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A SYSTEM TO COMBINE STRATIGRAPHIC AND ANNUAL MASS-BALANCE SYSTEMS: A CONTRIBUTION TO THE INTERNATIONAL HYDROLOGICAL DECADE*

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ABSTRACT. Mass-balance quantities at specific points on a glacier as defined in [IHD] (1970) relate either to annual maxima or minima in ice mass at that point (the *stratigraphic system*), or to values at the beginning and end of a hydrologic year (the *annual* or *fixed-date system*). Most quantities measured in the field relate to summer surfaces, which correspond to the annual minima at the measurement points. When stratigraphic system point values are integrated over a whole glacier, the result may be meaningless because annual maxima and minima and summer surfaces may form at different times at different places.

The combined system utilizes several kinds of data to derive meaningful area-average results that can be directly related to other hydrologic and meteorologic information. Measurements to summer surfaces at certain specific times, including the beginning and end of a hydrologic year, are added together with proper recognition of the types of material involved: old firn and ice, snow and superimposed ice of the year under study, new firn formed during that year, and late snow deposited toward the end of the year. Other "balance increment" terms relate values at the beginning and end of a hydrologic year to corresponding area-average balance minima. As a result, two types of "net balance" and many other terms are given precise meaning for a glacier as a whole. The scheme is sufficiently versatile to be used on any glacier, although the terms relating to summer surfaces are not defined on a glacier in which ablation or accumulation is continuous throughout a year.

RÉSUMÉ. *Une méthode pour combiner les méthodes stratigraphiques et annuelles d'estimation des bilans glaciaires: une contribution a la Décennie Hydrologique Internationale.* Les bilans matière en des points représentatifs sur un glacier, tels que définis par le document [IHD] (1970) sont établis soit à partir des maxima et minima annuels en ce point (méthode stratigraphique), soit à partir des valeurs au début et à la fin de l'année hydrologique (méthode annuelle ou à date fixe). La plupart des valeurs mesurées sur le terrain se rapportent à des surfaces d'été qui correspondent aux minima annuels aux points de mesure. Lorsque les valeurs ponctuelles de la méthode stratigraphique sont intégrées sur tout un glacier, le résultat peut être dépourvu de signification parce que les minima et maxima annuels et les surfaces d'été peuvent se former à différentes époques dans les différents endroits.

La méthode combinée utilise plusieurs sortes de données pour obtenir des résultats significatifs en moyenne par unité de surface qui peuvent être directement comparées aux autres informations hydrologiques et météorologiques. Des mesures de surfaces d'été à certaines époques déterminées, dont le début et la fin de l'années hydrologique, sont additionnées entre elles avec une reconnaissance détaillée des types de matériel concernés: vieux névé et glace, neige et glace de surimposition de l'année en cours, nouveau névé formé durant l'année et neige tardive déposée jusqu'à la fin de l'année. D'autres termes "d'augmentation du bilan" relient les valeurs au début et à la fin de l'année hydrologique au bilan minimum correspondant en moyenne à la surface. Il en résulte que deux types de "bilan net" et beaucoup d'autres termes peuvent recevoir une signification précise pour un glacier entier. Le schèma est suffisamment général pour être utilisé sur n'importe quel glacier, bien que les termes relatifs aux surfaces d'été ne sont pas définis sur un glacier dans lequel l'ablation (ou l'accumulation) se manifeste de manière continue pendant toute l'année.

ZUSAMMENFASSUNG. *Ein System zur Verbindung von stratigraphischen und jährlichen Massenbilanz-Systemen.* Massenbilanzwerte an spezifischen Punkten eines Gletschers, wie sie durch [IHD] (1970) definiert wurden, beziehen sich entweder auf Jahresmaxima oder -minima der Eismenge an diesem Punkt (*stratigraphisches System*) oder auf Werte zu Beginn und Ende eines hydrologischen Jahres (*Jahres-* oder *Festdatum-System*). Die meisten Messungen im Felde gehen von den Sommeroberflächen aus, die den Jahresminima am Messpunkt entsprechen. Wenn stratigraphische Punktwerte über den ganzen Gletscher intergriert werden, kann das Ergebnis bedeutungslos sein, weil sich die jährlichen Maxima bzw. Minima und die Sommeroberflächen zu verschiedenen Zeiten an unterschiedlichen Stellen bilden können.

Das kombinierte System benutzt verschiedene Arten von Daten, um sinnvolle Gebietsdurchschnittswerte zu erhalten, die direkt mit anderen hydrologischen und meteorologischen Informationen in Beziehung gebracht werden können. Messungen der Sommeroberfläche zu bestimmten spezifischen Zeitpunkten einschliesslich des Beginns und Endes eines hydrologischen Jahres werden zusammen mit einer sachgemässen Bestimmung des vorhandenen Materials herangezogen: Altschnee und -eis, Schnee und Aufeis des laufenden Jahres, neuer Firn, der sich während dieses Jahres gebildet hat, und Schnee, der gegen Ende des Jahres abgelagert wurde. Andere "Bilanz-Inkremente" verknüpfen Werte vom Beginn und Ende eines hydrologischen Jahre smit korrespondierenden gebietsdurchschnittlichen Bilanzminima. Als Folge erhalten zwei Arten von "Nettobilanz" und viele andere Ausdrücke eine präzise Bedeutung für einen Gletscher als Ganzes. Das Schema ist vielseiig genug, um für jeden Gletscher angewendet werden zu können, wenngleich die Grössen, die sich auf Sommeroberflächen beziehen, nicht für einen Gletscher mit ganzjähriger Ablation oder Akkumulation definiert sind.

