At the crossroads of astrophysics and cosmology: Period–luminosity relations in the 2020s
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Theoretical Stellar Pulsation Physics

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Abstract. Pulsating stars play a crucial role in the calibration of the cosmic distance scale as well as in tracing the properties of the associated stellar populations. In the era of large observational surveys and precise astrometric missions, it is crucial to rely on accurate stellar pulsation models able to predict the observed behaviors for different physical assumptions. Indeed, the relations currently used in the literature to derive individual and mean distances of mainly radially pulsating stars such as Cepheids and RR Lyrae are well physically understood, but are also known to depend on a number of often unknown parameters. Recent extensive sets of stellar pulsation models developed by various authors show how variations in the physical assumptions can affect the theoretical prediction of the instability strip boundaries, the morphology and amplitude of light and radial velocity curves, and the consequent Period-Luminosity, Period-Luminosity-Color and Period-Wesenheit relations. These aspects are discussed in the framework of current open problems in the field of classical pulsating stars.

Keywords. pulsating stars, distance scale, stellar pulsation, stellar parameters

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1. Introduction

Pulsating stars play a very important role in stellar astrophysics and not only there. They are easily identified thanks to their variability. Moreover, their main pulsation observables, namely the period and the amplitude of the magnitude variation, are unaffected by distance and reddening uncertainties. On the other hand, the acoustic nature of the oscillation implies a fundamental relation connecting the oscillation period to the mean stellar density, which in turn implies relations among the period, the luminosity, the mass, and the effective temperature, with a non-negligible dependence on chemical composition. This is the fundamental relation between pulsation and evolutionary stellar parameters (Bono et al. 2000a; De Somma et al. 2021) that is the basis of the well known Period-Luminosity (PL) relation obeyed by several classes of pulsating stars and, in turn, of their role as standard candles. At the same time the relation between the pulsation period and evolutionary properties allows us to test stellar evolution predictions through independent pulsational constraints. In this review, we discuss the physics on the basis of stellar pulsation and of PL relations (in Section 2). In Section 3 we illustrate the state of the art of the modeling of pulsating stars, whereas in Sections 4 and 5 we still discuss some recent results for Cepheids and RR Lyrae, respectively. The Conclusions and some future developments are finally summarized in Section 6.

2. Stellar pulsation physics

Since the pioneering investigations by Eddington (1918), Zhevakin (1963) and Cox & Hodson (1978), the physical basis of stellar pulsation has been progressively clarified. The evidence that all the “classical” pulsating stars lie within the same almost vertical region in the color-magnitude diagram (the instability strip) suggested the efficiency of driving pulsation mechanisms associated with the surface parameters and, in particular, with the effective temperature of the stars. This driving phenomenon was identified with a sort of valve mechanism that favoured an absorption of energy during the contraction phase and the release of this energy excess during the subsequent expansion phase in the form of mechanical work. This valve effect was early associated with variations of the stellar opacity (the so-called κ mechanism) and of the adiabatic exponents (the so-called γ mechanism) in the ionization regions of the most abundant elements of stellar envelopes, namely H, He and He^+ (Zhevakin 1963). It is then crucial to adopt sound physical ingredients, in particular accurate opacity tables and equation of state, to properly model pulsating stars.

2.1. *The physical basis of the instability strip*

As the driving pulsation mechanisms are associated with variations of the opacity and adiabatic exponents in the ionization regions of the most abundant chemical elements in the stellar envelope, in order to be efficient, they require that the effective temperature of the star not be too high. Indeed, for each stellar mass and luminosity level, only when the effective temperature is below a critical value, the ionization regions are deep enough in the stellar envelope to allow pulsation to occur, because the driving zones prevail over the damping ones. This is the explanation for the existence of a blue edge of the instability strip for each selected pulsation mode. On the other hand, the existence of a red edge is due to the increasing efficiency of convection as the effective temperature decreases. In fact, the convective phenomenon tends to quench pulsation, by making the κ and γ mechanisms less and less active. The effect of the coupling between pulsation and convection has been verified by investigating how the predicted light and radial velocity curves, as well as the shape and the width of the instability strip vary, increasing the efficiency of super-adiabatic convection. Several authors have shown that the pulsation amplitudes decrease, the blue edge tends to become mildly redder and the red edge significantly bluer (see e.g. Di Criscienzo et al. 2004; Fiorentino et al. 2007; Paxton et al. 2019; De Somma et al. 2020, 2021). These are all effects of the increased opacity and convective efficiency in the stellar envelope.

2.2. *The Physics of PL relations*

The combination of the period-mean density relation with the Stefan-Boltzmann law implies a relation connecting the period to the mass, the luminosity and the effective temperature of the star, for each selected chemical composition. This is the so-called van Albada & Baker relation, derived for the first time by these authors for RR Lyrae stars on the basis of their linear non-adiabatic pulsation models (van Albada & Baker 1971).

In the case of RR Lyrae stars, no PL relation exists in the optical bands, as these pulsating stars belong to the Horizontal Branch (HB) phase (see Catelan et al. 2004), with colors and periods varying at a roughly fixed (but metallicity-dependent) magnitude level. However, due to the dependence of Near-Infrared (NIR) bolometric corrections to the effective temperature for RR Lyrae models, accurate NIR (metal-dependent) PL relations are predicted, confirming observations (since Longmore et al. 1990). On the other hand, in the case of Classical Cepheids a mass-luminosity (-chemical composition) relation is predicted by stellar evolution models (see e.g. Chiosi et al. 1993; Bono et al. 2000a, and references therein), so that the mass can be removed in the previous van Albada & Baker relation, obtaining a Period-Luminosity-Temperature or a Period-Luminosity-Color (PLC) relation, that is foreseen to be obeyed by each individual Cepheid. The traditionally used Period-Luminosity (PL) relation of Classical Cepheids is the projection of the PLC relation onto the PL plane and is statistically derived as the average period at each fixed luminosity level. On this basis, reliable PL relations can be derived only if a statistically significant sample of pulsators is available and reflect the topology of the instability strip, in particular in the optical bands. As also shown by the observations (see e.g. Freedman & Madore 2010, and references therein), the intrinsic dispersion of PL relations decreases from the optical to the NIR bands; the metallicity and nonlinearity effects are predicted to do the same (see e.g. Caputo et al. 2000; Marconi et al. 2005). There are two possible approaches to reduce the PL systematics: i) to work in the NIR or Mid Infrared (MIR) filters (e.g. with the *James Webb Space Telescope*); ii) to adopt Period-Wesenheit (PW) relations (since Madore 1982) that are reddening-free formulations of PL relations, as they include a color term whose coefficient is exactly the ratio between the total and the selective extinction. The PW relations have the advantage of being insensitive to reddening and of partially correcting for the color extension of the strip. At the same time, they have the disadvantages of differing from true PLC relations and of relying on the assumption of an extinction law (e.g. Cardelli et al. 1989).

