	-0.126	-0.300	-0.282	-0.284	-0.289	-0.244	-0.244
7	-0.229	-0.303	-0.522	-0.637	-0.588	-0.586	-0.611	-0.548	-0.518
8	-0.624	-0.681	**-0.725**	**-0.753**	**-0.719**	**-0.719**	**-0.746**	**-0.699**	**-0.623**
9	**-0.824**	**-0.852**	-0.697	-0.616	-0.611	-0.611	-0.621	-0.637	-0.509
10	-0.781	-0.786	-0.433	-0.217	-0.273	-0.241	-0.237	-0.330	-0.209

leading to more significant correlations, essentially corresponding with the classic Pearson correlation.

In the next section we show how the correlation between the shifted fragments of the W and W' series was determined, and we also try to substantiate the physical meaning and the corresponding predictive potential of these shifts.

2. Correlation of shifted series and the physical meaning of the maximal shifts

Unlike a previously published study by Starchenko & Yakovleva (2022), in the present work we determine a correlation between the complete series W and W', which seems more reasonable in terms of future forecasting. The correlation coefficient K is calculated using the Pearson equation where corresponding values W are used together with their mean values $\overline{W}$ and $\overline{W'}$:

$$\mathrm{K} = \sum_{i=1}^{\mathrm{I}} (W_i - \overline{W})(W_i' - \overline{W'}) \Big/ \sqrt{\sum_{i=1}^{\mathrm{I}} (W_i - \overline{W})^2 \sum_{i=1}^{\mathrm{I}} (W_i' - \overline{W'})^2} \tag{1}$$

Here i counted (from the past to the present) from the I-th value in the past for the complementary series and from the (I+$|Z|$)-th for the leading series.

All obtained correlation coefficients K are presented in Table 1 for fragments of series with lengths I = 11, 14, 22, 33, 44, 66, 88, 176 (years) and the entire available series with I = 322 (years). The most significant correlations/anti-correlations are highlighted in bold.

A comparison of correlation coefficients K for fragments of different lengths showed, that the most significant (0.85–0.87) correlation coefficients are associated with the shifts of 2 or 3 years for fragments covering the last 11–14 years. For longer fragments, the coefficients remain significant (0.76–0.84) at the same shifts. Therefore, the main phase shift between W and W' is approximately a quarter of a solar cycle, which physically corresponds to the preferential connection of spots with magnetic energy in accordance with the simplest dynamo model (see next paragraph). There is also a significant shift by 8-9 years, characterizes by anti-correlation coefficients from -0.70 to -0.85. Summarizing the resulting 2–3-year (one quarter) and 8–9-year (the remaining three quarters) shifts, we obtain the observed variety of dynamo magnetic energy periodicity from 10 to 12 years. Figure 1 shows the evolution of the Wolf numbers and their derivatives for the interval of 1976.5–2021.5 (magnetic solar cycles XXI–XXIV), supplemented with fragments of W shifted to 3 and 9 years ago. In particular, these plots allow us to estimate the predictive potential of the approach proposed here.

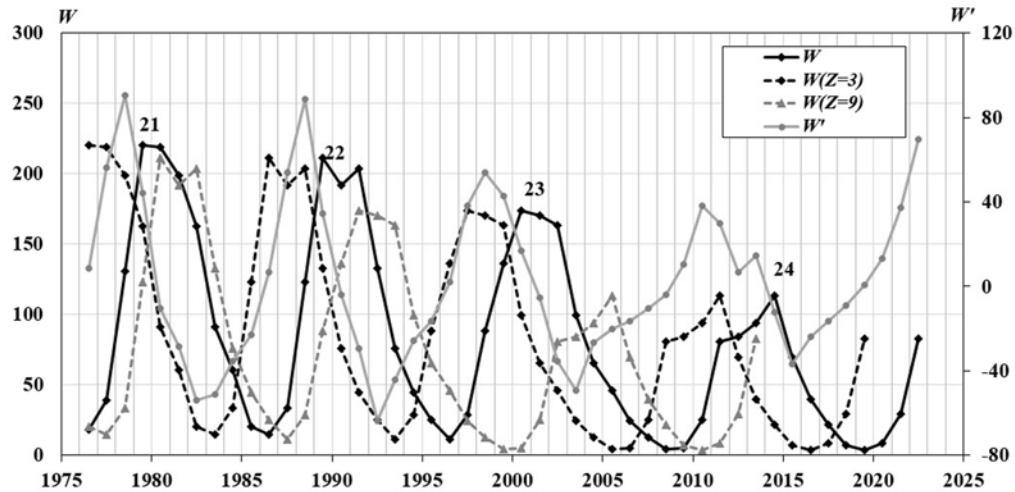

Figure 1. Displacement plots of W with respect to W' for Z = 3 and Z = 9 based on calculations for the magnetic solar cycles XXI-XXIV.

We use the idea of our previous study (Starchenko & Yakovleva (2022)) that the magnetic field variation is a sinusoidal: $\sin(\pi t/T_c)$, where T_c is the length of the solar activity cycle (about 11 years). Then, taking into account that the energy is quadratic to the field, the dimensionless normalized functions of the corresponding energy (essentially Wolf numbers) and derivative (essentially power) minus the average are $-\cos(2x)$ and $\sin(2x)$ respectively, where $x = \pi t/T_c$ is a phase variable. Obviously, the phase shifts (up to complete correlation or anti-correlation) correspond to $1/4$ and $3/4$ of the T_c. This conclusion is consistent with the maximum correlations/anti-correlations mentioned above. It should be noted that such a physical reason for anti-correlation was not emphasized in (Starchenko & Yakovleva (2022)) and we are filling this gap here.