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MOST glacier mass-balance data are collected through the use of stakes, pits, cores, or probing to or from a reference horizon. This is normally a summer horizon—either a winter snow/ice interface in the ablation area as measured in spring, or a snow/firn interface in the accumulation area as measured in spring or summer. This interface, here termed *summer surface*, may form at different times of the year in different parts of the world or even on the same glacier. A summer surface is normally recognized as a discontinuity between units of differing density or other physical property, or as a diagnostic, correlatable horizon such as a depth-hoar layer. On some glaciers it may not be possible to recognize summer surfaces; in this case conventional mass-balance measuring procedures cannot be used and the system presented here is not applicable.

In a pit or core in the accumulation area, the mass of ice material between two consecutive summer surfaces can be measured. This mass, in Mg/m^2 or meters of water equivalent, may be the *balance* (the difference between accumulation and ablation) at that point for the time interval between the formation of the two summer surfaces. However, in the percolation or soaked facies (Benson, 1962 p. 24–25) an appreciable part of the material deposited in this time interval may have been melted and subsequently redeposited (refrozen) in lower layers, below the lower of the two summer surfaces of interest. Detection of this problem is not easy, and analysis of the resulting balance may be even more difficult. We assume here that any appreciable mass redeposited below a summer surface of interest can be calculated from repeated depth–density profiles and added to the balance above the summer surface.

A more difficult problem, especially when relating mass-balance quantities to meteorologic and hydrologic quantities as in the Combined Heat, Ice, and Water Balances Program of the International Hydrological Decade stems from the fact that summer surfaces may form at different times in different places. This means that a simple integration over the glacier of mass-balance data related to summer surfaces produces a result that has no clear meaning with respect to time. Thus these data cannot be directly related to heat or mass-flux data obtained by other techniques.

This situation is somewhat analogous to the geologic problem of dealing with rock formations that were deposited at different times in different places, for example, a narrow zone of beach sand deposited as sea-water slowly encroached on a landmass over a span of geologic time. In geologic nomenclature one can speak of rock types, rock–stratigraphic units (such as a formation) which are correlatable units independent of time concepts; time–stratigraphic units which are the rocks deposited during a specific time interval; and time units (American Commission on Stratigraphic Nomenclature, 1961). Snow, firn and ice can be used either as rock-type or rock-stratigraphic terms. Confusion enters when a term such as firn is incorrectly used as a time–stratigraphic unit for a whole glacier.

The fact that a late snow layer and a summer surface may be time-transgressive (forming at different times in different places) can be illustrated by a typical sequence on a temperate Northern Hemisphere glacier (Table I).

Mass-balance terms based on observable summer surfaces were proposed by Meier (1962). The *stratigraphic system* (Anonymous, 1969; [IHD], 1970) is a modification in terminology of the basic mass-balance concepts. This stratigraphic system works well for individual points. However, in order to compare ice-balance data with hydrologic (water-balance) data, these point values must be integrated over a whole glacier or drainage basin. If, as is usually the case, the summer horizons are not formed synchronously over the whole area, this integration is an invalid measure of snow and ice storage. Therefore, a different system, the *annual system* (*fixed-date system*), has been conceived to relate glaciological data to hydrological data. Unfortunately, glaciological programs using only the annual system cannot take advantage of convenient reference horizons in the field, so the field work may be extremely difficult or exorbitantly expensive. These two systems are described in [IHD] (1970) and in Anonymous (1969); but no attempt was made to show how they might be combined.

We feel that to document properly the relation of glaciers to climate, and to provide a check on mass-balance measurements, it is necessary to combine these two systems into a unified whole. The vital key to a combination of these systems is identification of the material under consideration. This identification is, of course, useful additional information for any description of the meteorological–hydrological environment. We define four types of material which may be found on a glacier in one specific year—*snow*, *old firn and ice*, *late snow* and *new firn*—as follows: the highest (most recent) summer surface found in a pit dug in winter (or early spring) before the beginning of appreciable, continuous melting is termed ss_0. The

TABLE I. HYPOTHETICAL MASS-BALANCE HISTORY FOR A TYPICAL GLACIER

January	Entire glacier covered with snow, snowfall continues, little if any melt anywhere on glacier	Snow mass increasing Old firn and ice mass constant Total mass increasing
May	Snow accumulation continues at higher altitudes, melt begins at terminus removing winter snow and then ablating ice	Snow mass increasing Old firn and ice mass decreasing Total mass relatively constant
July	Melt occurs over whole glacier, ablating snow in upper regions and ice in lower regions. Little if any snowfall anywhere on glacier	Snow mass decreasing Old firn and ice mass decreasing Total mass decreasing
August	"Late snow" falls in upper regions burying melt surface. This buried melt surface now identified in pits as the "*summer surface*" and the older snow below it now becomes "new firn" by definition. Ablation of ice continues in lower regions	Late snow mass increasing Older snow mass decreasing both by melting and by conversion to new firn Old firn and ice mass decreasing Total mass decreasing
October	Late snow accumulation extending over greater area, thus the buried *summer surface* is extending to lower and lower altitudes as time proceeds. However, melt continues at the lowest altitudes; here the *summer surface* has not yet formed	Late snow mass increasing Older snow mass decreasing mainly by conversion to new firn Old firn and ice mass decreasing Total mass increasing
January	Entire glacier covered with late snow, snowfall continues, melting at the terminus has ceased. Now the formation of the *summer surface* is complete over the whole glacier	Late snow mass increasing, identified as "snow" of the new year Older snow mass completely melted, or converted to firn which becomes "old firn" of the new year Old firn and ice mass, increased by a new increment of firn, now is again constant Total mass increasing

material above ss_0 is termed *snow* and the material below it is *old firn and ice*. The highest (most recent) summer surface found in a pit in the upper regions of the glacier late in the same year after the beginning of snow accumulation following a period of melting in summer is termed ss_{I}. The material above ss_{I} is termed *late snow* and the material below ss_{I} yet above ss_0 is *new firn*.