3. The state of the art of the modelling of pulsating stars

Several efforts have been made in the last few decades to model the observed properties of Cepheids and RR Lyrae among the various classes of pulsating stars. Recent theoretical studies of Cepheids and RR Lyrae are mainly based on the following three approaches: i) Linear non-adiabatic hydrodynamical models; ii) Nonlinear convective hydrodynamical 1D models; iii) Multidimensional hydrodynamical simulations. In linear non-adiabatic hydrodynamical models, non-adiabatic effects related to the driving mechanisms are taken into account but the hydrodynamic equations describing the pulsation phenomenon are linearized. In this scenario, only the periods and the instability strip edges can be estimated. Among the most recent results obtained from this kind of pulsation modeling, we can quote the prediction of Cepheid instability strip boundaries and period-age relations as a function of both metallicity and the efficiency of rotation, by Anderson et al. (2016). More recently, interesting results for the prediction of double mode RR Lyrae periods and period ratios have been obtained by Kovacs & Karamiqucham (2021) on the basis of their extensive sets of linear nonadiabatic models. In nonlinear convective hydrodynamical 1D models the hydrodynamic equations describing the pulsation phenomenon are not linearized, so that not only the periods and the instability strip edges but also the pulsation amplitudes (full amplitude variation of all the relevant quantities along the pulsation cycle) can be predicted. Several authors developed nonlinear convective pulsation models of Cepheids and RR Lyrae (Gehmeyr 1992; Bono & Stellingwerf 1994; Yecho et al. 1997; Kolláth et al. 2002; Bono et al. 1999; Szabó et al. 2004; Smolec & Moskalik 2008; Paxton et al. 2019). The bolometric light curves are then usually transformed into a variety of photometric filters through the adoption of model atmospheres (e.g. Castelli & Kurucz 2003), so that mean magnitudes and colors can also be derived. These are used, together with the pulsation periods to obtain theoretical PL, PLC and PW relations. The derived accurate topology of the instability strip with the blue and, in particular, the red edge location driven by the coupling between pulsation and convection allows us to derive synthetic PL relations.These are obtained by populating the region between the predicted boundaries with an assumed mass distribution (see e.g. Caputo et al. 2000; Fiorentino et al. 2007, for details). Figure 1 shows synthetic PL relations in the *HST* filters F555W, F184W and F160W as derived on the basis of the nonlinear convective pulsation models by De Somma et al. (2022) with metallicity varying from $Z = 0.004$ to $Z = 0.03$ (see labels). In multidimensional hydrodynamic simulations, the 1D assumption is released and the pulsation-convection coupling is treated in 2 or 3 dimensions (see e.g. Mundprecht et al. 2013, 2015, for a 2D approach to pulsation). These simulations provide more realistic predictions and useful

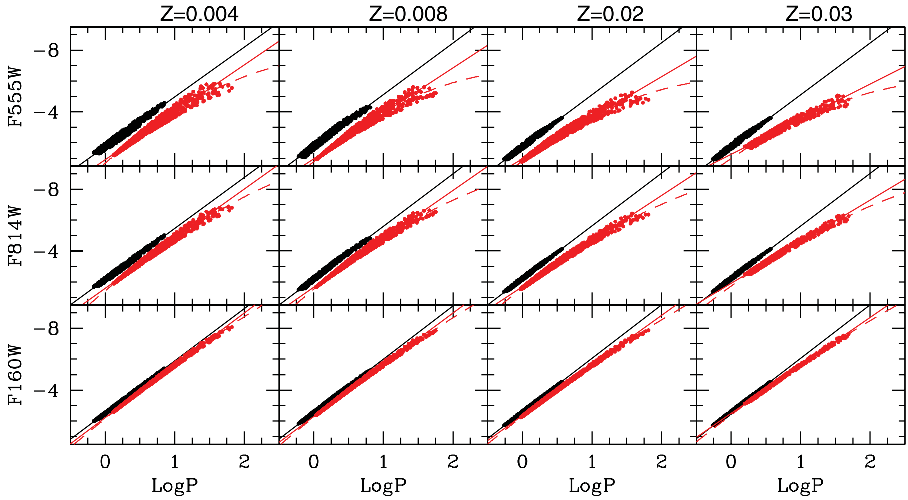

Figure 1. An example of synthetic PL relations for F-mode (red symbols) and FO-mode (black symbols) Cepheids based on nonlinear convective pulsation models for the labelled metallicities.

guidelines for developing descriptions of convection to be applied in 1D modeling (see also Deupree 2021, and reference therein)

4. Recent results for Classical Cepheids

The modelling of Classical Cepheids through non linear convective pulsation codes provides useful information for understanding the role of these pulsators as distance indicators and relatively young stellar populsation tracers. The recent nonlinear convective pulsation model results obtained through the radial stellar pulsation (RSP) module in the MESA code (Paxton et al. 2019), provided a new atlas of light and pulsation velocity curves, as well as period-amplitude diagrams, for both Cepheids and RR Lyrae, investigating the dependence of metallicity at fixed convective model parameters and of various model assumptions at fixed metallicity. Extensive model grids of Classical Cepheids have also been computed using an updated version of the original Stellingwerf's code (see Bono & Stellingwerf 1994; Bono et al. 1999, for details) for various assumptions about the efficiency of super-adiabatic convection, the mass-luminosity (ML) relation and the metal content (see De Somma et al. 2020, 2022, for details). In the quoted hydrocodes, the full amplitude behaviour of Cepheid models is obtained by integrating the pulsation equations together with those describing heat transfer and the coupling between pulsation and convection till the stability of the pulsation cycle. Then accurate predictions for the instability strip boundaries, and the variations of all relevant quantities along the pulsation cycle, can be derived for each assumed pulsation mode and combination of stellar parameters.

