CORRESPONDENCE.

ON THE D AND N FORMULA FOR A TERM ASSURANCE.

To the Editor of the Journal of the Institute of Actuaries.

SIR,—I have not found in any of our treatises the application of the ordinary Commutation formula, when expressed in terms of Columns N and D, to *term* assurances, and would therefore point out its convenience in the computation of foreign rates, those for joint lives and others, the tables for which may be but partially complete.

The annual premium for n years' assurance on x being $\frac{M_x - M_{x+n}}{N_{x-1} - N_{x+n-1}}$ is convertible to $\frac{D_x - D_{x+n}}{N_{x-1} - N_{x+n-1}} - (1-v)$*; calculation by the second being little more laborious than by the first.

Subject to the following modification, the formula is amenable to the practical application treated of by Mr. Peter Gray in the *Journal*, vol. x. p. 118.

I. Were the 1 assured made due at the beginning of the first instead of at the end of the last year of the transaction, the equivalent payment for it would be the "term" annuity due which 1 would purchase, say $\frac{D_x}{N_{x-1} - N_{x+n-1}}$; but

II, this not being so, from each payment falls to be deducted the difference between 1 and its present value due a year hence, $(1-v)$; and finally,

III, the stipulated 1 not being payable at all should the assured be alive at the end of the term (n years), the annual premium for such an endowment, $\frac{D_{x+n}}{N_{x-1} - N_{x+n-1}}$, has likewise to be subtracted from the foregoing, the complete formula being the result.

I am, Sir,
Your most obedient servant,

H. AMBROSE SMITH.

* This form of expressing such a premium was one of a number contributed by myself to the *Journal* in January 1859 (vol. viii, p. 117), and is referred to now to correct a misprint occurring in that communication. The *single* premium for the benefit in question (Example 5) should be

$$1 - \frac{(1-v)(N_{x-1,y-1} - N_{x+n-1,y+n-1})}{D_{x,y}}.$$

2 *King William Street, E.C., London,*
11 *March* 1875.

ON THE APPROXIMATE VALUE OF A COMPLETE ANNUITY PAYABLE BY INSTALMENTS.

To the Editor of the Journal of the Institute of Actuaries.

SIR,—In his report on and valuation for the Madras Medical Fund in 1840, the late Mr. Griffith Davies gave a very simple method for

L 2

the addition to be made to the value of an annuity payable yearly and ceasing with the last payment, when payable half-yearly and continued up to the date of death; and as I am not aware that any method on the same principle has been given when the annuity is payable any number of times in the year, will you kindly allow me space in your columns for the following formula, as expressing such addition to be made.

Let the annuity be payable at n equal intervals in the year.

Then it is manifest that, on account of the annuity being payable at the end of every $\frac{1}{n}$ th part of a year, the annuitant would gain in interest alone every year $\frac{i}{n}\cdot\frac{1}{n}(\overline{n-1}+\overline{n-2}+\ldots\ldots+1)=\frac{n-1}{2n}\cdot i$. In addition to this, the sum of the chances of the annuitant receiving the 1st, 2nd, &c., $(n-1)$th payments of the annuity $=\frac{1}{n}(\overline{n-1}+\overline{n-2}+\ldots\ldots+1)=\frac{n-1}{2}$, and the value of such payments $=\frac{n-1}{2n}$, which added to the interest above obtained is equivalent to a sum in present money of

$$\frac{n-1}{2n} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (a)$$

Again, $\frac{1}{2n}$ denotes the fraction of a year, which the annuitant fails to survive to complete the half year, and therefore $\frac{1}{2n}$ will denote that fraction of the annuity, which the annuitant's representatives will be entitled to at end of the half year. Now, $\dfrac{1-ia_x}{1+\frac{i}{2}}$ is the present value of £1 to be received in the middle of the year in which the life fails;—consequently, the present value of this residuary portion of the annuity is

$$=\frac{1}{2n}\,\frac{1-ia_x}{1+\frac{i}{2}}=\frac{1-ia_x}{n(2+i)} \quad . \quad . \quad . \quad . \quad (\beta)$$

The sum of a and β is the total addition required to be made, and coincides with the value given by Mr. Milne (*vide* vol. i, arts. 484 and 496).

I remain, Sir,
Your obedient servant,

28 *Notting Hill Square, W.* HENRY HOSKINS.
18 *February* 1875.