These terms define units which can be correlated over large areas of a glacier in any one year; however, they may be formed in different times at different places. Thus they correspond to "formations" (rock–stratigraphic units) in the geological sense. A year later, a different set of formations is described, each unit dropping back one step:

First year		*Next year*
ss_0	→	not used
ss_{I}	→	ss_0
late snow	→	snow
new firn old firn and ice }	→	old firn and ice

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Another important requirement of combining the two systems is making measurements of stratigraphic units at the beginning and end of a fixed-date hydrologic year.

The variation in these units of the mass balance may be illustrated by graphs showing the changing balance with time, $b[t]$, at specific points on a glacier, expressed as mass per unit area (Mg/m^2) or simply in water equivalent (m) (Fig. 1).* The balance quantities are designated by the letter b with qualifying symbols, as follows: the subscript $_0$ refers to the initial measurements made at or near the beginning of the year to relate fixed-date system measure-

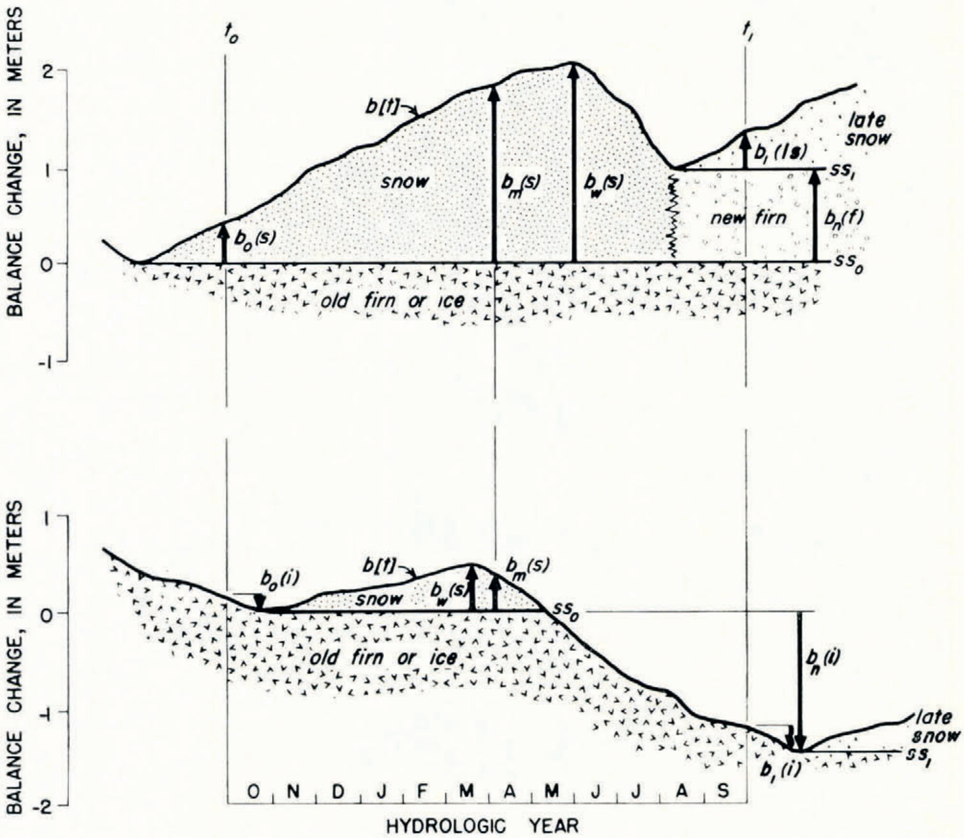

Fig. 1. Balance quantities as measured at specific points on a glacier using a stratigraphic system referenced to summer surfaces. Shown here are typical balance curves b[t] for points in the accumulation area (top diagram) and in the ablation area (bottom diagram). The vertical scale is meters of water equivalent, with an arbitrary zero. The zero may be placed in any other position depending upon the needs of the data analysis. This and succeeding illustrations use the following sign convention: balance terms which increase with time, or represent an increment of mass added to the system, are considered positive and are shown with arrows pointing up. Note the formation of summer surfaces, and how they form at different times at the two points. Measurements at the beginning (t_0) and end (t_1) of the hydrologic year, in spring, and at a time after the hydrologic year (perhaps next spring) define these quantities $b_0(s)$ *: the* initial snow balance; $b_0(i)$, *the* initial ice balance; $b_m(s)$, *the* measured winter snow balance; $b_1(ls)$, *the* final late snow balance; $b_1(i)$, *the* final ice balance; $b_n(f)$, *the* net firnification; *and* $b_n(i)$ *the* net ice balance. *Also shown is* $b_w(s)$, *the* maximum winter snow balance *which usually is not measured, being dependent on knowledge of* $b[t]$ *at each point. Note that the* maximum winter snow balance *occurs at different times at different points.*

* The vertical scale on these graphs is strictly the mass per unit area referenced to a lower horizon or the bottom of the glacier (the scale is arbitrary) in a coordinate system moving with the glacier (Lagrangian coordinates), and ignores strains due to glacier flow and compaction. This qualification is unimportant in most cases as the measurements are usually made in pits or stakes and the amount of vertical strain in a year's time is usually smaller than the error in mass-balance measurements.

ments to stratigraphic units; the subscript $_1$ refers to the final measurements made at or near the end of the year to relate the two systems; the subscript $_a$ refers to certain measurements made (or calculated) exactly at the end of the hydrologic year, and the subscript $_n$ refers to measurements related to the minimum firn and ice or the minimum total mass near (but not necessarily at) the end of a hydrologic year; the letter $_x$ identifies balance quantities at the time of the maximum total balance in the hydrologic year. Letters in parentheses following the b indicate the material being measured: snow (s), old firn and ice (i), late snow (ls), and new firn (f). If superimposed ice accumulates or melt water is redeposited in lower layers it is added to either the snow (s) or new firn (f), to whichever the superimposed ice is related. A lack of parentheses following the b indicates that the total mass (undifferentiated) is considered. The hydrologic year, usually defined as 1 October through 30 September, runs from t_0 to t_1. Arrows pointing up indicate an addition of mass as time proceeds; the corresponding balance quantities are taken as positive.