4.1. *Classical Cepheids as distance indicators*

Figure 2 shows the predicted instability strip as a function of the assumed chemical composition. Confirming previous results by Bono et al. (1999), we notice that the theoretical instability strip gets redder as the metallicity increases. This trend directly affects the predicted PL relations, especially in the optical bands, in the sense that more metal-rich Cepheids are expected to be fainter at a fixed period. On the other hand, the predicted dependence of the zero point of PW relations on metallicity is in the opposite

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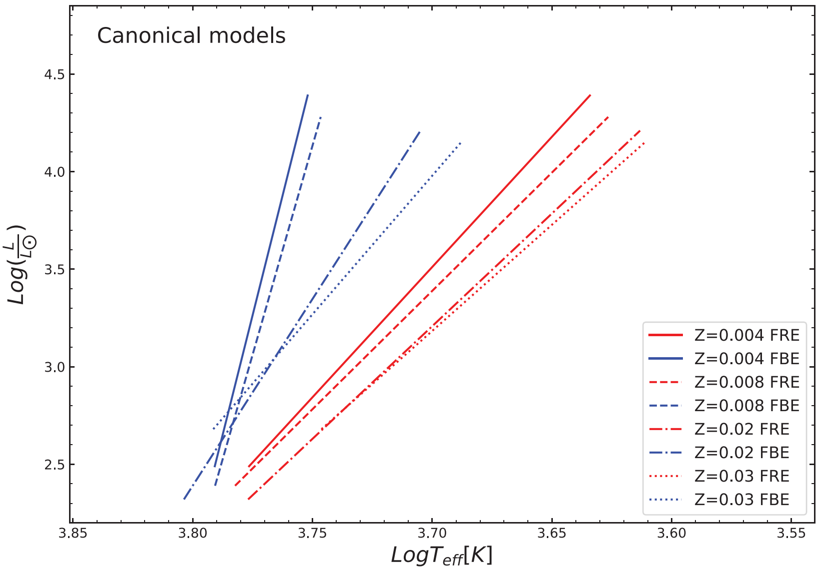

Figure 2. The predicted instability strip as a function of the assumed metallicity (see labels).

sense. Indeed fixing the period and the color, metal-rich Cepheids are predicted to be brighter (see e.g. Tables 19-22 in De Somma et al. 2022). In particular, in the *Gaia* filters, metal-dependent PW relations point towards a metallicity effect on the zero point varying from ~ -0.1 dex to ~ -0.2 dex for the Fundamental (F)-mode relations and from ~ -0.1 dex to ~ -0.3 dex for the First Overtone (FO)-mode relations. By applying these PWZ relations to a sample of Galactic Cepheids in the *Gaia* Data Release 3 (DR3) with available spectroscopic metal abundance measurements (see De Somma et al. 2022, for details), accurate theoretical parallaxes can be obtained. These results were directly compared with *Gaia* DR3 values. The agreement was found to be good, pointing towards a zero point parallax offset in agreement with Riess et al. (2021) results but dependent on the assumed convective efficiency and ML relation. In particular, if the effect of the assumed mixing length parameter is almost negligible, the effect of a brighter ML relation by 0.2 dex implies that the corresponding PWZ relations predict shorter distances by about 5 %. In principle a percentage variation in the inferred distance scale directly reflect in an equivalent variation in the resulting Hubble constant, thanks to the Hubble law. This point is interesting in view of improving our understanding of the so called Hubble constant tension, between early universe measurements and determinations and the values inferred from the extragalactic distance scale (see e.g. Verde et al. 2019; Riess et al. 2022, for detail).

4.2. *Classical Cepheids as age indicators*

Thanks to the relations between the stellar age and mass and between the stellar mass and luminosity, combined with the PL relation, accurate period-age (PA) and period-age-color (PAC) relations can be derived for Classical Cepheids, combining stellar evolution predictions for intermediate mass evolutionary tracks crossing the modeled instability strip (see e.g. Bono et al. 2005; Marconi et al. 2006; Anderson et al. 2016; De Somma et al. 2020, and references therein) with the Period-Mass-Luminosity-Temperature relations based on pulsation models. Using an extensive set of stellar evolution and nonlinear convective pulsation models for several chemical compositions, De Somma et al. (2021)

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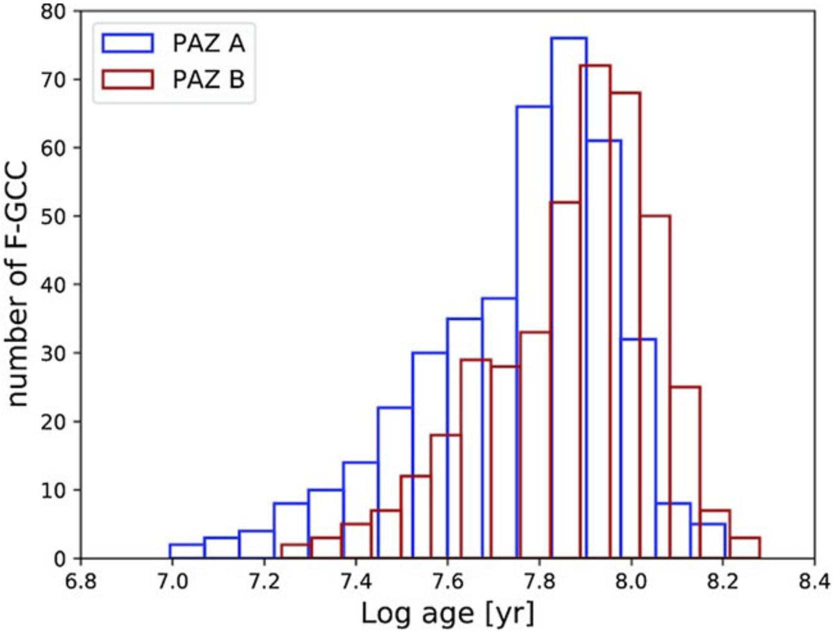

