Hill's Equation, by W. Magnus and S. Winkler. Interscience, New York, 1966. viii + 127 pages. $8.50.

It is surprising that a topic that belongs to the classical theory of differential equations should still be of great current interest. The bibliography shows that about half the citations date since the end of World War II. Furthermore a perusal of this work indicates that there are still numerous open questions regarding Hill's equation.

Basically Hill's equation is a second order, linear, formally self-adjoint, homogeneous differential equation with periodic coefficients. It is named after the American astronomer G. W. Hill, who encountered equations of this type in his studies of the motion of the lunar perigee. His work was first published in 1877. Subsequently Lyapunov developed the theory considerably further. But his work remained relatively unknown until it appeared in a French translation in 1907. An English translation has been promised for the near future.

What makes Hill's equation interesting to workers in many branches of mathematics are the many applications, in addition to the purely mathematical questions that are raised. Applications have been found in connection with parametric amplifiers, quantum mechanics, linear accelerators and stability theory of oscillations in non-linear periodic systems. The most famous special case of Hill's equation, namely the Mathieu equation has many applications and an enormous literature devoted to it already.

In the past two outstanding survey articles by Krein and Starzinskii have appeared, devoted to this subject. Both are available in English translation. The book under review, although quite complete and self-contained, presents a wealth of material without serious overlap with the above survey articles or other material available in the book literature. There are two parts in the present book. The first one consists of two chapters and is primarily devoted to the general theory. The second part consists of five chapters devoted to more specific results. It would have been impossible to provide complete proofs for all the theorems cited. But in every case where a theorem is stated without proof a literature reference is provided.

The primary focus of Magnus and some of his co-workers has been the structure of the discriminant of Hill's equation. This is a function of a parameter that generally appears in the equation and is essentially an eigenvalue parameter. A full knowledge of the discriminant tells one immediately for which values of the parameter $\lambda$ periodic solutions exist, and also whether for any particular value of $\lambda$ the solutions are stable or unstable. One of the big unknown questions regarding the discriminant is the following. To what extent does a full knowledge of the discriminant allow one to reconstruct the differential equation? Some theorems that pertain to particular types of discriminants due to Borg and Hochstadt are discussed. These in turn are intimately related to the nature of the stability intervals to which a full chapter is devoted. Many asymptotic results due to Borg are discussed in detail.

Magnus has made extensive studies of the discriminant, and in particular its dependence on the Fourier coefficients of the periodic coefficient in Hill's equation. These are summarized and discussed fully. Another chapter is devoted to the general coexistence problem. This concerns itself with the questions of when can a Hill's equation have two linearly independent periodic solutions. For the Mathieu equation for example it is known that coexistence cannot arise, unless the equation degenerates to a harmonic equation. A number of classes of Hill equations are discussed, where either partial or complete answers can be provided. A final chapter is devoted to a number of special Hill equations for which detailed information is available.

This book provides a wealth of information regarding Hill's equation, most of which was up to now only available in the periodical literature. The authors promise another volume that will be devoted to the many interesting applications of Hill's equation in the physical sciences. It is to be hoped that we will not be kept waiting long for this companion volume.

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Handbook of Laplace transformation, by Floyd E. Nixon. Prentice-Hall, Applied Mathematics series. Englewood Cliffs, N.J., 1965. xii + 260 pages, second edition.

According to the preface 'this book is intended as a practical guide or reference for those already familiar with the Laplace transform and a self-study book for those who want to learn the Laplace transformation.'

The book consists of 8 chapters.

Chapter 1 deals with determinants and determinant manipulation techniques. Chapters 2, 3 and 4 give root finding methods and frequently used identities including the derivation of many important Laplace transform theorems. Chapter 5 contains a newly added extensive discussion on recurring wave forms which evolves the theory required for understanding practical design problems and mechanical vibrations due to inputs and electronic filtering wave forms. In chapter 6 selected Laplace transforms examples are given. Finally in chapters 7 and 8, tables are given for Laplace transform operations and Laplace transform pairs.

The book should be very useful for Engineers and Physicists.

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