Measurements made at specific points define the following quantities:

1. $b_0(\text{s})$, the *initial snow balance*, is the snow at the beginning of the hydrologic year. Often $b_0(\text{s}) = 0$ in the ablation area, and sometimes even in the accumulation area. It is measured by probing or in pits by field work at the beginning of the hydrologic year or by computation using other field results and interpolation, perhaps using weather records.

2. $b_0(\text{i})$, the *initial ice balance*, is the old firn and ice loss after the start of the hydrologic year and before ablation ceases in winter and is usually measured by observing stakes in early winter or the next summer before snow has disappeared from the ice. Always $b_0(\text{i}) = 0$ in the accumulation area and sometimes in the ablation area also.

3. $b_{\text{m}}(\text{s})$, the *measured winter snow balance*, is the snow above the summer surface ss_0 as measured directly by field work in late spring as near as possible to the time of greatest glacier mass (not necessarily the time of greatest mass at any one point). Normally $b_{\text{m}}(\text{s})$ is measured over the whole glacier at about the same time.

4. $b_1(\text{ls})$, the *final late snow balance*, is the late snow at the end of the hydrologic year, the same as $b_0(\text{s})$ for the year following.

5. $b_1(\text{i})$, the *final ice balance*, is the old firn and ice loss after the end of the hydrologic year before ablation ceases for the next winter, the same as $b_0(\text{i})$ for the year following.

6. $b_{\text{n}}(\text{f})$, the *net firnification*, is the amount of new firn formed at about the end of the hydrologic year (either just before or just after). It is therefore the mass between the successive surfaces ss_0 and ss_1, and is usually measured in pits well after its time of formation.

7. $b_{\text{n}}(\text{i})$, the *net ice balance*, is the corresponding change in mass between ss_0 and ss_1 in the ablation area where this change is negative; thus it records the loss of ice and old firn from the end of one melt season to the end of the next.

8. $b_{\text{w}}(\text{s})$, the *maximum snow balance*, is the hypothetical maximum mass of the snow during the hydrologic year. This value will occur at a different time at each place and thus will probably not be measured directly.

Stratigraphic balance quantities at a point have been defined previously (Meier, 1962; Anonymous, 1969; [IHD], 1970) and their measurement is straightforward. The quantities and symbols presented here are consistent with the last two references. Difficulty appears only when one attempts to integrate these point values over a whole glacier. For instance, if values of maximum winter snow balance, $b_{\text{w}}(\text{s})$, are determined at numerous points on a glacier, a map of these values represents a time-transgressive maximum which is greater than the total amount of snow on the glacier at any moment. This peculiar "snow flood crest" map has no real meaning for most hydrologic balance computations. Similar problems exist with $b_0(\text{i})$, $b_1(\text{i})$, $b_{\text{n}}(\text{i})$, and $b_{\text{n}}(\text{f})$, but not with $b_0(\text{s})$, $b_1(\text{ls})$, and $b_{\text{m}}(\text{s})$.

We may now consider mass-balance data in terms of diagrams showing the area-average balance curve $\bar{b}[t]$ (Figs. 2–4). Alternatively, one could plot the total balance curve $B[t]$

using a different vertical scale of m^3 instead of meters of water equivalent. In the material to follow, a bar over a symbol indicates an area average, as in [IHD] (1970).

Figure 2a is a plot of $\bar{b}[t]$ on which are shown the area-averaged terms $\bar{b}_w$ (*winter balance*) and $\bar{b}_n$ (here termed the *total mass net balance*) which are analogous to the stratigraphic system point terms b_w and b_n in [IHD] (1970). The time interval t_0' to t_1' is a *balance year* which is unique to each glacier and each year, and is not necessarily 365 days long. Figure 2b is a plot of $\bar{b}[t]$ showing the area-averaged *annual balance* $\bar{b}_a$, which is analogous to the fixed-date system point term b_a in [IHD] (1970). Also included in this figure is a new term $\bar{b}_x$, the *maximum balance* [of the hydrologic year]. In order to relate $\bar{b}_n$ to $\bar{b}_a$, or vice versa, two additional terms are proposed: $\bar{b}_0$, the *initial balance increment*, and $\bar{b}_1$, the *final balance increment*. These curves show mass-balance quantities that can be directly related to other hydrologic balances.