Figure 3. Inferred age distribution obtained by applying the canonical (A) and non-canonical (B) PAZ relations to the quoted *Gaia* F-mode Galactic Cepheid sample. Adapted from De Somma et al. (2021).

derived accurate metal-dependent PA (PAZ) and PAC (PACZ) relations (the latter in the *Gaia* filters). These relations were applied to the same sample of Galactic Cepheids in *Gaia* DR3 mentioned above and the theoretical age distribution was inferred for both F-mode and FO-mode pulsators (see De Somma et al. 2021, for details). The effect of the assumed efficiency of super-adiabatic convection is negligible, whereas a moderate overluminosity with respect to the canonical ML relation, implies that moderately older ages are predicted from PAZ and PACZ relations (see De Somma et al. 2021, for details). For example, in Figure 3 we show the inferred age distribution obtained by applying the canonical (A) and non-canonical (B) PAZ relations to the quoted *Gaia* F-mode Galactic Cepheid sample.

4.3. *The model fitting of Cepheid light and radial velocity curves*

One of the advantages of nonlinear convective hydrodynamic codes is the capability to predict the variation of luminosity, radius, effective temperature, gravity and radial velocity along a pulsation cycle. These theoretical variation curves can be directly compared with their observational counterparts. The first application of this model fitting technique was performed by Wood et al. (1997) for a Cepheid in the Large Magellanic Clouds. Several applications followed both in the Milky Way (see e.g. Natale et al. 2008; Gaia Collaboration et al. 2017) and in the Magellanic Clouds (see e.g. Bono et al. 2002; Keller & Wood 2002, 2006; Marconi et al. 2013a,b, 2017; Ragosta et al. 2019). In Figure 4 an example of model fitting of the observed light and radial velocity curves is shown for a few Galactic Cepheids in the *Gaia* DR3 database. Theoretical light curves were fitted to the observational ones, by shifting the former along the magnitude axis, in order to match the mean observational magnitude and by searching for the best-fit model matching the pulsation periods, amplitude and light curve morphology. On the other hand, theoretical pulsational velocity curves were fitted to observational radial velocities, by shifting the

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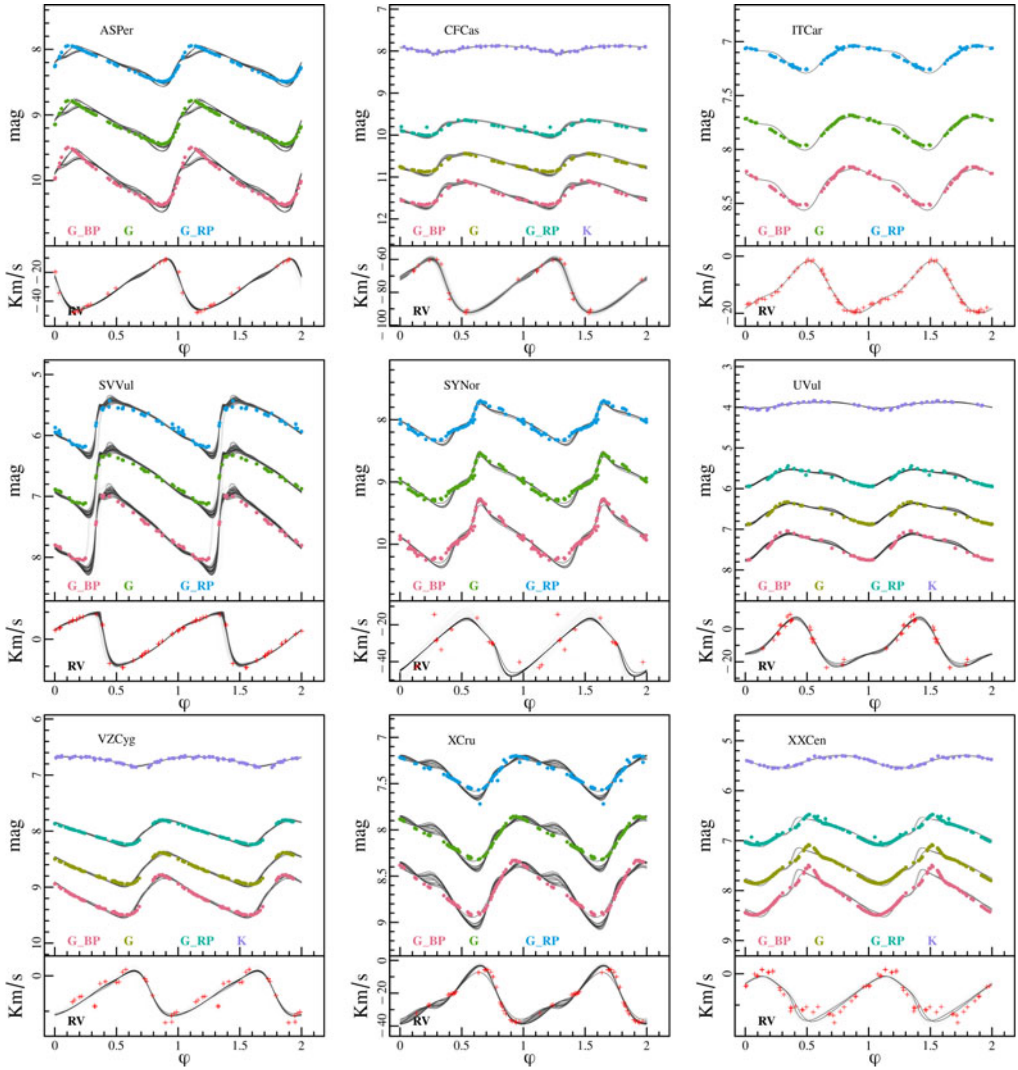

