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Fluid Mechanics: the quintessential complex system

Raymond T. Pierrehumbert†

Department of Physics, University of Oxford, Oxford OX1 3PJ, UK

The 2021 Nobel Prize in Physics recognizes advances in the understanding of complex systems, and underscores that 'complex' does not mean 'imponderable'.

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1. Introduction

The 2021 Nobel Prize in Physics was awarded to Syukuro ('Suki') Manabe, Klaus Hasselmann and Giorgio Parisi. Manabe was recognized for his foundational work on the physics of global warming, Hasselmann for his work on detecting climate change signals and Parisi for his wide-ranging work on statistical physics, notably the theory of spin glasses. The common theme knitting together the work of all three is that their work lies in the area of understanding complex systems. Such systems have many interacting components. The physics of each component is generally well understood from the microscopic standpoint. The challenge for physics in such systems is to understand the emergent phenomena that arise from the collective behaviour of the interacting subsystems. The award of a prize for work of this sort is a first for the Nobel Prize, and it is to be hoped that it opens the door to recognition of a broader variety of breakthroughs in the physical sciences.

In my view, fluid mechanics constitutes the quintessential complex system. The relevant phenomena will be very familiar to the *Journal of Fluid Mechanics* readership, although many who have worked in fluids did not grow up thinking of it as such, and indeed study of fluid mechanics long predates the currency of the term 'complex systems'. Consider the incompressible case, for which we do not even need to bring in an equation of state; it is just a matter of Newton's laws written many times over for each point in space, accompanied by a Lagrange constraint enforcing incompressibility and molecular viscosity to remove energy at small scales. There is no structure built into this system, but yet it spontaneously gives rise to an awe-inspiring variety of emergent coherent structures and phenomena. Leonardo da Vinci himself was fascinated by the almost organic forms created by fluid flow, as witnessed by his exquisite drawings of turbulent fluids, such as figure 1 (Colagrossi *et al.* 2021; Marusic & Broomhall 2021).

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† Email address for correspondence: raymond.pierrehumbert@physics.ox.ac.uk

Figure 1. One of Leonardo da Vinci's many drawings of fluid turbulence (Royal Collection Trust/© Her Majesty Queen Elizabeth II 2022).

The unique challenge for physicists posed by nonlinear spatially distributed systems like fluids was highlighted by Feynman, Leighton & Sands (1977, chap. 41):

> We have written the equations of water flow. From experiment, we find a set of concepts and approximations to use to discuss the solution–vortex streets, turbulent wakes, boundary layers. When we have similar equations in a less familiar situation, and one for which we cannot yet experiment, we try to solve the equations in a primitive, halting, and confused way to try to determine what new qualitative features may come out, or what new qualitative forms are a consequence of the equations. Our equations for the sun, for example, as a ball of hydrogen gas, describe a sun without sunspots, without the rice-grain structure of the surface, without prominences, without coronas. Yet, all of these are really in the equations; we just haven't found the way to get them out.
>
> There are those who are going to be disappointed when no life is found on other planets. Not I – I want to be reminded and delighted and surprised once again, through interplanetary exploration, with the infinite variety and novelty of phenomena that can be generated from such simple principles. The test of science is its ability to predict. Had you never visited the earth, could you predict the thunderstorms, the volcanos, the ocean waves, the auroras, and the colorful sunset? A salutary lesson it will be when we learn of all that goes on on each of those dead planets – those eight or ten balls, each agglomerated from the same dust cloud and each obeying exactly the same laws of physics.

The sentiment expressed in Feynman's second paragraph is the animating principle of my own take on planetary climates (Pierrehumbert 2010). Feynman did not live to see the current explosion of progress in exoplanetary climate, but his remark is prescient with regard to what we have discovered about the diversity of planetary climates, although they all emerge from essentially the same physical building blocks as used in Manabe's work on global warming.

Statistical thermodynamics also deals with macroscopic behaviour arising from interaction of many particles. What distinguishes thermodynamics from complex systems is that predictions in thermodynamics can rely on an assumption of ergodic behaviour on an energy surface. The necessary calculations can be challenging, but the underlying principle is clear. For complex systems involving dissipative structures (and even some that do not) no such ready formalism for obtaining the probabilistic behaviour is available.