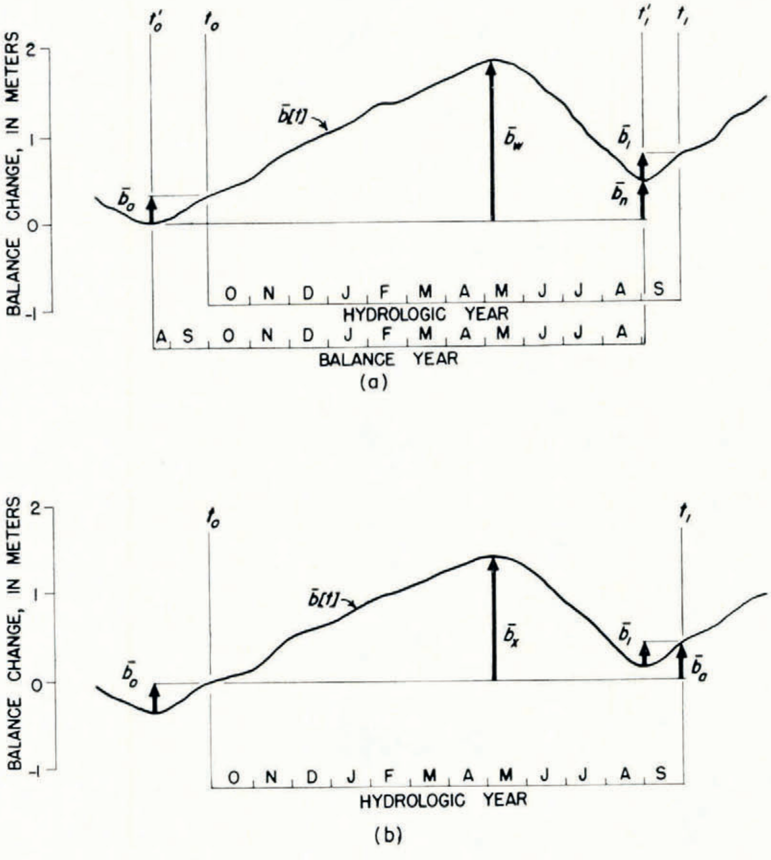

Fig. 2. Area-averaged balance quantities shown without relation to summer surfaces or type of material measured. The vertical scale is meters of water equivalent averaged over the entire area of the glacier, with an arbitrary zero (the scale could just as well be in kilograms of mass or cubic meters of water equivalent). Sign convention as in Figure 1. Figure 2a shows area-averaged terms equivalent to the stratigraphic system terms of [IHD] (1970), including b_w, the winter balance, *and $\bar{b}_n$, the* total mass net balance. *The period between two balance minima in successive years is the* balance year. *Figure 2b shows the area-averaged term $\bar{b}_a$, the* annual balance, *and $\bar{b}_x$, the* maximum balance, *which are related to the annual (fixed date) system of [IHD] (1970). Also shown are the terms $\bar{b}_0$ and $\bar{b}_1$, the* initial *and* final balances, *which relate the two systems. Field determination of all these quantities except $\bar{b}_a$ is difficult because knowledge of the area-averaged balance curve $\bar{b}[t]$ is required to know magnitude* and *timing of these balance quantities. However, the curve $\bar{b}[t]$ is directly related to the changing mass storage, equal approximately to precipitation minus run-off in many environments.*

Unfortunately the scheme shown in Figure 2a and b is *not* fully workable in practice. This is because the terms $\bar{b}_0$, $\bar{b}_w$, $\bar{b}_n$ and $\bar{b}_1$ have no relation to the corresponding values measured relative to summer surfaces at the individual points. This scheme is workable only if point data are integrated in the following way:

Suppose that curves such as those shown in Figure 1 have been obtained at a large number of stakes on a glacier in a given year. The balance curve for the whole glacier, $B[t]$, might then be obtained by summing the individual balance curves ($b_1[t]$, $b_2[t]$ . . .), multiplying each by a weighting factor (a_1, a_2 . . .) according to its proportion of the total area (A):

$$B[t] = \sum_{j=1}^{n} b_j[t]a_j = A\bar{b}[t]$$

where $\bar{b}[t]$ is the area-average balance curve for the whole glacier. Note especially that the balance curves $\bar{b}_j[t]$ are combined to obtain $\bar{b}[t]$, not the individual point measurements such as $b_0(\text{i})$ which might have been made at different times in different places.

This computation procedure requires more measurements than can normally be made. Needed is a scheme in which the point data, taken only a limited number of times during a year and usually referenced to summer surfaces, can be combined directly. In order to do this, the summer surfaces must be included in the area-average diagrams. This is done by dividing the balance curve $\bar{b}[t]$ into its four components: old firn and ice, snow, new firn and late snow(Fig. 3). The largest mass, old firn and ice, is plotted at the bottom—it can only decrease or remain constant during any one year. Above it is plotted snow, which increases during the first part of the year; during the last part of the year it decreases due to ablation and is converted to new firn. Toward the end of the year, late snow is deposited on top of a melt surface, causing the snow below the surface to be converted to firn. The amount of new firn after all of the snow is eliminated and/or converted remains relatively constant. The interface between snow plus new firn and old firn and ice is summer surface ss_0. The interface between new firn and late snow is summer surface ss_1. The interface between snow and new firn is shown as a jagged line; it has little physical significance.

The point data taken at specific times during the field season (or determined after the fact) can now be averaged over the glacier and shown on the area-average balance diagram (Fig. 3a–c). Now several important balance terms are determined. One is the *annual balance* $\bar{b}_a$. Another important balance quantity is the difference between old firn and ice melt, $\bar{b}_n(\text{i})$, the *net ice balance*, and snow which lasts through the melt season and is preserved as new firn, $\bar{b}_n(\text{f})$, the *net firnification*. This difference is here called the *firn and ice net balance*, $\bar{b}_n(\text{fi})$. This is in fact the quantity most often reported by glaciologists as "net balance" but is *not* the area-average net balance $\bar{b}_n$ (Fig. 2) indicated in [IHD] (1970).

On some glaciers, all of the snow is normally converted to firn before the end of the hydrologic year, and it is convenient to measure firn and ice balances at this time. We therefore define three additional terms: the *annual firnification* $\bar{b}_a(\text{f})$, the *annual ice balance* $\bar{b}_a(\text{i})$, and the *firn and ice annual balance* $\bar{b}_a(\text{fi})$ (Fig. 3b). These units are analogous to $\bar{b}_n(\text{f})$, $\bar{b}_n(\text{i})$ and $\bar{b}_n(\text{fi})$, respectively.

In some situations, snow melt may continue after the end of a hydrologic year. One can measure the residual snow on the glacier surface on 1 October, but only part of this (that part which is buried under new snow) is properly called firn. Thus the quantities $\bar{b}_a(\text{f})$ and $\bar{b}_a(\text{fi})$ cannot be defined (Fig. 3c). If it is considered necessary to compute a quantity like $\bar{b}_a(\text{fi})$, one can redefine terms so that $\bar{b}_a(\text{f})$ [or $\bar{b}_a(\text{f+s})$] relates to the new firn and residual snow and not material which everywhere underlies late snow. The quantity $\bar{b}_a(\text{i})$ is always definable.