Figure 4. Model fitting of the observed light and radial velocity curves is shown for a few Galactic Cepheids in the *Gaia* DR3 database.

former along the velocity axis, in order to fit the mean observational velocity, and by stretching (through the p-factor) them in order to match the observational peak-to-peak radial velocity amplitude. A set of 100 bootstrap simulations was performed to compute the errors on the fitted parameters. In particular, the observational light and radial velocity curves were resampled (allowing data point repetitions) and the fitting procedure was repeated for every simulation. A statistical analysis of the obtained parameters of the fit allowed us to give an estimate of their errors. This investigation is part of a more complete analysis involving a few tens of Galactic Cepheids in the *Gaia* database (Molinaro et al. in prep).

From the model fitting, the individual distances and the intrinsic stellar parameters can be derived. In Figure 5 a comparison between the theoretical parallaxes inferred from the model fitting shown above, and the *Gaia* DR3 values, is displayed for the investigated stars. The good agreement obtained seems to support the predictive capability

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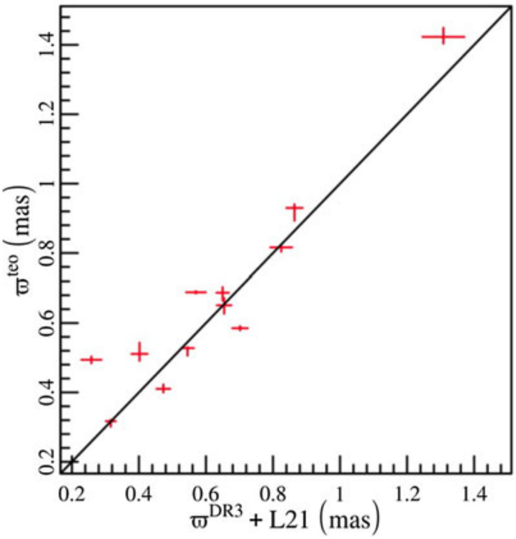

Figure 5. Comparison between the theoretical parallaxes inferred from the model fitting method aapplied to a subset of the considered *Gaia* Cepheids and the corresponding *Gaia* DR3 values.

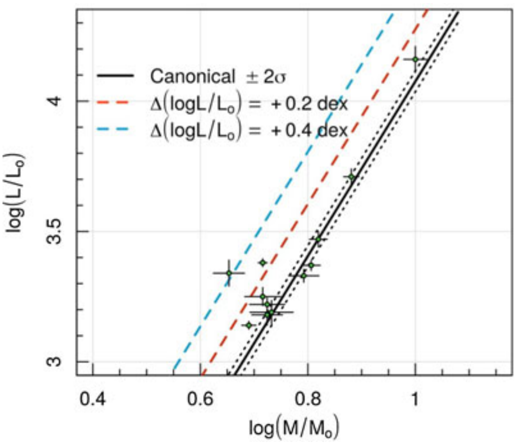

Figure 6. The inferred ML relation obtained from the model fitting of the investigated Cepheids, compared with the canonical (black solid line), moderately (red dashed line) and fully (cyan dashed line) non-canonical ML relations.

of current theoretical scenario. Similarly, in Figure 6, we show the inferred ML relation obtained from the model fitting method compared with the canonical (black solid line), moderately (red dashed line) and fully (cyan dashed line) non-canonical ML relations. From inspection of this plot, we notice that most of the studied stars seems to obey to a mildly non-canonical ML relation, with some of them lying on the canonical (no mass loss, no overshooting, no rotation) theoretical relation.

4.4. *The Hertzsprung progression*

This Hertzsprung progression (HP) phenomenon, discovered about one century ago (Hertzsprung 1926), consists of a relationship between the position of the bump along the

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light (and radial velocity) curves and the pulsation period, occurring in the period range from ~ 6 to ~ 16 d. The bump is observed on the descending branch for periods from ~ 6 to ~ 9 days, close to the true maximum of light/radial velocity curves for periods from ~ 9 to ~ 12 d, and on the rising branch for longer periods. The behaviour of pulsation amplitudes and Fourier parameters R_{21} and Φ_{21} of the bump is often considered to illustrate the phenomenon as these quantities show peculiar minima at the HP center (see Bono et al. 2000b, for details)

In Figure 7 we show an example of light (left panels) and radial pulsation velocity (right panels) curves for models of bump Cepheids varying the effective temperature and the period (see labels) at fixed mass ($M = 6.2 M_{\odot}$) and luminosity ($\log L/L_{\odot} = 3.38$). We notice how the bump phase evolves from the descending to the rising branch and the curve becomes almost flat at the HP center.

As the period of the HP center is expected to depend on the stellar parameters, including the assumed ML relation and chemical composition, the comparison with observed data can provide interesting clues on the intrinsic move of observed bump Cepheids.

At the same time the interpretation of the evidence of the HP phenomenon in bump Cepheids observed by the *Gaia* satellite can be an important benchmark for pulsation models.

In Figure 8 we show the picture adapted from the ESA *Gaia* Image of the Week of May 27, 2022, showing the HP for a sample of *Gaia* Galactic Cepheids. According to this plot, the center of the progression seems to correspond to a period close to 8 days. The circle marks the center of the light curve HP, where the bump and the maximum are more or less at the same brightness level.

