COMPUTER SIMULATIONS OF GALAXY CLUSTERING

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1. INTRODUCTION AND INITIAL CONDITIONS

The aim of the present work (performed in collaboration with J.R. Gott and E.L. Turner) is to account for the observed distribution of galaxies in terms of the gravitational instability picture. Specifically we assume that all the matter is contained in galaxies. The evolution of such a system can then be studied by N-body simulations once the initial conditions are specified. Our approach is essentially experimental; a variety of models are computed and the results are compared with observations.

The models are characterized by the following set of parameters: N, n, m, β, Ω_o, ε, R_o. Here N denotes the total number of galaxies; the index n is used to describe the initial density fluctuations $\delta\rho/\rho \propto M^{-\frac{1}{2}-n/6}$ (n = o corresponds to a random distribution, whereas n = - 1 gives rise to a 'flat' fluctuation spectrum favoured by Gott and Rees (1975)). The galaxy masses m are usually taken to be equal; alternatively we select two mass groups or a general mass function. A randomized component of kinetic energy, $T_p \equiv \beta T_H$, may be superimposed on the initial Hubble velocity flow $\underline{v}_i = H \underline{r}_i$ with total kinetic energy T_H (β = 0 denotes a 'cold' universe). We adopt the standard Friedmann cosmology with $\Lambda = 0$ and concentrate on two values of the final density parameter $\Omega_o \equiv 8\pi G\rho_o/3H_o^2$, where ρ_o is the present mean density of the universe: (i) a closed universe with $\Omega_o = 1$ (parabolic case) and (ii) an open universe with $\Omega_o = 0.1$. The calculations are assumed to start at a red-shift z_{st} when the primordial density fluctuations have reached a value $\delta\rho/\rho \sim 1$ on galactic mass scales. Associating this epoch with a galaxy collapse time $\sim 10^9$ yrs then gives an expansion parameter at the present epoch of $R_o = 1 + z_{st}$, whereupon the initial value of Ω may be evaluated if $\Omega_o \neq 1$. The scale of the system is set from considerations of the total luminosity; we adopt a final radius R_o = 50 Mpc for a Hubble constant H_o = 50 km sec^{-1} Mpc^{-1}.

Two separate computer codes are used, depending on whether the galaxies are assumed to be mass-points or extended distributions of

M. S. Longair and J. Einasto (eds.), The Large Scale Structure of the Universe, 189-196.

https://doi.org/10.1017/S0074180900144523 Published online by Cambridge University Press

characteristic size ε. In the latter case the galaxies are represented by a 'soft' potential of the form $Gm/(r^2 + \varepsilon^2)^{\frac{1}{2}}$. Treating close mass-point pairs by a two-body regularization technique, we have computed several models with N = 1000 galaxies. A soft potential permits more particles to be studied; here we report on two models with N = 4000 galaxies and a softening parameter $\varepsilon/R_o = 1 \times 10^{-3}$ (i.e. ε = 50 kpc). The equations of motion for each galaxy are advanced according to the Ahmad-Cohen (1973) integration scheme which treats neighbours and distant particles on different time-scales. We consider a spherical region of space in isolation and therefore neglect the effect of external galaxies. Any galaxy which approaches the expanding boundary is reflected with decreased radial velocity. This procedure preserves the mean density in the co-moving frame and has the effect of cooling the peculiar motions near the boundary. The boundary itself is assumed to obey an appropriate Newtonian equation of motion throughout.

The calculations give rise to an extensive data bank which will be used for a variety of tests and comparisons. In this paper we report briefly on the covariance function and discuss some properties of the clustering process. More detailed considerations will be published elsewhere (Aarseth, Gott and Turner (1977), henceforth referred to as AGT).