Other area-averaged terms shown on Figure 3, exactly equivalent to the corresponding values shown on Figure 1, are $\bar{b}_0(\text{s})$, the *initial snow balance*; $\bar{b}_0(\text{i})$, the *initial ice balance*; $\bar{b}_1(\text{ls})$, the *final late snow balance*; $\bar{b}_1(\text{i})$, the *final ice balance*; and $\bar{b}_m(\text{s})$, the *measured winter snow balance*.

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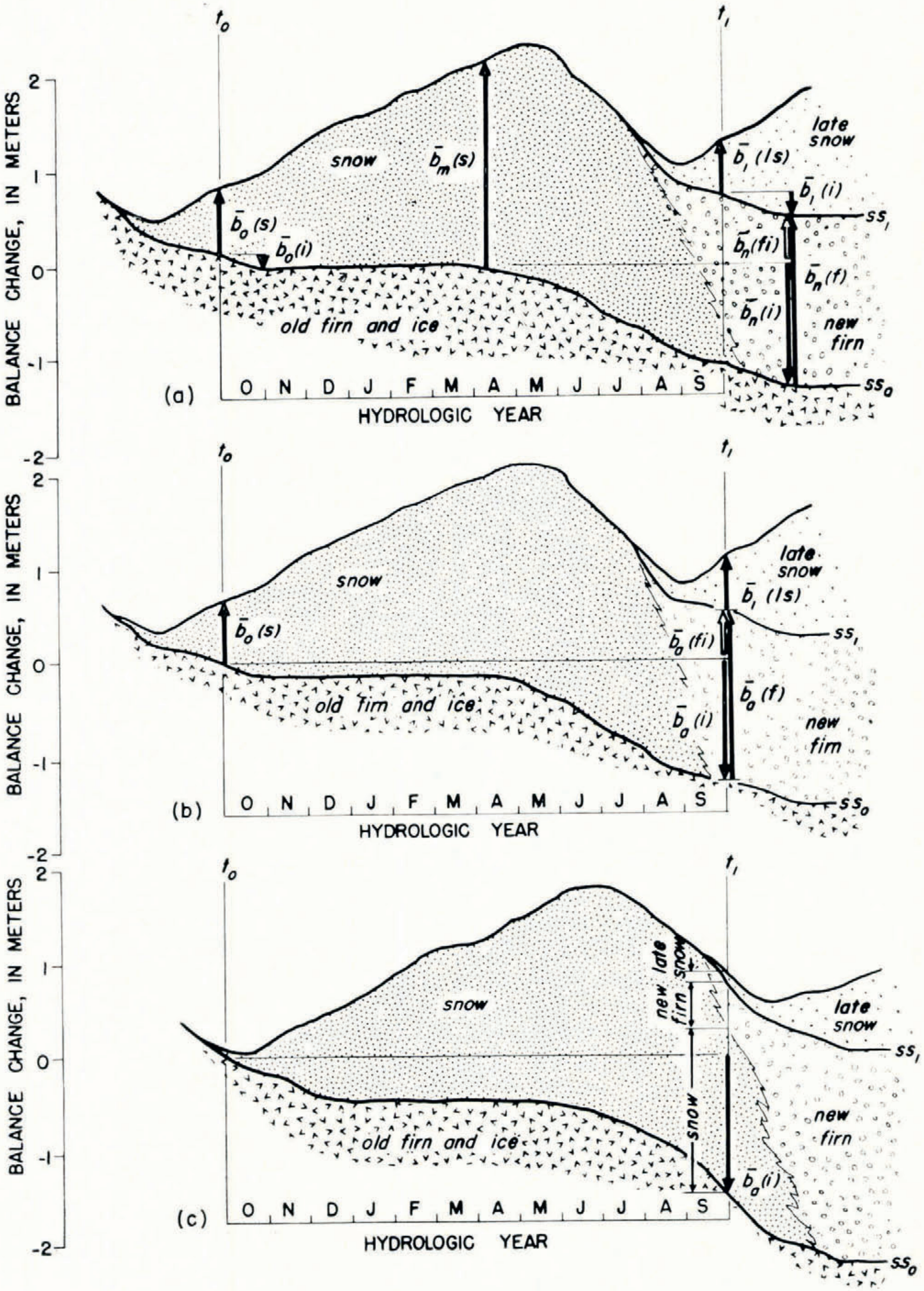

Fig. 3.

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The annual balance, $\bar{b}_a$, is an important quantity because it represents the total change in storage (of snow, firn and ice) during a hydrologic year. Thus this value can be compared directly with the difference between precipitation as snow and melt-water run-off if net evaporation/condensation and net changes in liquid water storage are negligible. $\bar{b}_a$ can be measured directly, or computed as $\bar{b}_a = \bar{b}_a(\mathrm{f}) + \bar{b}_a(\mathrm{i}) - \bar{b}_0(\mathrm{s}) + \bar{b}_1(\mathrm{ls})$ if $\bar{b}_a(\mathrm{f})$ is definable. The annual balance can also be calculated, more indirectly, from balance year (stratigraphic system) quantities as $\bar{b}_a = -\bar{b}_0(\mathrm{s}) + \bar{b}_0(\mathrm{i}) + \bar{b}_n(\mathrm{i}) + \bar{b}_n(\mathrm{f}) - \bar{b}_1(\mathrm{i}) + \bar{b}_1(\mathrm{ls})$.

It is important to define the *maximum winter snow balance* $\bar{b}_w(\mathrm{s})$ on the glacier during the year, because it is often impossible to measure the actual winter snow accumulation. Our term $\bar{b}_m(\mathrm{s})$, a measured but lower value at *about* the right time of year, can often be used as a basis for computing $\bar{b}_w(\mathrm{s})$ if sufficient supplementary meteorological data are available. The same statement can be made about $\bar{b}_w$ (Fig. 2a) and $\bar{b}_x$ (Fig. 2b). Although $\bar{b}_w$ and $\bar{b}_x$ occur at the same instant in time, $\bar{b}_w(\mathrm{s})$ can occur at a later date. Note that $\bar{b}_x + \bar{b}_0 = \bar{b}_w$. Normally, only one or two of these four balance quantities [$\bar{b}_w(\mathrm{s})$, $\bar{b}_w$, $\bar{b}_x$, or $\bar{b}_m(\mathrm{s})$] would be reported. It must be stressed that none of these can be calculated by averaging point values of b_w (winter balance) because the balance reaches a maximum at different times at the different points.

