theory of normal operators and specialised to apply to a number of particular cases such as planar surfaces. The authors' special interests are treated in the fourth chapter, which deals with the classification of open Riemann surfaces of different classes and the derivation of inclusion relations between them. The final chapter on differentials contains the classical results, obtained by Hilbert space theory and Weyl's lemma, and culminating in the Riemann-Roch theorem and the theory of Weierstrass points. There is a very comprehensive bibliography and index.

The treatment is almost completely self-contained. Possibly because of the great wealth of material included, study of the book demands considerable concentration; a fair amount of work is also required of the reader since arguments are not always given in complete detail. For this reason the student beginning the study of Riemann surfaces may find it advantageous to read first a less comprehensive and sophisticated account, such as is given, for example, by G. Springer's *Introduction to Riemann Surfaces* (Addison-Wesley, 1957). For the research worker, however, the book by Ahlfors and Sario is indispensable and is likely to remain the standard text for some time.

R. A. RANKIN

N. BOURBAKI, *Éléments de Mathématique XXVI. Groupes et Algèbres de Lie: Chapitre* 1. Algèbres de Lie (Hermann et cie), 148 pp., 21 NF.

Chevalley completed two volumes of his well known work on Lie groups before he embarked on a systematic account of Lie algebras. In this latest volume of Bourbaki, which is the first in a series on Lie groups and algebras, we are first introduced to the theory of Lie algebras. A hundred pages of theory is balanced by over thirty pages of exercises. The theory in the case of a Lie algebra over a ring of prime characteristic is sketched in a number of the exercises. For the moment we confine our attention to those Lie algebras that are finite dimensional vector spaces over a field of characteristic zero.

A number of criteria are obtained for a Lie algebra to be one of the following types: soluble, nilpotent, semi-simple. Varying soluble and nilpotent radicals play an important role. The soluble radical has a complement—a Levi subalgebra—which is semi-simple; two Levi subalgebras are transformed into each other by a special automorphism (Theorem of Levi-Mal'cev). The theory reaches its climax in the Theorem of Ado. Every Lie algebra has a faithful representation of finite dimension such that the image of the nilpotent radical consists entirely of nilpotent elements.

The enveloping algebra of a Lie algebra over a commutative ring with unit element is considered in detail and the general form of the Poincaré-Birkhoff-Witt Theorem on the isomorphism between the symmetric algebra and the graded algebra associated with the filtered enveloping algebra is obtained.

The well known theory of Lazard on the connection between groups and Lie algebras is confined to a sketchy mention in the exercises and the following topics receive no attention: free Lie algebras, the Campbell-Hausdorff formula and basic monomials.

The volume is almost self contained and can be recommended as an introduction to the theory of Lie algebras.

S. MORAN

BORSUK, KAROL, AND SZMIELEV, WANDA, *Foundations of Geometry* (North Holland Publishing Co., Amsterdam, 1960), 400 pp., 90s.

This book is concerned with the foundations of Euclidean, Bolyai-Lobachevskian (hyperbolic) and real projective geometries and carries the development of each to the point at which the system of axioms can be shown to be categorical as well as consistent.

The introduction contains an account of the history of geometry up to the discovery by Hilbert at the beginning of this century of satisfactory sets of axioms for both Euclidean and Bolyai-Lobachevskian geometries. It also contains an account of the parts of set theory and topology which are required for an understanding of the rest of the book.

A feature of the book is that a topology is introduced into each geometry at the earliest possible stage and that an interest in this topology is maintained throughout the development of the geometry. In the axiom system used for each geometry, lines and planes are regarded as sets of points.

Part I deals with Euclidean and Bolyai-Lobachevskian geometries. The first four chapters, comprising almost half of the whole book, are devoted to *absolute* geometry, which is developed to the point at which Euclidean and Bolyai-Lobachevskian geometries can each be obtained by the addition of a single axiom.

The axioms of absolute geometry consist of four groups—the axioms of incidence, order, congruence and continuity. The axioms of order and congruence deal respectively with the primitive relations of betweenness and equidistance; the axiom of congruence ensures that an open segment, a half-line and a line all possess the Dedekind property.