2. Manabe on physics of global warming

The essential ingredients of planetary climate are radiative transfer, thermodynamics (including thermodynamics of phase change) and Newtonian mechanics as embodied in the laws of fluid motion. Manabe made significant contributions to the accurate computation of radiative transfer for CO_2 and H_2O in an N_2/O_2 background atmosphere, but his signal achievement is in explicating the emergent behaviour arising from the interaction of the aforementioned ingredients, first in one dimension and later in three dimensions. This enabled the first prediction of the effect of doubling CO_2 on climate which was based on fully correct physical principles. The greenhouse effect produces warming only to the extent that the atmospheric temperature goes down with altitude over a substantial layer of the atmosphere, and the heat transport by convection plays an essential role in determining the vertical structure of planetary atmospheres. In the case of Earth, the vertical structure is strongly affected by the latent heat released by condensation of water vapour in the course of convection. Manabe's breakthrough work (Manabe & Wetherald 1967) coupled an accurate radiative transfer calculation (including stratospheric heating arising from absorption of ultraviolet by ozone) to a representation of convection based on an assumption that convection relaxes an unstable gradient to a state of neutral stability – moist convective adjustment. Water vapour radiative feedback was handled through an assumption that relative humidity should remain fixed as temperature changes, much as done earlier by Arrhenius. One of Manabe's key insights in his analysis of the resulting one-dimensional radiative–convective model was that surface temperature is primarily controlled by the top-of-atmosphere energy balance, not the surface energy balance. This insight was missing from earlier attempts to use accurate radiative calculations to determine the amount of warming induced by doubling atmospheric CO_2. Besides the key calculation of climate sensitivity, the one-dimensional model revealed that the signature of CO_2-induced surface warming is that it is accompanied by stratospheric cooling. (A form of convective adjustment without the effects of condensation had long been used in the theory of equilibrium stellar structure, notably in the work of Karl Schwarzschild. Research on radiative convective modelling appears to have undergone parallel development in astrophysics and Earth climate modelling for many years, with only sporadic contact between the communities until the blossoming of interest in modelling planetary atmospheres beginning roughly in the 1970s.)

Years later, Manabe and his team coupled the same physics into a three-dimensional model of the Earth's general circulation, which incorporated the geographical redistribution of heat by large scale fluid motions (Manabe & Wetherald 1975). The convective adjustment scheme was refined by incorporating an adjustment to the moist adiabat rather than an empirical lapse rate. A key advance in the three-dimensional simulation is that the water vapour feedback, which approximately doubles the direct CO_2 warming comes out as an emergent property of the system, rather than being built in via a fixed relative humidity assumption. Water vapour in the atmosphere is out of thermodynamic equilibrium with the moisture source at the ocean surface – in equilibrium, it would be saturated everywhere. It is kept from equilibrium by an intricate interplay of turbulent mixing and removal by condensation, which ultimately determines the probability distribution of humidity (Pierrehumbert, Brogniez & Roca 2007). (In Manabe's three-dimensional simulation, relative humidity was still imposed in the representation of convection, but most of the atmosphere is not actively convecting and atmospheric humidity is strongly controlled by large scale fluid motions. Later representations of convection adopted a more prognostic approach to humidity in convecting regions. For that matter, convective adjustment itself is an emergent property of fully three-dimensional convection, and only an approximation to reality. Representation of convection in global circulation models is an area of considerable current research interest.) Other key emergent properties from the three-dimensional model include amplification of polar warming relative to the global mean, and the contrast between vertical structure of polar warming (bottom heavy) vs tropical warming (top heavy).

For further discussion of where the breakthroughs of Manabe and co-workers fit into the long quest to understand global warming, see Archer & Pierrehumbert (2011).