The terms defined and shown on Figures 2 and 3 can be combined on one diagram for reference and comparison (Fig. 4). The apparent complexity of this diagram is somewhat misleading; only half to two-thirds of these terms need be reported in any given study.

These terms define glacier balance with sufficient generality to report any condition of varying balance conditions at different points. At many glaciers, some of the correction terms, such as $\bar{b}_0$, $\bar{b}_0(\mathrm{s})$, $\bar{b}_1(\mathrm{s})$ and $\bar{b}_0(\mathrm{i})$, will be zero or small, and if so can be neglected to simplify the calculations.

Many of the terms taken at the end of one year automatically become the important initial terms for the next year. The symbols change, as follows:

First year		*Second year*
t_1	→	t_0
t_1'	→	t_0'
b_1	→	b_0
$b_1(\mathrm{ls})$	→	$b_0(\mathrm{s})$
$b_1(\mathrm{i})$	→	$b_0(\mathrm{i})$

The combined terms allow a description of the hydrology of a glacier involving process, material and time. Thus we can list initial, final and annual changes in snow cover and ice storage, and relate these to the hydrologically relevant quantities of precipitation and run-off.

Fig. 3. Area-averaged balance quantities that can be conveniently measured in the field. The vertical scale is meters of water equivalent averaged over the entire area of the glacier, with arbitrary zero. Sign convention is as in Figure 1. Figure 3a shows terms in the stratigraphic system including $\bar{b}_0(\mathrm{i})$, *the* initial ice balance; $\bar{b}_m(\mathrm{s})$, *the* measured winter snow balance; $\bar{b}_n(\mathrm{i})$, *the* net ice balance; $\bar{b}_n(\mathrm{f})$, *the* net firnification; *and* $\bar{b}_1(\mathrm{i})$, *the* final ice balance. *Nearly all the firn and ice ablation at this time (after* t_1'*) will occur on* ss_0 *rather than* ss_1. *However, the final ice-balance decrease,* $\bar{b}_1(\mathrm{i})$, *is shown on* ss_1 *and not* ss_0 *because, if no snow ablation occurs after* t_1', $\bar{b}_1(\mathrm{i})$ *will automatically become* $\bar{b}_0(\mathrm{i})$ *for the succeeding year. The terms* $\bar{b}_0(\mathrm{s})$ *and* $\bar{b}_1(\mathrm{ls})$, *the* initial snow balance *and* final late snow balance, *relate stratigraphic and annual (fixed-date) systems. Figure 3b shows in the annual system, including in addition to those above,* $\bar{b}_a(\mathrm{i})$ *the* annual ice balance *and* $\bar{b}_a(\mathrm{f})$ *the* annual firnification; *the latter is defined only if new firn formation is complete by the end of the hydrologic year. As in Figure 1, these terms can all be measured in a continuing program involving only two trips per year: one at the beginning of the hydrologic year and one in the spring. Two additional terms (open arrows) can be calculated easily from observed quantities: the* firn and ice net balance $\bar{b}_n(\mathrm{fi}) = \bar{b}_n(\mathrm{f}) + \bar{b}_n(\mathrm{i})$, *and the* firn and ice annual balance $\bar{b}_a(\mathrm{fi}) = \bar{b}_a(\mathrm{f}) + \bar{b}_a(\mathrm{i})$. *Figure 3c shows the situation if residual snow, not yet completely covered by new snow, remains at the end of the hydrologic year. In this case* $\bar{b}_a(\mathrm{f})$ [*and* $\bar{b}_a(\mathrm{fi})$] *cannot be defined.*

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This scheme appears, unfortunately, to be rather complicated. However, it cannot be simplified without omitting important features of glacier hydrology. One must recognize that some of these terms may be appreciable in magnitude at one glacier while not even definable or measurable at another. Lack of recognition of these terms has made analysis of some previous mass-balance data difficult, if not impossible. If ice-balance data are not analyzed together with heat or water-balance data, a reporting scheme which appears to be simple and concise can be devised. But such a system may be either useless or invalid for comparison with any kind of meteorological or other hydrological data.

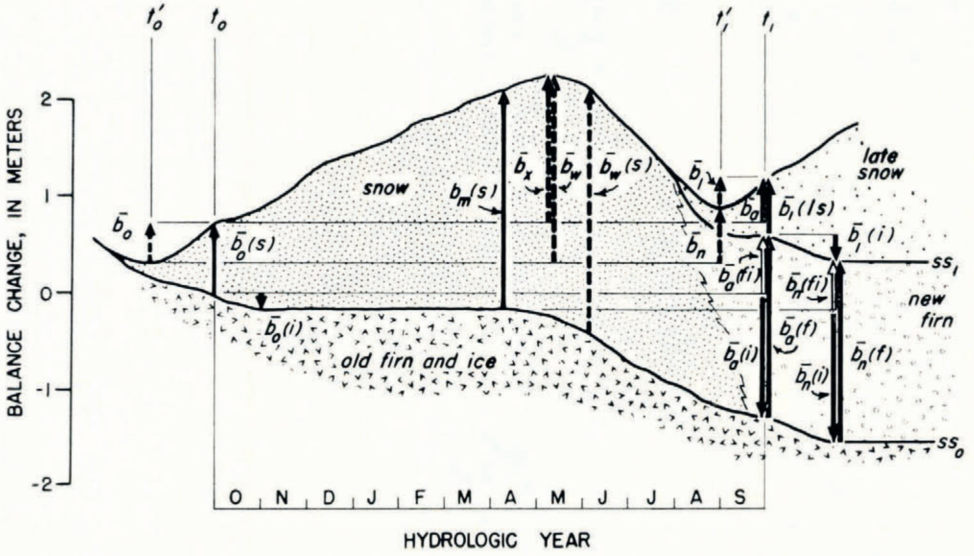

Fig. 4. All area-averaged quantities shown together on one diagram for reference purposes. No one program would expect to report all of these possible terms, but only those selected as most relevant to the particular environment and observation scheme. The vertical scale is meters of water equivalent averaged over the entire area of the glacier, with arbitrary zero. Conveniently measurable quantities (solid arrows) are those shown in Figure 3. Quantities determined through knowledge of the area-averaged balance curve $\bar{b}[t]$ (dashed arrows) are those shown in Figure 2. Calculated quantities are shown with open arrows.