2. THE COVARIANCE FUNCTION

The covariance function ξ provides a precise tool for comparing the final models with observations. It is defined in terms of the probability dP of finding a galaxy within a volume dV at the distance r from a randomly chosen galaxy: $dP = n_g\ (1 + \xi(r))\ dV$, where n_g is the number density of galaxies. Peebles (1974) finds $\xi(r) \simeq 68\ r^{-1.8}$ for $0.06 \lesssim r \lesssim 60$, r being measured in Mpc, where the amplitude is uncertain by a factor of ~ 2. The 64 rouble question is then whether any of the models can reproduce a satisfactory power-law covariance function over the observed range.

Analysis of models I and II yields covariance functions $\xi(r) \simeq 110\ r^{-1.9}$ and $\xi(r) \simeq 190\ r^{-1.9}$, respectively, over the range $\xi(r) \sim 10^4$ to $\xi(r) \sim 1$ for the scaling R_o = 50 Mpc. Here Model I is a closed universe with parameters $\Omega_o = 1$, $n = 0$, $\beta = 0.027$, $1 + z_{st} = 13.8$, whereas the open Model II is characterized by $\Omega_o = 0.1$, $n = -1$, $\beta = 0$, $1 + z_{st} = 30.9$. Both models contain N = 1000 galaxies of equal mass which have been treated as mass-points ($\varepsilon = 0$). The amplitudes which are somewhat high may be reduced by using smaller values of z_{st}, whereas the exponent is very close to the observed value for both models.

It is a striking feature of the mass-point models that the covariance function continues to increase (with a slightly steeper slope) at smaller distances, reaching $\xi(r) \gtrsim 10^8$ at $r/R_o \sim 2 \times 10^{-5}$ in both models. This property is accounted for by a number of extremely close binaries. No such small scales are present in the initial distributions where the closest binaries are $\sim 10^{-2}$ of the corresponding scale factor R. The net shrinkage is mainly due to relaxation effects between binaries and single galaxies which are well understood from theoretical considerations (Heggie 1975). In the case of soft potentials the form of $\xi(r)$ is similar to the mass-point models over the observational range but the maximum amplitude is considerably smaller.

Models with two mass groups m_1 and m_2 have also been considered. In one model with $m_2 = 2m_1$ we find that the amplitude of $\xi(r)$ for the heavy mass component is about twice that for the light component, whereas the respective slopes are 2.1 and 1.9. We may associate the heavy component with E and SO galaxies which show some evidence of having twice the M/L value of spirals (Turner 1976) but similar luminosity functions (Shapiro 1971). Our result is then in qualitative agreement with covariance functions determined separately for ellipticals, lenticulars and spirals (Davis and Geller 1976). The mass effect in the simulated models is another manifestation that significant two-body relaxation has taken place. A further indication of relaxation is provided by an initial feature in the covariance function at large r which disappears during the evolution.

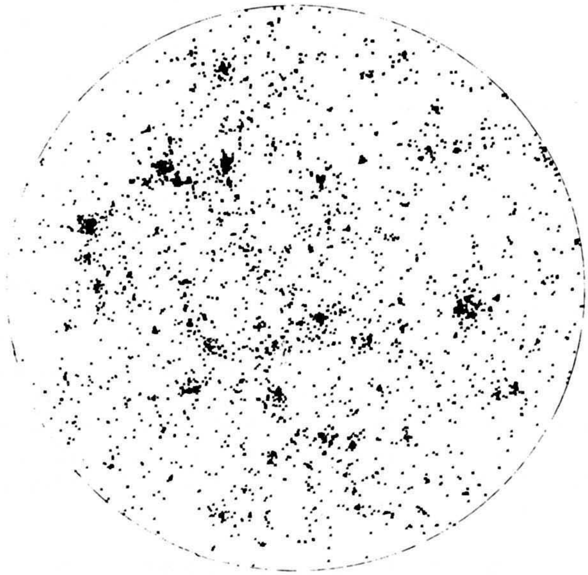
To conclude this discussion, it appears that different cosmological models (i.e. closed vs open) cannot be distinguished from the analysis of $\xi(r)$ for the present range of parameters. However, the N-body simulations do demonstrate that the gravitational instability theory can account for the observed galaxy clustering.

