STELLAR WIND EQUATIONS IN A NEW STEADY STATE

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Abstract:
A set of equations of stellar wind in a new steady state in spherically symmetry is presented. This equations are available also for the deep interior of stars, whereas the usual equations can be applied only to the surface region. The new equations have a variable mass flux which becomes zero at the inner boundary of the mass flow. The velocity also reduces zero, whereas it diverges at r=0 in the usual continuity equation $4\pi r^2 \rho v$=constant. In the surface region, the present equations approach the usual equations.

1. Introduction

In the usual steady state

$$\partial/\partial t|_r=0 \qquad (1)$$

the equation of mass continuity becomes

$$4\pi r^2 \rho v=\text{const.} \qquad (2)$$

This equation has been widely applied to X-ray bursts and nova outbursts in which the acceleration occurs inside the photosphere (Kato 1983a,b, 1986). There is, however, a problem that the velocity could not be zero at the surface of the degenerate stars. Equation (2) gives the finite velocity at the surface of the degenerate stars, in spite of no matter actually flowing out from the interior of the degenerate stars.

Moreover if we apply equation (2) to the deep interior of stars (r=0), eather the velocity or the density should become infinitely large at r=0. Therefore we cannot get any normal stellar structures. This means that equation (2) is inadequate to the interior part of stars. This difficulty comes from the steady-state approximation (1). The mass flux must reduce zero at the center of the stars or the surface of the degenerate stars. The interior flow therefore should be described by another steady states, not by equation (1). Therefore we will present a new steady-state approximation and derive mass-loss equations which is available also to the deep interior of stars.

2. Equations in a new steady state.

We define the q-coordinate as

$$q = (M_r-M_c)/(M-M_c) , \qquad (3)$$

where M_c and M are the mass within the inner boundary and the total mass of the star. We derive the equations in a new steady state

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$\partial v/\partial t|_q=0$, instead of usual steady state $\partial v/\partial t|_r=0$. The equation of motion becomes

$$\frac{v}{Q}\frac{dv}{dr} + \frac{GM_r}{r^2} + \frac{1}{\rho}\frac{dP}{dr} = 0 \quad , \qquad (4)$$

where the steady state factor Q is defined by

$$Q = 4\pi r^2 \rho v/|\dot{M}|q \quad . \qquad (5)$$

The equation of motion (4) approaches the equation of hydrostatic balance near the inner boundary. The velocity also becomes smoothly zero; $v=0$, $dv/dr=0$ at $r=R_c$. With the assumption $\partial\rho/\partial t|_q=0$ the equation of continuity becomes

$$\frac{1}{Q}\frac{d\ln\rho}{d\ln r} + \frac{d\ln v}{d\ln r} + 2 = 0 \quad . \qquad (6)$$

This is integrated analytically as

$$4\pi r^2 \rho v = |\dot{M}|q - \frac{4\pi r^3 \rho |\dot{M}|}{3(M-M_c)}\left(1 + \frac{C}{r^3}\right) \quad , \qquad (7)$$

where C is the integral constant. Note that the mass flux $F= 4\pi r^2 \rho v = |\dot{M}|qQ$ is not constant but a function of r, whereas usual equation (2) gives a constant mass flux. The inner boundary conditions are that the mass flux F is zero at the inner boundary of the mass flow R_c.

$$C = -R_c^3 \; , \quad r = R_c \qquad \text{at } q_c=0. \qquad (8)$$

The inner boundary radius R_c is the radius of the neutron star for X-ray bursts and is zero in the normal stars.

The equation of energy conservation defined by $\partial s/\partial t|_q=0$ becomes

$$|\dot{M}|Tq\frac{ds}{dr} = 4\pi r^2 \rho_n - \frac{dL_r}{dr} \quad . \qquad (9)$$

The equations of mass continuity and the energy transport are unchanged.

The present equations will be useful when the acceleration occurs deep interior or when the gravitational energy release is important in the energy conservation equation. In such cases we will have different structures from that of the usual solutions. Further investigation will be desired in many cases such as red giants or very massive stars.

references

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