The authors, wisely, make no attempt to reduce the number of primitive notions and axioms to a minimum, but as an example of an independence proof, they deal with the case of the axiom of continuity, establishing its independence of the axioms of the other three groups by showing that any system satisfying all four sets of axioms is homeomorphic with $\mathbf{C}_3$, real Cartesian space of three dimensions, but that there is a denumerable subset of the points of $\mathbf{C}_3$ which provided a system satisfying the axioms of the first three groups. The existence of the model $\mathbf{C}_3$ having established the consistency of the axioms of absolute geometry, the construction of a second model, the Klein-Beltrami space $\mathbf{K}_3$, is used to show that these axioms are not categorical; it is proved that of the two possible contradictory properties of parallelism called the axioms of Euclid and Bolyai-Lobachevski, $\mathbf{C}_3$ possesses the first and $\mathbf{K}_3$ possesses the second. Finally in Chapters V and VI, devoted respectively to Euclidean and Bolyai-Lobacheoskian geometry, it is shown that the axiom systems of these geometries, each obtained from the axiom system of absolute geometry by the addition of the appropriate parallel axiom, are categorical as well as consistent.

Part II gives a similar treatment of the real prospective geometries of the plane and of space, based on axioms of incidence, order and continuity, the axioms of order being concerned with the primitive relation of division (separation).

It is surprising that the non-categoricity of the system of axioms in the case of the plane is established by constructing the rather elaborate Hilbert model when there is available the beautifully simple non-Desarguesian plane geometry of Moulton (see, for example, Hodge and Pedoe, *Methods of Algebraic Geometry*, vol. I, p. 213 (Cambridge University Press, 1946).

The book is very well printed and has an excellent set of diagrams. The exposition is extremely clear and the English style is so fluent and natural that it is difficult to believe that the original book was not written in English. The use of the combined-letter notations **HL** and **HP** for half-line and half-plane makes for great clarity in the statement of propositions and proofs.

Among a small number of errors noted and not included in the errata are: About the middle of p. 105, *open* segment appears for *free* segment; on pp. 158 and 159, figs. 142 and 143 are interchanged; on p. 263, sentence S4 appears twice and sentence S5 is omitted; on p. 375, line 3 from the foot, U_2 appears for $U_1 \cap U_2$ and at the top of p. 376 there are several incorrect or missing suffixes and misplaced primes.

The book is stated in the preface to have been "planned, at first, as a textbook adapted to the present program of studies in this subject at the Polish universities" though its scope has clearly been extended in the writing. It is impossible to imagine that in the study of the foundations of geometry in any British University, Euclidean and hyperbolic geometries could occupy anything like so considerable a place. But because of the important part that the search for a satisfactory system of postulates for Euclidean geometry has played in the history not only of geometry but of the development of the idea of mathematics as a logical structure, it is desirable that every mathematician should know what is involved in the construction of such a system of postulates; and the proper place for this is within the logical structure of the family of geometries of which Euclidean geometry is a member. As a complete and clear account of this structure, this book can be confidently recommended.

T. S. GRAHAM

NICOLSON, M. M. (edited by D. R. Hartree and D. G. Padfield), *Fundamentals and Techniques of Mathematics for Scientists* (Longmans, 1961), 526 pp., 45s.

The first draft of this book was almost completed when the author, Dr. M. M. Nicolson, was killed in a tragic accident. Thereafter, Professor D. R. Hartree undertook to edit the manuscript and prepare it for publication with the assistance of Dr. Daphne Padfield but, unfortunately, Professor Hartree died suddenly in 1958. Consequently, much of the editorial work and the task of providing chapters on essential topics not treated by Dr. Nicolson has fallen on Dr. Padfield. It is gratifying that the work has now been completed and the fruits of Dr. Nicolson's labours, which show evidence of considerable pedagogic skill, put into permanent form.

The book is eminently suitable for honours students of physics, who are not mathematical specialists, in their penultimate year of undergraduate study and, besides covering the calculus which they require, includes chapters on vector calculus, determinants and matrices, functions of a complex variable, Laplace transforms, special functions, and eigenvalue problems. The style of writing is smooth and discursive and there is plenty of motivation throughout. The book has one major defect from the student's point of view, namely, the lack of exercises; a few easy examples are interspersed in the text, but nothing more. Should a new edition of the book be called for in due course, surely this defect could be remedied.

D. MARTIN

JEFFREYS, H., *Cartesian Tensors* (Students Edition) (Cambridge University Press, 1962), 8s. 6d.

This is a paper-backed edition of the book which was originally published in 1931. The contents consist of some of the properties of cartesian tensors, followed by some applications of them in the fields of Geometry, Dynamics, Statics, Elasticity and Hydrodynamics. Considering the date at which the book was written the treatment is still remarkably modern on the whole. It is a book which should be very useful for colleges and universities, and at a price of 8/6 it should be within the reach of most students.

G. EASON