In a forthcoming paper (Marconi et al. in prep) we will present a very fine grid (decreased step in effective temperatures and mass) of nonlinear convective pulsation models for bump Cepheids, with metallicities ranging from 0.004 to 0.03, also increasing the helium abundance at fixed solar metallicity and different ML relations. Some preliminary results of this analysis are shown in Figure 9, where models across the HP are shown for the labelled chemical compositions and physical parameters. We notice how the period of the HP center decreases as the metallicity increases, with the $Z = 0.02$ case giving the best agreement with *Gaia* observations (see above).

5. Recent results for RR Lyrae stars

A combination of Horizontal Branch evolutionary predictions and pulsation modelling for central He burning stars is the key to understand the observed behaviours of RR Lyrae stars. As previously noted, RR Lyrae do not obey PL relations in the optical bands but are known to follow strict metal-dependent PL (PLZ) relations in the wavelength range from Cousins R to the Mid-Infrared *Spitzer* filters (see e.g. Marconi et al. 2015; Neeley et al. 2017; Marconi et al. 2022, and references therein). These PLZ relations, including a [Fe/H] term, can be inverted to derive the metallicity distributions of RR Lyrae that either share the same distance or have independently known individual distances. This approach to infer the metal abundance distribution has been applied by several authors both to Galactic Globular Clusters (see the case of ω Cen; Braga et al. 2016) and to satellites of the Milky Way (see e.g. the cases of Sculptor and Eridanus by Martínez-Vázquez et al. 2016, 2021, respectively). Metal-dependent PW relations have also been derived for RR Lyrae, with the minimum metallicity effect corresponding to some combination of optical filters, such as B,V, that is predicted to be metal-insensitive, or g,r (see Marconi et al. 2015, 2022, for details). Moreover, the predicted metallicity dependence of RR Lyrae NIR PL and PW relations is in agreement with observations in Galactic globular clusters (see Bhardwaj et al. 2023). As for the helium content, current models predict an effect on PL and PW relations in the sense that for an enhanced

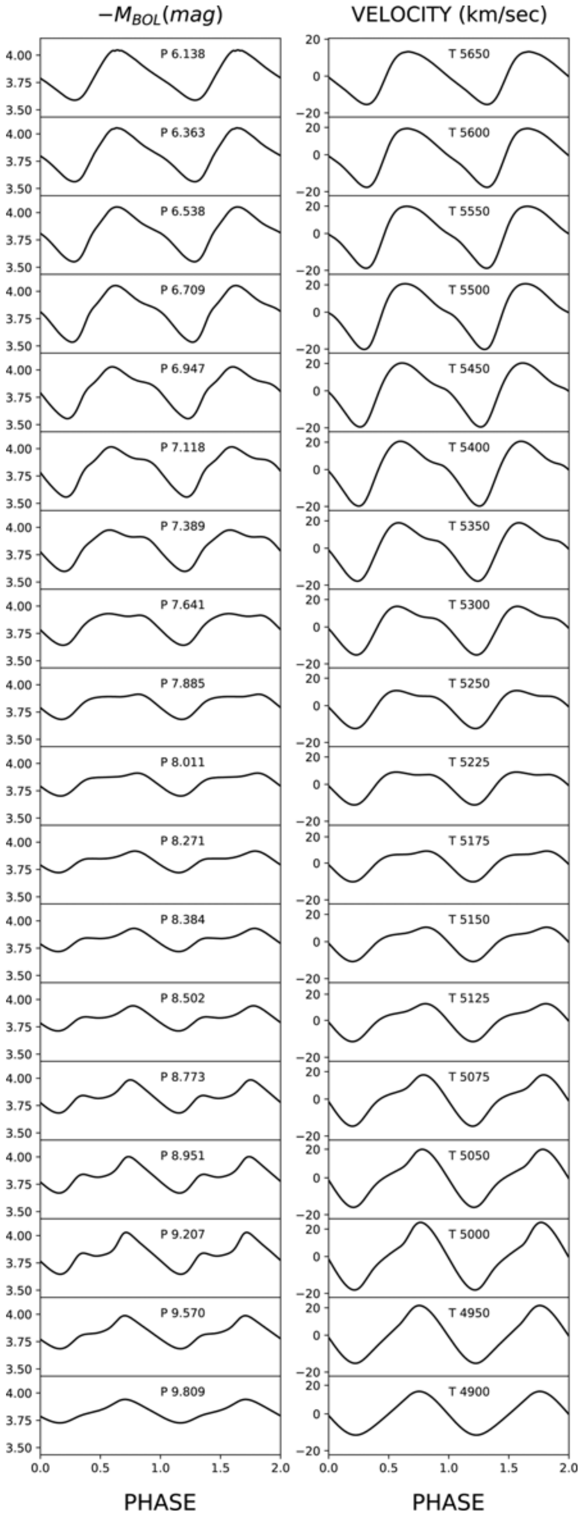

Figure 7. Light (left panels) and radial pulsation velocity (right panels) curves for models of bump Cepheids with $M = 6.2M_{\odot}$, $\log L/L_{\odot} = 3.38$. The effective temperatures and periods are labelled.