3. THE GROWTH OF CLUSTERING

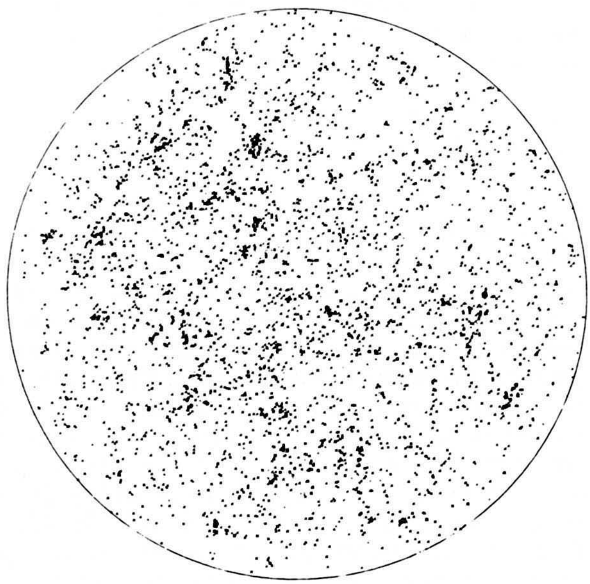
Pictures of the final models reveal a significant amount of galaxy clustering (cf AGT) and it is therefore of interest to understand the clustering process. The growth of clustering is illustrated by a 16 mm time sequence movie which shows the projected xy-distribution of N = 4000 galaxies in co-moving coordinates. This is an open model with basic parameters $n = -1$, $\beta = 0$, $\Omega_0 = 0.1$, $\varepsilon = 50$ kpc, $1 + z_{st} = 32$. We employ a general mass spectrum derived from the luminosity function of galaxies in small groups (Turner and Gott 1976), where the initial density fluctuations $\delta\rho/\rho$ are comparable to a similar distribution of N= 1000 equal galaxy masses. Although the movie only displays the two-dimensional development, the apparent clustering is quite pronounced. Two snap-shots of the evolution are displayed in Figure 1; there is some indication of the clusters grouping together into super-clusters as suggested by the observations. One cluster forms at the boundary and survives throughout the calculation; hence the boundary effects do not appear to suppress the clustering unduly.

A proper analysis of the clustering process is best carried out in three dimensions. Some preliminary results have been obtained using a simple approach based on spherical symmetry. For this purpose we employ an operational cluster definition as follows. The integration scheme

https://doi.org/10.1017/S0074180900144523 Published online by Cambridge University Press



R = 15.4



R = 4.3

Figure 1 Projected distribution in the xy-plane for the model Ω_0=0.1, N=4000 at expansion factors R=4.3 and R=15.4.

https://doi.org/10.1017/S0074180900144523 Published online by Cambridge University Press

provides information about the number density of neighbours (i.e. neighbour sphere radius and corresponding membership). For simplicity we assume that the galaxy with the highest associated density is located at a cluster centre. We then include the mass M_s within spheres of increasing radius R_s until the corresponding density contrast $C \equiv R^3 M_s / N R_s{}^3$ falls below a prescribed value C_{cl} (N is also the total mass). A density contrast of $C_{cl} = 5.5$ represents turn-around in an Einstein-de Sitter universe, but alternative values may also be considered. The properties of such a cluster can then be analysed as for an isolated system, whereupon the next cluster is identified in a similar manner subject to the condition that no galaxy is counted more than once.

Table I gives the frequency distribution of identified clusters with more than 12 members for $C_{cl} = 5.5$. The first line refers to the $\Omega_o = 0.1$ model discussed above and the second line is for a similar closed model with $1 + z_{st} = 10.7$.

