Les fonctions combinatoires et les isols, by J. C. E. Dekker. Paris, 1966. Collection de logique mathematique, série A No. 22. Gauthier-Villars. 74 pages.

This booklet is a report on the work by Myhill and Nerode on combinatorial functions (see below) and on the work by Dekker on isols (see below). It is well written and easy to read for a non-expert; only the second part requires some rudimentary knowledge of recursive functions, at least the definition.

Let $f: N \to N$ be a function from the set of natural numbers (≥ 0) into itself; then there exists a unique sequence of integers $c_0, c_1 \ldots$ such that

$$f(n) = \Sigma_{i=0}^{n} c_i \binom{n}{i} .$$

This is proved in a fairly complicated way on page 18, using a construction of the c_i by recurrence. The reviewer wishes to point out that, as a simple exercise in the umbral notation of Lucas, one may show that for each $f: N \to Z$ there exists a unique $g: N \to Z$ such that

$$f(n) = \Sigma_{i=1}^{n} (-1)^i g(i) \binom{n}{i} ,$$

to wit

$$g(i) = \Sigma_{n=0}^{i} (-1)^n f(n) \binom{i}{n} .$$

The function $f: N \to N$ is called combinatorial if and only if all $c_i \geq 0$. In the present treatment this a theorem, not the definition. The latter is rather sophisticated and depends on the possibility of extending f to a function from the set of all subsets of N into itself which satisfies certain conditions.

Two sets A and B of natural numbers are said to be recursively equivalent if there is a partial recursive function whose domain contains A and which establishes a one-one correspondence of A onto B. Assume that A has no recursively enumerable subset, then the class of all B recursively equivalent to A is called an isol. In particular, a finite number n, identified with the set $\{0, 1, \ldots, n-1\}$, is an isol. Addition and multiplication of naturals may be extended to isols. The resulting system satisfies the usual laws and may be embedded into a ring, the so-called ring of isolic integers.

Every combinatorial function may be extended canonically to a function of the set of isols into itself. Aside from this result, it is not clear to the reviewer why the two topics are treated under one cover.

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Truth functions and the problem of their realization by two-terminal graphs, by A. Ádám. Akadémiai Kiadó, Budapest, 1968. 206 pages. U.S. $7.80.

This book consists of two parts, the first being a survey of the mathematical theory of Boolean functions. Chapter 1 introduces basic definitions, various normal forms, prime implicants, and symmetric functions. In an appendix to Chapter 1, an unpublished and non-trivial (combinatorial) theorem of Bakos is given. (This theorem yields a uniform construction of Gray codes as a consequence.) Chapter 2 gives the standard theory of minimality. Applications to special cases such as monotonic or symmetric functions are given. Chapter 3 discusses inter-relationships between conjunctive and disjunctive normal forms. Chapter 4 deals with functional completeness and the Post-Yablonsky theorem is proven. Some applications to finite automata are given. Chapter 5 is concerned with the decomposition of truth functions.

Chapter 6 on numerical problems is particularly good. Groups are used to classify truth functions. A form of Pólya's theorem is given and the work of Pólya, Slepian and the reviewer is presented. A number of special cases are worked out including some of the results of Povarov. Chapter 7 on linearly separable functions gives a number of characterizations.

In summary, Part I gives a rather complete survey of the mathematical properties of truth functions.

Part II relates these mathematical properties to the graphs or networks which realize Boolean functions. Chapter 8 is concerned with two terminal graphs. Particular attention is given to series-parallel graphs and both Trakhtenbrot's and Ádám's theorems are proven. Chapter 9 is devoted to several kinds of realizations, particularly repetition-free realizations in which the assignment of variables to edges is one-to-one. Various problems concerning these realizations are stated and proven. Chapter 10 is devoted to optimal realizations. Unfortunately, little is said about this interesting topic and the book closes with some open problems.

The book is a fine vehicle for the mathematician or computer scientist who wants a concise and accurate survey of switching theory. The book could even be used as a graduate text, although it was not written for this purpose and does not contain problems. The style is excellent and the arguments are clear. The nomenclature generally follows western usage. It is a valuable addition to the literature as it makes available a number of results which had not been published before in English.

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Combinatorial identities, by John Riordan. Wiley, New York, 1968. xii + 256 pages. $15.00.

The title is taken to include "all identities phrased in terms of recognized combinatorial entities, such as the old-fashioned permutations, combinations, variations, and partitions, and the numbers arising in their enumeration" or, more generally, "any identity with combinatorial significance". This book is not a dictionary of identities, in fact the reader is explicitly warned "that he may not expect his identity of the moment, however fascinating it seems, to be listed and verified"; it is instead a survey of certain methods for finding and verifying combinatorial identities.

Many identities can be obtained by iterating in different ways the basic recurrence for binomial coefficients