3. Application of correlation of shifted series for Wolf numbers forecasting

For potential prognostic estimates based on the W series, we chose the correlation coefficient from Table 1 for K(I=11) and K(I=14). Based on four values of W corresponding to the beginning of the cycle XXV (2019.5-2022.5) and using the least squares, we calculate the coefficients of the equation $y = A + Bx$, where y are the values of W shifted by 3 years ago, and x are the values of unshifted W'. So we get $W = A + BW'$. Using coefficients A and B, we can test the accuracy of the approximation and make a forecast for 3 years based on the known values of the derivative W' (see Figure 2). To estimate the quality of approximation of the reconstructed series, the determination coefficient R^2 was calculated. In our case of linear regression, the coefficient of determination is equal to the square of the Pearson correlation coefficient similar to one shown in (1).

As an example, we present the equations calculated for several options for four W:

(a) $W = 70.524 + 3.228\ W'$; R^2=0.89 or K=0.94 for the interval 2019.5-2022.5;

(b) $W = 75.829 + 3.680\ W'$; R^2=0.87 or K=0.92 for 2 pairs of four W for the intervals 2008.5–2011.5 2019.5–2022.5, respectively;

Tests have shown that the calibration coefficient calculated for the same phase of the previous cycles (XXI–XXIII) is the most effective. The maximum accuracy of pairwise correlation is 0.94 for option (a).

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Table 2. Example of approximations using four W for the interval 2019.5–2022.5 (option (a)).

Years	Initial fragment W	Derivative W'	Shifted fragment W	Approximated W	Predicted W
2016.5	39.8	-24.05	3.6	7.12	
2017.5	21.7	-16.4	8.8	17.58	
2018.5	7.0	-9.05	29.7	41.31	
2019.5	3.6	0.9	83.1	73.43	7.12
2020.5	8.8	13.05		**112.65**	17.58
2021.5	29.7	37.15		**190.46**	41.31
2022.5	83.1	69.65		**295.38**	73.43
2023.5					**112.65**
2024.5					**190.46**
2025.5					**295.38**

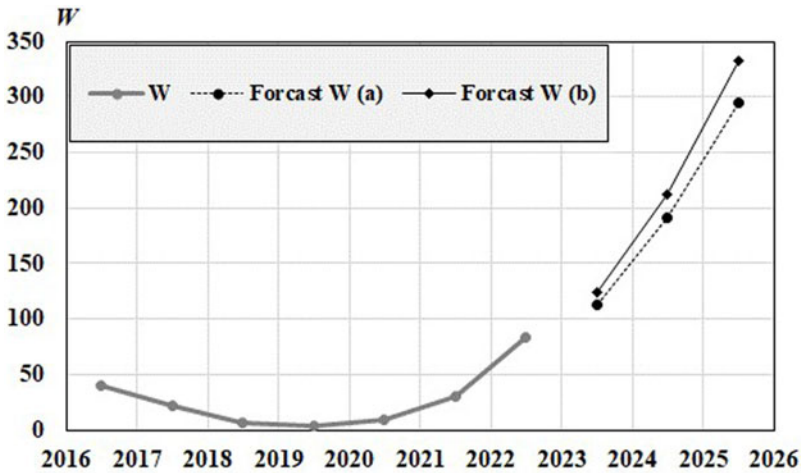

Figure 2. An example of a forecast for three years ahead for four W at the beginning of cycle XXV(a) and for 2 pairs of four W corresponding to the beginning of the cycles XXIV and XXV(b).

We presented an example in Table 2 to illustrates the calculation techniques. First, a fragment of W of the required length was shifted back 3 years, then the linear regression coefficients A and B were calculated using the least squares method. Then, the approximated series W was calculated using the known values of W', and finally, the resulting series was shifted forward to its place, thereby making a forecast for three years.

So, we are expecting a relatively long growing phase with sufficiently high maximum and extended duration of the current cycle. Apparently, this forecasting method is better suited for forecasting intermediate and average annual (or monthly average) values, rather than for determination of maximum amplitude in solar cycle and the time of it's occurrence. We are currently working on this problem. At the same time, the predicted values of W (option (a)) for the coming years confirm the results of the forecast of a number of researchers (Bhowmik & Nandy (2018), Du & Du (2006), Okoh *et al.* (2018)). This includes the work that particularly interested us (Pishkalo (2008)), which used the correlation between cycle parameters to predict maximum and minimum Wolf numbers for the solar cycle XXV. According to the study by Pishkalo (2008), the maximum amplitude in solar cycle XXV will be 112.37 ± 33.4 in mid-2023. At the same time, the predicted time of occurrence of the upcoming maximum covers a fairly large range from mid-2023 to end-2025 (Nandy (2021)).

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4. Conclusions

The correlations of the annual means of W and their time derivatives W' were studied with shifts of the series fragments relative to each other. The most significant (up to 0.87 and -0.85) correlation and anti-correlation coefficients are obtained with shifts of 3 years and 9 years for fragments of 11 and 14 years. Fragments of 22, 33, 44, 66, 88, 176, and 322 (the entire series) years have also been studied. Their coefficients are at the levels of -0.8 for shifts of 2–3 years and 8–9 years.

The corresponding phase shifts between the Wolf numbers and their derivatives are approximately 1/4 and 3/4 of the solar cycle, which is consistent with the predominant relationship between the dynamics of solar spots and the variability of magnetic energy.

Based on the high correlation coefficients between the shifted fragments of the W relative to W', the forecast of Wolf numbers for 2023.5–2025.5 is made with the maximum accuracy of pairwise correlation is 0.94. Moreover, this accuracy remains approximately at the same level with the joint participation of similar fragments of neighboring cycles.

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