LINEAR AND NONLINEAR PULSATIONS OF β CEPHEI STARS

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The best known β Cephei variable is a α Virginis (Spica) because it is a binary star (P = 4.01 days) and has a interferometer measurement of its angular diameter. In this study to see if the pulsations can be caused by mixing of hydrogen and helium in the semiconvective zone, the parameters we take are 11.5 $M_{\odot}$, 26,300 K, 6.5 x 10^{37} erg/s with an envelope composition of Y = 0.28, Z = 0.02. These data agree within the errors of observations as reported recently by Odell (1980), and the theoretical radial fundamental mode period of 0.171 day compares well with the single observed pulsation period of 0.174 day.

An interesting and maybe unique aspect of the α Virginis pulsations is that it has been observed to have stopped pulsating in 1972, even though in 1969 it had the same period seen also in 1956 and 1934 by radial velocity observations (see Shobbrook, Lomb and Herbison-Evans 1972). The first author personally observed this star over several hours in January 1951 and saw no light variations of more than a few thousandths of a magnitude. It appears that the star has a pulsation decay time of about six years as linear theory gives if there is no persistent pulsation driving and only the known radiative damping.

Figure 1 discusses semiconvection in B stars. In the simple homogeneous composition case on the right, the stellar envelope is stable because its existing local $\nabla = d\,\ell n\,T/d\,\ell n\,P$ is less than the adiabatic value of $\nabla_{ad} = \Gamma_3 - 1/\Gamma_1$. The homogeneous composition core, though, has a gradient above the Schwarzschild value $\nabla - \nabla_{ad}$ and is in turbulent convection. However, if there is a composition gradient due to nuclear burning, turbulent elements have a restraining force against convection and will not have turbulent convection unless the gradient ∇ is above the Schwarzschild level plus $\beta/(4-3\beta)\ d\,\ell n\ \mu/d\,\ell n\,P$, that is, above the Ledoux level. Kato (1966) showed that eddies oscillate if the existing gradient is super-Schwarzschild but sub-Ledoux. Shibahashi and Osaki (1976) showed that for large ℓ values, global nonradial pulsation modes will eventually grow, and presumably mixing will occur to increase the local helium content enough to make the semiconvection zone eventually sub-Schwarzschild. Over long time scales the gradient

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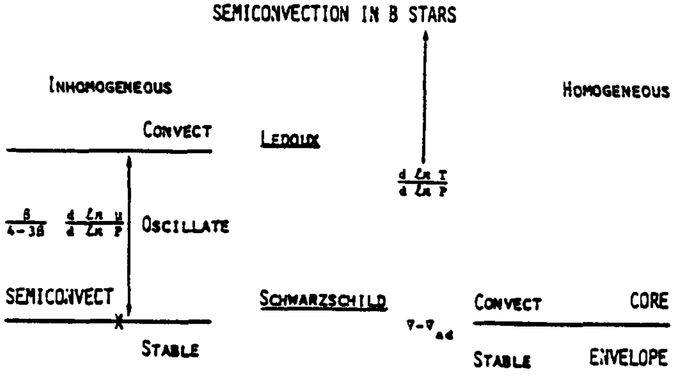

Figure 1

will be given by the x at the Schwarzschild level, but it is possible during evolution, with always increasing luminosity, to move upward into the unstable μ gradient region. We investigate the possibility that this unstable μ gradient gives rise to both the unobservable high ℓ nonradial modes and the radial fundamental seen in β Cephei variables like α Virginis.

Table 1 gives details of our model studied both in linear and nonlinear theory. This model has a semiconvective zone 9 at a Y = 0.449 with an envelope zone 10 of Y = 0.28 and a convective core (ℓ/H_p = 1.0) zone 8 of Y = 0.70. The central 6 percent of the mass within the central 8 percent of the radius produces 87 percent of the total luminosity by CNO cycling at T > 30.2 x 10^6 K.

TABLE 1

ALPHA VIRGINIS MODEL ($0^d.174$)
60 LAGRANGIAN ZONES

11.5 $M_\odot$ 26,300 K 6.5 x 10^{37} ERG/S Y_s = 0.28 Y_{cc} = 0.70 Z = 0.02 ℓ/H_p = 1.0

LAST MASS 8 x 10^{23} G PHOTOSPHERE RADIUS 4.37 x 10^{11} CM

MASS RATIO 1.53 FOR 49 ZONES AND 0.769 FOR 11 ZONES

SEMICONVECTIVE LAYER OUTER SURFACE: q = 0.39 x = 0.20 T = 18.4 x 10^6 K

CONVECTIVE CORE: q = 0.24 x = 0.15 T = 23.5 x 10^6 K

CENTRAL BALL SURFACE: q = 0.06 x = 0.08 L_c/L = 0.87 T = 30.2 x 10^6 K

Π_0 = 0.171 η_0 = - 8 x 10^{-5} Π_0^{-1}

More details of the internal structure are given in Figure 2 with log T, log κ and log ρ plotted against the logarithm of the external mass fraction. The semiconvection zone and convective core can be seen in the central third of the mass as a steeper ρ and T gradient and a decrease in κ.

The linear theory eigenfunctions for the first three radial modes are given in Figure 3. The semiconvection shell lies between radius fraction x = .15 and x = .20 where clearly the fundamental mode has a larger amplitude than all higher modes. Semiconvection driving would couple better with that mode.