Table I

Model	> 400	400 - 200	200 - 100	100 - 50	50 - 25	25 - 13
$\Omega_o = 0.1$	1	0	4	6	9	31
$\Omega_o = 1$	1	1	3	4	8	31

The table contains 51 and 48 clusters with respective total populations of 2422 and 2748 galaxies. Adopting $C_{cl} = 22$ instead we find similarly 29 and 21 clusters with a corresponding total membership of 1731 and 1804 galaxies. The average mass of the cluster galaxies in Table I tends to be somewhat higher than the total average (typically $\sim$ 20 - 30 per cent), but only in one case does the excess reach a factor of 2. On the other hand, a few clusters show a small deficiency of heavy members; this is perhaps not surprising considering that only about 26 per cent of all the galaxies exceed the average mass. In any case it should be emphasized that the relatively small mass differences between cluster galaxies and field galaxies in these models do not allow for the possibility of mergers, which would be more likely in clusters.

Further analysis may show whether some of the richest clusters contain independent sub-clusters not identified by the present procedure. In principle at least the stability of hierarchical clustering would provide a test for distinguishing between closed and open models. All clusters in Table I have significant negative binding energies and are therefore quite stable. This is still the case (with one exception) after subtraction of the negative energy stored in close pairs, equivalent to evaluating the total energy in the two-body approximation. Most of the members are located inside half the estimated cluster radius, indicating a significant core-halo structure. The associated crossing times are defined by $t_{cr} = M_s{}^{5/2}/(2|E_{cl}|)^{3/2}$, where E_{cl} is the corrected cluster energy. Typical values of the dynamical age t/t_{cr} range from $\sim$ 2 to $\sim$ 10, hence there has been time for significant internal relaxation.

Tidal interactions between the clusters may also be of dynamical importance, with shear motions generating angular momentum of opposite sign. The effectiveness of such tidal torques may be tested by evaluating the dimensionless quantity $\gamma \equiv \underline{J}^2/2TI$, where $\underline{J}$, T and I are the total angular momentum, kinetic energy and moment of inertia with respect to the cluster centre. For most clusters in Table I, $\gamma \lesssim 0.01$ and hence the externally induced rotational energy is very small. This result may also have implications for the tidal origin of rotation inside galaxies; to be effective this process requires the galaxies to be relatively closer to each other than is the case for the clusters.

In conclusion, the present dynamical models can account quite well for the observed covariance function and are also capable of producing clusters with a wide range of membership. The insensitivity of these quantities to the initial conditions (particularly Ω) is somewhat surprising and needs to be understood better. However, the velocity distribution provides more sensitive tests for distinguishing between cosmological models, as discussed by Gott elsewhere in this volume.

REFERENCES

Aarseth, S.J., Gott, J.R., and Turner, E.L. 1977, in preparation.
Ahmad, A., and Cohen, L. 1973, J. Comp. Phys. 12, 389.
Davis, M., and Geller, M.J. 1976, Astrophys. J. 208, 13.
Gott, J.R., and Rees, M.J. 1975, Astron. & Astrophys. 45, 365.
Heggie, D.C. 1975, Monthly Notices Roy. Astron. Soc. 173, 729.
Peebles, P.J.E. 1974, Astrophys. J. Letters, 189, L51.
Shapiro, S.L. 1971, Astron. J. 76, 291.
Turner, E.L. 1976, Astrophys. J. 208, 304.
Turner, E.L., and Gott, J.R. 1976, Astrophys. J. 207, 6.

DISCUSSION

Peebles: I have produced similar films, but have always called them "propaganda films" because it seemed to me that it is very dangerous to compare them too closely to the real Universe. There are two important lengths in the model, the radius R, and $R/N^{1/3}$, where N is the number of particles, the latter being the radius at which the initial fluctuations are non-linear. Since we do not know how to model the non-linear part of the mass distribution, I felt that one should only examine the structure that develops on physical scales $\gtrsim R/N^{1/3}$. Unhappily, since N is limited to ~ 1000, this leaves only a very restricted range of scales. You have taken a bold step in going to much smaller scales, and you might be right, but I think it is a little dangerous.

de Vaucouleurs: Could you offer some comments - possible reasons - for the facts that (1) your calculations match satisfactorily the observed two point covariance function, but (2) the galaxy and cluster distributions shown in your film does not resemble the real Universe as depicted by Peebles from the Lick counts?