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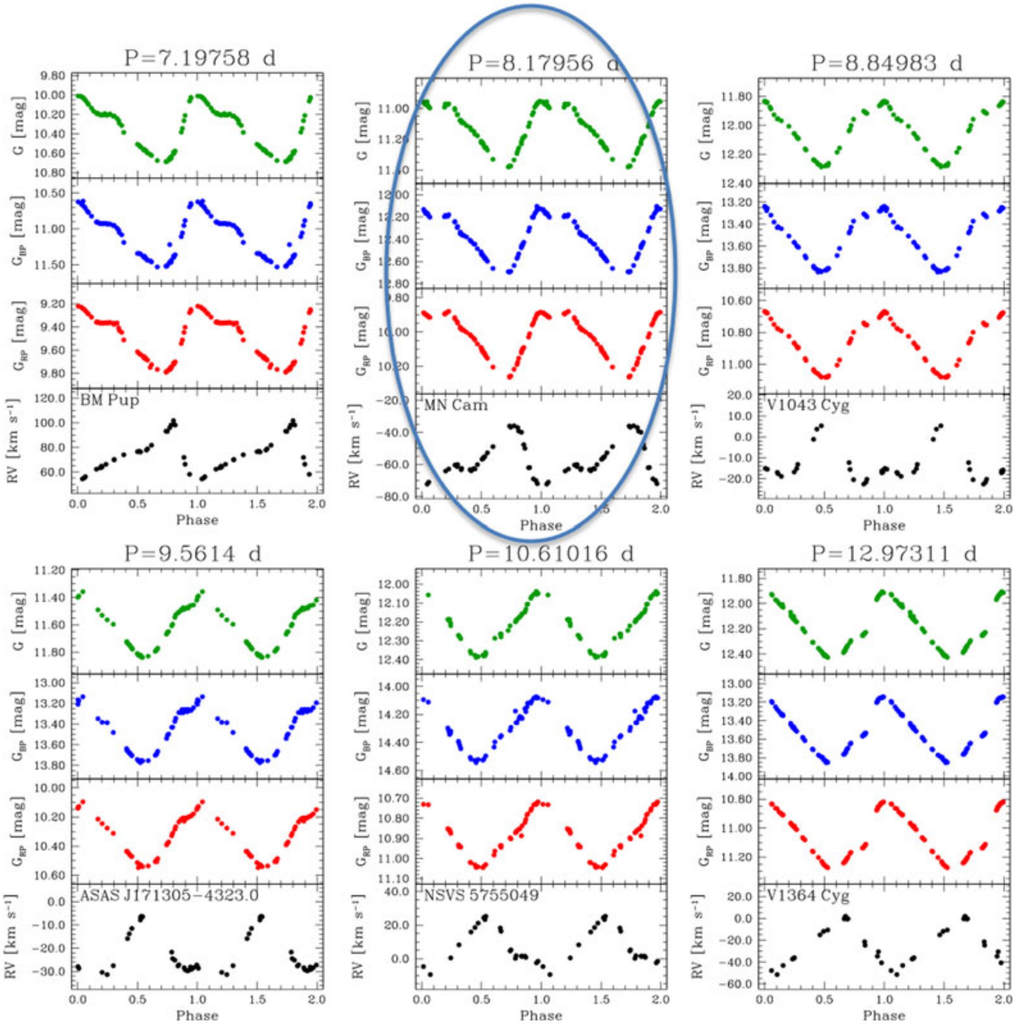

Figure 8. The HP for a subsample of *Gaia* Galactic Cepheids. The plot has been adapted from the ESA *Gaia* Image of the Week of May 27, 2022. The circle marks the center of the light curve HP.

Helium abundance longer periods and brighter magnitudes are expected on the basis of the increased luminosity (see e.g. Marconi et al. 2018, 2021, and references therein) due to the more efficient CNO-burning shell in the Horizontal Branch phase. Predicted parallaxes based on Period-Wesenheit-[Fe/H] relations were also successfully compared with EDR3 *Gaia* values (mean magnitudes from EDR3, [Fe/H] from spectroscopic literature data) for a sample of Galactic RR Lyrae (see Marconi et al. 2021, for details).

5.1. *The Bailey diagram*

Variations in the metal abundance are also expected to affect the distribution of RR Lyrae in period-amplitude planes (the so-called Bailey diagram, see Marconi et al. 2021, 2022, and references therein), in the sense that more metal-rich RR Lyrae are predicted to have shorter periods as an effect of the fainter Horizontal Branch level, and smaller amplitudes as an effect of the increased opacity and convection efficiency. This trend

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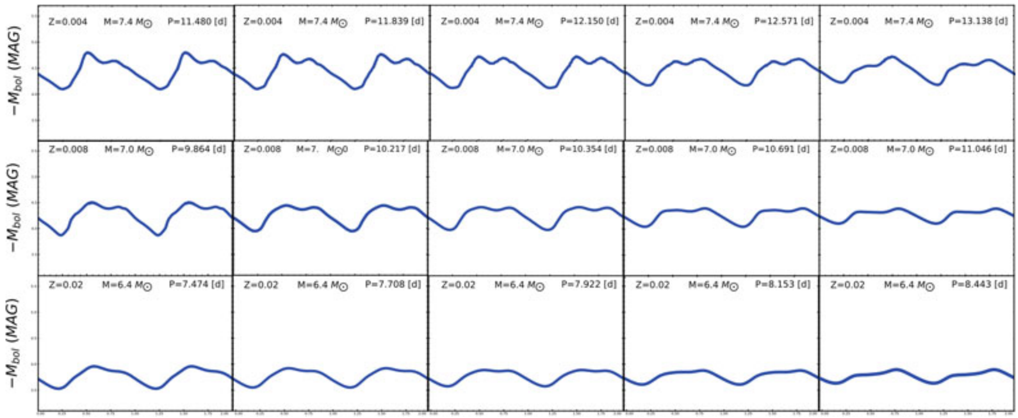

Figure 9. The HP for a subset of computed pulsation models across the HP center for the labelled chemical compositions and physical parameters.

is found to be in excellent agreement with the observational Bailey diagram based on *Gaia* data (see Clementini et al. 2022). In Figure 10 we show the quoted *Gaia* DR3 observational Bailey diagram for Galactic RR Lyrae with a metallicity color-coded scale, compared with theoretical predictions based on current nonlinear convective pulsation models for the labelled metal abundances. Solid lines are for Zero Age Horizontal Branch (ZAHB) models, dashed lines correspond to evolved RR Lyrae by 0.1 dex in $\log L/L_{\odot}$. We notice that the same trend with metallicity is followed by theoretical results and observational data, even if the predicted amplitudes seem slightly higher than the observed ones. This effect can be due to an underestimated convective efficiency in the adopted pulsation models. Indeed increasing the mixing length parameter would reduce the pulsation amplitudes, as widely discussed in Di Criscienzo et al. (2004). However, a fine-tuning in the mixing length parameter is needed in order to simultaneously reproduce the observed instability strip width and pulsation amplitudes.