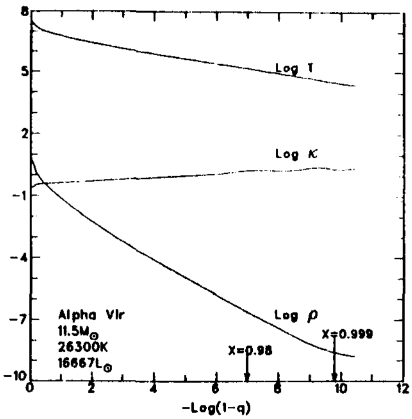

Figure 2

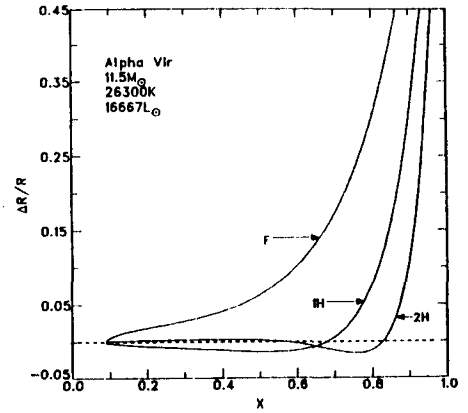

Figure 3

Figure 4 gives the linear theory damping for the first three radial modes. Driving at 150,000 K actually exists in zone 44 (x = 0.98, $1-q = 10^{-7}$) but it was too small to be considered by the computer-made plot. The most damping zones, 38, 39, and 40, at temperatures of almost 300,000 K occur at $1-q \sim 10^{-6}$ and x = 0.96 where all three modes considered have a large pulsation amplitude as shown in Figure 3. The reason for the increase by a factor of roughly 10 in damping as one goes to each higher mode seems to be related to the steeper gradient and increased nonadiabaticity of the higher mode eigenfunctions. The well known result that these stars seem pulsationally stable in linear theory is evident.

Various nonlinear theory calculations have been made to test destabilization due to time varying convection, and continuous, periodic and impulsive mixing of H and He in the semiconvection zone. At the outer edge of the convective core (interface 7) convection carries only about 20 percent of the local luminosity and it varies by only about 10 percent. Further, the convective luminosity peaks just after the time when the star is at maximum radius. Thus this extra luminosity, if it has any effect at all, impedes contraction and tends to damp pulsations.

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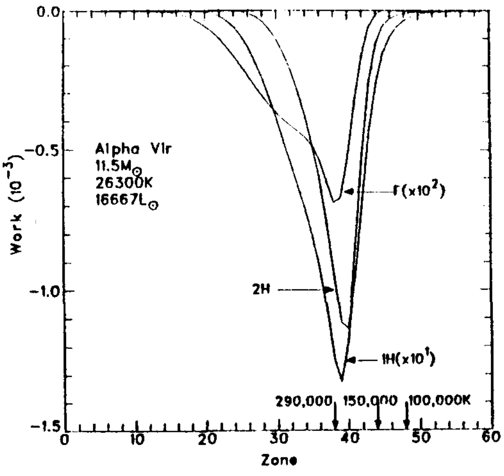

Figure 4

Calculations to date indicate that continuous or periodic mixing, using as a diffusion coefficient the mixing length times the convective velocity, give no destabilization. Helium is gently moved outward, even periodically, but this semiconvective driving is easily damped in the outer dissipative regions.

Figure 5 shows the result of a sudden increase of H in the semi-convective zone by one part in 10^4. A background radial velocity of ± 1 km/s (± $0^m.02$) is present in this nonlinear calculation. The disturbance at the surface is about ± $0^m.1$ as the model rings. Since the overtones die rapidly, it may be that this single event can put the model into the fundamental mode after 10^3 first overtone periods. Maybe occasional sudden mixing can keep the star pulsating indefinitely. More studies are in progress.

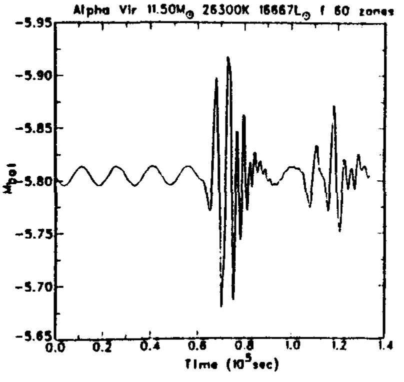

Figure 5

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DISCUSSION

PERCY: Can Odell remind us of the Q value for α Virginis?

JERZYKIEWICZ: If you use photometric indices you get the radial second overtone. If you use data by Code and others you get close to the fundamental.

A. COX: If you use the numbers given by Odell you will come up with radial first overtone. The radial fundamental has a better chance of being excited by semiconvective mixing because its eigenfunction is large in that zone. If you change the temperature within the observational error bar, you can put it in a fundamental mode. That means the observed temperature is too low by over 1500 K.

ODELL: I am surprised that the result of changing the temperature by that little is a different mode.

A. COX: The period goes like T^{-3}, and if you increase T_e by 7-8% you get a 25% reduction in period.

J. COX: What theory of time dependent convection are you using?

A. COX: The convective velocity is incremented by $\Delta\rho/\rho$ (the bouancy) times the acceleration due to gravity times the hydrodynamic time step. It is not allowed to change faster than that nor is it frozen-in with no change. Otherwise, the convective luminosity formulas are given by Cox and Giuli.

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