3. Hasselmann on predictability amidst chaos

If climate is governed by a nonlinear chaotic system, how can we predict the response to a change in atmospheric CO_2 concentration? And if we do detect a trend, how do we know it is really due to a change in the climate forcing rather than natural low-frequency variability? This, in a sense, is where Hasselmann's contribution comes in. Linear systems are much easier to understand than nonlinear ones, so one of the techniques used in dynamical systems theory is to represent internally generated chaos as a stochastic noise forcing a linear subsystem with slow response time (think oceans or glaciers). This is the approach taken in Hasselmann's influential paper on stochastic climate models (Hasselmann 1976). The mathematical methods used were far from original to this paper, but the insights as to their implications for the origins of red noise in the climate system were, at the time, novel. The original paper did not actually treat the question of forced vs free response, but it did engage the critical ideas that were ultimately to give rise to methods for detecting forced trends. A shortcoming of the approach is that it does not allow for nonlinear effects in the slow-response component of the system, although such effects were later taken up by others, exploring such phenomena as sub-exponential temporal correlations, bimodality and stochastic resonance. Nonetheless, Hasselmann's work in considerable measure laid the foundation for what has become a large enterprise in identifying statistically significant fingerprints of human-caused climate disruption, an enterprise in which he has been an active participant himself (Hasselmann 1993).

4. Parisi on collective behaviour: spin glasses to bird flocks

Parisi represents a different kind of attack on complex systems, and one that is more rooted in traditional methods of statistical physics – but with a stunning twist. His signature accomplishment lies in the theory of spin glasses. A spin glass is a partly disordered state of a set of particles on a lattice whose spins interact via magnetic coupling. Although the system is described by a Hamiltonian, it exhibits strongly non-ergodic behaviour, in which not all states on the energy surface can be reached from a given instance; in this sense, spin glasses are not 'thermodynamic'. The non-ergodicity gives rise to a fascinating mix of order and disorder, as well as a variety of anomalous non-exponential temporal behaviours. Parisi's solution of the idealized Sherrington–Kirkpatrick spin glass model by the replica mean field method (Parisi 1979), and subsequent analysis of the properties of the solution (Mézard *et al.* 1984) unlocked a treasure trove of progress on spin glasses and related complex systems. Parisi and co-workers also developed innovative mathematical techniques for dealing with a range of other complex systems, including self-organized behaviour of bird flocks and multifractal measures for characterization of self-similarity in spatio-temporal structure.

Parisi did not specifically work on fluids, but his work on spin glasses has deep affinities with problems arising in two-dimensional incompressible fluids. The inviscid case has a Hamiltonian structure in principle amenable to thermodynamic analysis, as shows up most clearly in Onsager's point-vortex gas model (Eyink & Sreenivasan 2006). The continuous vorticity case is further associated with an infinite set of Casimir invariants associated with material conservation of vorticity, and these intricately carve up the energy surface into mutually inaccessible domains. Non-ergodicity, in part associated with Arnol'd-stable vortices in the continuous case, lead to a rich variety of order and non-thermodynamic behaviour, even in the absence of dissipation.

5. Concluding remarks

The work of Manabe and Hasselmann underscores that even in a system as complex and chaotic as climate, successful predictions can be made. As late as 1997, decades after Manabe's pioneering work, the signal of anthropogenic global warming was considered just marginally detectable. Now, the attribution is considered incontrovertible, and most of the fingerprints of global warming initially predicted by Manabe have emerged from the background of climate noise. This is not to say 'the science is settled', as there is still important ongoing work to determine the precise climate sensitivity, and in particular to improve the understanding of clouds and convection. Nonetheless, the inclusion of Manabe and Hasselmann's work in the 2021 Nobel Prize in Physics underscores that the alarm rung about the climate crisis rests on sound physical principles. The science may not be 'settled', but it is 'settled enough' to justify redoubled efforts to decarbonize the world economy.

Feynman concludes his thoughts on what we now call complex systems as follows:

> The next great era of awakening of human intellect may well produce a method of understanding the qualitative content of equations. Today we cannot. Today we cannot see that the water flow equations contain such things as the barber pole structure of turbulence that one sees between rotating cylinders. Today we cannot see whether Schrödinger's equation contains frogs, musical composers, or morality – or whether it does not. We cannot say whether something beyond it like God is needed, or not. And so we can all hold strong opinions either way.

There is still no automated discovery method for complex systems, and I would guess there never will be. Like Tolstoy's unhappy families, every complex system is complex in its own way. That is part of the joy of this kind of science, and it guarantees there will never be 'an end of physics', even if a Grand Unified Field Theory is discovered and physicists run out of new microscopic laws of nature to discover. Complex systems may be hard to understand, but with creativity, physical insight, simulation and a good dose of innovative mathematics, progress and even predictions, can be made. The 2021 Physics laureates have lifted their lanterns, and helped show the way.

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