6. Conclusions and next steps

Classical pulsating stars are important distance indicators and stellar population tracers. Several theoretical efforts have been made to understand the pulsation mechanism and model the observed properties. The physical bases of stellar pulsation, of the existence of the instability strip and of PL and PLC relations are well known. Current nonlinear convective pulsation models are able to predict most of the relevant properties of classical pulsating stars, in particular Cepheids and RR Lyrae, generally in good agreement with the observations. However, model predictions depend on a number of physical and numerical assumptions that need to be constrained through the comparison with independent results. In order to improve current nonlinear convective pulsation models we need to work on three main aspects: i) To improve the input physics and the adopted atmosphere models; ii) To improve the treatment of convection, also taking into account results of 2D and 3D simulations; iii) to compare results of different pulsation codes to cross-calibrate physical and numerical assumptions; iv) to consider still open issues. Concerning the first point, accurate and updated opacity and equation of state tables or analytical formulations have to be adopted, consistently with the physical assumptions of stellar evolution models that provide the input parameters for the static envelope structures. As for the step forward in the treatment of convection, the limitations of current approaches are evident from the failure of the model fitting of observed light curves, e.g. for RR Lyrae models, close to the red edge of the instability strip (see

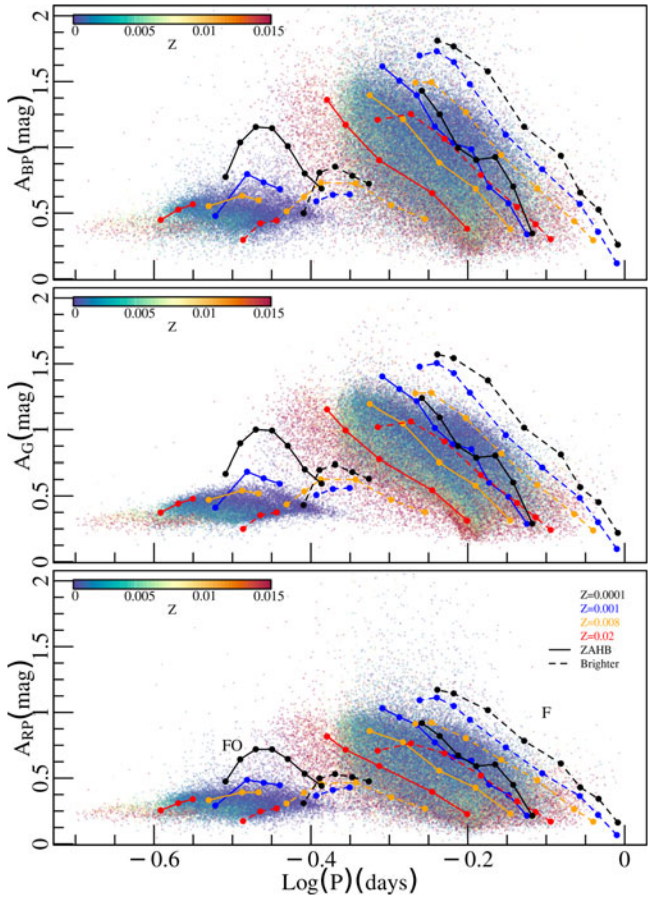

Figure 10. *Gaia* DR3 observational Bailey diagram for Galactic RR Lyrae with a metallicity color-coded scale, compared with theoretical predictions based on current nonlinear convective pulsation models for the labelled metallicities. Solid lines are for ZAHB models, dashed lines correspond to evolved RR Lyrae by 0.1 dex in $\log L/L_{\odot}$.

e.g. Marconi & Degl'Innocenti 2007), in spite of the general success of the model fitting technique. 2D and 3D simulations can provide useful inputs to model in a more realistic way the coupling between pulsation and convection in pulsating stars. Concerning the third aspect, namely the comparison among different model predictions, we think this is an important approach to better constrain the free parameters involved in the various hydrocodes and refer the interested reader to the paper by Kovács et al. (2023) who compare the results of the Budapest-Florida code with the ones obtained from the MESA RSPs module (but see also Smolec & Moskalik 2008). Finally, future models should be able to take into account open issues such as the Blazhko effect for RR Lyrae stars (see e.g. Kolenberg et al. 2011; Netzel et al. 2018) and the difficult prediction of double mode pulsators among classical Cepheids (see e.g. Smolec & Moskalik 2008).

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7. Discussion

Question (Anderson): Does the relatively strong temperature-dependence of the instability strip boundaries on metallicity in the Marconi/De Somma/Bono models explain why Period-Wesenheit relations (which combine one filter with one color) predict a negative sign to the metallicity effect, while each of their individual filter PL-relations have positive sign?

Answer: Exactly.

Comment (Anderson): Geneva models exhibit negative metallicity effects in each of the individual bands analyzed as well as the Period-Wesenheit relations. The dependence of the instability strip boundaries in these models is very small (Anderson et al. 2016).

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Answer: The IS dependence on metallicity reflects on the PL dependence on metallicity. If you predict that the IS depends mildly on metallicity, we will have PL relations mildly dependent on metallicity. In our cases, we find quite an important metallicity effect that is in some case counterbalanced by helium variations. These have not been taken into account so far in the De Somma et al. models, but Fiorentino et al. models had already shown that if you also increase the helium content at solar or super-solar metallicity, the effect is exactly the opposite: the IS is shifted to the blue. In any case, when you correct for the color term, building Period-Wesenheit relations you are making things more accurate because you are locating the star where it should be and the fact that a negative coefficient of the metallicity term is found is in agreement with several results in the literature. The PL dependence on metallicity reflects the IS dependence; so it is important to understand what is the effect of the adopted opacity, helium-to-metal abundance ratio, there are a lot of things that affect this dependence and maybe the convective treatment should also be differential depending on the models that you are building. Certainly an important aspect to investigate, I agree with you.

Comment (Anderson): Detailed comparison with CMD with the models will help to clarify more.