# Appendix E
# Model of the single-particle strength function

In the extreme single-particle model of nuclear structure, single-particle states are either occupied or empty. As explained in Chapter 9 the coupling of single-particle motion to vibrations changes this situation and single-particle states near the Fermi surface are partially occupied. The spectroscopic factor is a measure of the occupancy of a level. This appendix presents a simple model ('picket fence' model) which relates the spectroscopic factor to the $\omega$-mass (see Bohr and Mottelson (1969), Mahaux *et al*. (1985)).

We consider a two-level model where the pure single-particle state $|a\rangle$ couples to a more complicated state $|\alpha\rangle$ of which a possible representation could be a single-particle state coupled to a vibration (see Fig. E.1). We want to diagonalize the Hamiltonian (see Bohr and Mottelson (1969)),

$$H = H_0 + v, \tag{E.1}$$

where

$$H_0|a\rangle = E_a|a\rangle \tag{E.2}$$

and

$$H_0|\alpha\rangle = E_\alpha|\alpha\rangle. \tag{E.3}$$

The interaction $v$ couples these states. Assuming

$$\langle a|v|a\rangle = \langle \alpha|v|\alpha\rangle = 0, \tag{E.4}$$

and calling

$$v_{a\alpha} = \langle a|v|\alpha\rangle, \tag{E.5}$$

one can write the secular equation associated with $H$ as

$$\begin{pmatrix} E_\alpha - E_i & v_{a\alpha} \\ v_{a\alpha} & E_a - E_i \end{pmatrix} \begin{pmatrix} c_\alpha(i) \\ c_a(i) \end{pmatrix} = 0, \tag{E.6}$$



https://doi.org/10.1017/9781009401920.017 Published online by Cambridge University Press

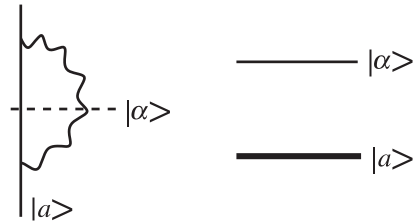


Figure E.1. The coupling between the state $|a\rangle$ and the intermediate state $|\alpha\rangle$ associated with the process in which a nucleon excites a vibrational mode to reabsorb it at a later time.

leading to the two equations

$$(E_\alpha - E_i)c_\alpha(i) + v_{a\alpha}c_a(i) = 0, \tag{E.7}$$

$$v_{a\alpha}c_\alpha(i) + (E_a - E_i)c_a(i) = 0. \tag{E.8}$$

From the first equation one obtains

$$c_\alpha(i) = -\frac{v_{a\alpha}}{E_\alpha - E_i}c_a(i). \tag{E.9}$$

From this relation and from

$$c_a^2(i) + c_\alpha^2(i) = 1, \tag{E.10}$$

one obtains

$$c_a^2(i) = \left(1 + \frac{v_{a\alpha}^2}{(E_\alpha - E_i)^2}\right)^{-1}. \tag{E.11}$$

Inserting equation (E.9) into equation (E.8) leads to

$$-\frac{v_{a\alpha}^2}{E_\alpha - E_i} + (E_a - E_i) = 0. \tag{E.12}$$

Thus

$$E_i = E_a - \frac{v_{a\alpha}^2}{E_\alpha - E_i}, \tag{E.13}$$

which is, within the present model, the self-consistent Dyson equation.

In other words, the single-particle self-energy is

$$\Delta E_a(E) = E - E_a = -\frac{v_{a\alpha}^2}{E_\alpha - E}\ . \tag{E.14}$$

Defining the $\omega$-mass by

$$\frac{m_\omega}{m} = 1 - \left.\frac{\partial \Delta E_a}{\partial E}\right|_{E=E_i}, \tag{E.15}$$

https://doi.org/10.1017/9781009401920.017 Published online by Cambridge University Press

and making use of the above equation we get

$$\frac{m_\omega}{m} = 1 + \frac{v_{a\alpha}^2}{(E_\alpha - E_i)^2}. \tag{E.16}$$

Comparing with equation (E.11) for $c_a^2(i)$ we see that the spectroscopic factor associated with the quasi-pure single-particle state

$$|\tilde{a}\rangle = c_a(i)|a\rangle + c_\alpha(i)|\alpha\rangle \tag{E.17}$$

is

$$Z_\omega = c_a^2(i) = \left(\frac{m_\omega}{m}\right)^{-1}. \tag{E.18}$$

https://doi.org/10.1017/9781009401920.017 Published online by Cambridge University Press