Aarseth: In the film each galaxy is shown with the same intensity, whereas the masses are in fact selected from a broad Schechter function. A proper representation would certainly improve the visual impression of the simulated cluster picture. In any case this question cannot be settled by a subjective inspection. What is needed, is a quantitative comparison using methods described at this meeting.

Petrovskaya: What is the time unit in the flim and what may one say about the time scale of the clustering process?

Aarseth: The initial epoch is taken to be 10^9 years and the final epoch is the present time, corresponding to an expansion factor of 32. A significant amount of clustering can already be seen after four or five expansion times.

Fall: My question is related to Prof. Peebles' comment about characteristic scales and their possible effect on the reliability of your calculations. As he has pointed out, one might worry that the point-mass calculation would not accurately reflect the clustering of real galaxies on scales smaller than the initial mean intergalaxy separation or, equivalently, the characteristic size of a galaxy. Also, one might worry that the real distribution of galaxies was already clustered and had significant velocities at epochs, corresponding to the beginning of your calculations. If the final distribution of galaxies in your calculations depended sensitively on galaxy sizes, velocities etc., then the interpretation would be complicated. My question is simply, do you find any evidence that the final distribution depends sensitively on these effects?

Aarseth: We have done calculations with mass-points and soft potentials as well as different initial position and velocity distributions, and find no significant differences in the final correlation functions.

Zeldovich: The initial peculiar velocities if they are different in different places are equivalent to temperature. The Jeans mass grows and small clusters are prevented from forming. But various peculiar velocities increase the amplitude of perturbations, each one on its own mass scale. They can even lead to clustering in the absence of initial density fluctuations.

Davis: Could you comment on the growth rate of the cosmic potential energy T or the cosmic potential energy U in your simulations? In the self-similar $\Omega = 1$ models, T and U will scale as $R^{(1-n)/(3+n)}$, where R is the cosmic factor and n is the initial spectral index of perturbations. If this is not so in your simulations, then non self-similar effects are dominating the solution.

https://doi.org/10.1017/S0074180900144523 Published online by Cambridge University Press

Aarseth: We plan to check the growth-rates of the peculiar energies, but have not done so yet.

Ostriker: One of the most interesting aspects of Aarseth's simulations is that the amplitude of the correlation functions for masses $M = 2M_1$ are twice as large as those with $M = M_1$. Can I ask all those who have made these numerical simulations whether they would have expected this result or does it indicate that the calculations are wrong?

Peebles: I have made N-body calculations to test this point. The simulations start with a clustering hierarchy with points distributed according to the empirical 2, 3 and 4-point correlation functions. The velocities are chosen so that on taking statistical averages the distribution will be kept in equilibrium, and the model is then run forward in time. It is found that the 2 and 3 point correlation functions do not change appreciably with time. If there is a mass distribution, the cross-correlation function for different masses does not change with time, i.e. the light particles do not float up to the dense spots.

Ostriker: Have you answered my question?

Peebles: If the simulations had resulted in the clustering distribution we observe in the Universe, I would not have expected to see mass segregation.

Gott: We were not at all surprised that our simulations with two mass groups ($M_2 = 2M_1$) gave the results that the galaxies of mass $2M_1$ had a covariance function of just twice the amplitude of the galaxies of mass M_1. The three point correlation results of Peebles and Groth show that a tight binary galaxy has just twice the covariance function of a normal galaxy, essentially because each of the two galaxies in the binary galaxy brings the expected average number of companions with it. Of course, one galaxy with mass 2M is dynamically indistinguishable from a tight binary of mass (M_1+M_1), and should, therefore, have a similar covariance function.