Two examples of the use of the combined scheme to report data from glaciers are given in Table II. These early results from an International Hydrological Decade study illustrate a typical measurement and computation program (on other glaciers it may be necessary or desirable to measure values listed as computed or vice versa). Some of the values shown are redundant and would not be given in a routine report; they are shown here to illustrate all of the terms we have defined.

Note that the terms $\bar{b}_0$, $\bar{b}_0(\mathrm{s})$, $\bar{b}_0(\mathrm{i})$, $\bar{b}_1(\mathrm{ls})$ are small or zero and $\bar{b}_\mathrm{a}$, $\bar{b}_\mathrm{n}$, $\bar{b}_\mathrm{n}(\mathrm{fi})$ are similar in magnitude for South Cascade Glacier. However, the term $\bar{b}_1(\mathrm{ls})$ for Gulkana Glacier is appreciable, and the balances $\bar{b}_\mathrm{a}$ and $\bar{b}_\mathrm{n}(\mathrm{fi})$ are not even of the same sign. Obviously, on a glacier like Gulkana it is meaningless to express the "state of health" by a net- or annual-balance value unless it is absolutely clear exactly what is meant. This scheme, although appearing somewhat cumbersome, gives the author a code for expressing whichever units he prefers in exact, definable and comparable terms.

TABLE II. MASS-BALANCE QUANTITIES FOR GULKANA GLACIER, ALASKA, AND SOUTH CASCADE GLACIER, WASHINGTON, 1966

(Except for times, all quantities are reported in meters water equivalent averaged over the glacier. Data for Gulkana Glacier are based on a reconnaissance study and contain large uncertainties.)

Symbol	*Gulkana Glacier*	*South Cascade Glacier*	*Name of term*	*Directly measured or computed*	*Explanation*
Time					
t_0	1 Oct. 1965	1 Oct. 1965		—	Beginning of hydrologic year
t_0'	15 Sept. 1965	2 Nov. 1965		Computed	Time of minimum glacier mass near beginning of hydrologic year, derived from graph of $\bar{b}[t]$
t_1	30 Sept. 1966	30 Sept. 1966		—	End of hydrologic year
t_1'	1 Sept. 1966	16 Oct. 1966		Computed	Time of minimum mass near end of hydrologic year, derived from graph of $\bar{b}[t]$. $t_1'-t_0'$ is the *balance year*
Stratigraphic system					
$\bar{b}_m(s)$	1.00 at 18 May 1966	2.52 at 14 May 1966	Measured winter snow balance	Measured	Balance measured to the summer surface (ss_0) in late winter or spring, measured in pits, cores, and by probing
$\bar{b}_w(s)$	1.25 at 10 June 1966	2.59 at 22 May 1966	Maximum winter snow balance	Computed	Maximum of snow mass during the balance year, computed from graph of $\bar{b}_s[t]$ or graphs of $\bar{b}[t]$ and $\bar{b}_i[t]$ before and after time of $\bar{b}_m(s)$
$\bar{b}_w$	1.10 at 1 June 1966	2.47 at 22 May 1966	Winter balance	Computed	Maximum value of the balance in relation to balance at t_0'; amplitude of mass change during the balance year. Computed from changes in $\bar{b}[t]$ before and after time of $\bar{b}_m(s)$
$\bar{b}_n(f)$	0.55 at 20 Aug. 1966	0.09 at 5 Nov. 1966	Net firnification	Measured	The increment of new firn in the accumulation area, as measured after ablation ceases in fall in pits or cores. Date ablation ceased indicated under value
$\bar{b}_n(i)$	−0.76	−1.16	Net ice balance	Measured	Old firn and ice melt in the ablation area of a single melt season, measured with stakes, once during period when ice is covered by snow and again after ablation ceases
$\bar{b}_n(fi)$	−0.21	−1.07	Firn and ice net balance	Computed	Change in mass of firn and ice during a single melt season; the mass between two consecutive summer surfaces. $\bar{b}_n(fi) = \bar{b}_n(f) + \bar{b}_n(i)$
$\bar{b}_n$	0.01	−1.03	Total mass net balance	Computed	Change in snow, firn, and ice storage between times of minimum mass; net change in mass during one balance year. $\bar{b}_n = \bar{b}_0 - \bar{b}_1 + \bar{b}_a$
Terms relating annual and stratigraphic systems					
$\bar{b}_0$	0.05	−0.08	Initial balance increment	Computed	Change in balance between first time of minimum balance (t_0') and t_0; computed from graph of $\bar{b}[t]$
$\bar{b}_0(s)$	0.10	0	Initial snow balance	Measured	Snow accumulated on summer surface (ss_0), measured at t_0 with pits or cores
$\bar{b}_0(i)$	0 at 15 Oct. 1965	−0.20 at 22 Nov. 1965	Initial ice balance	Measured	Old firn and ice melt in the ablation area after t_0 and before melt begins the following spring, measured by ablation stakes at t_0 and during period when ice is covered by snow
$\bar{b}_1$	0.10	−0.17	Final balance increment	Computed	Change in balance between time of minimum mass (t_1') and the end of the hydrologic year (t_1), computed from graph of $\bar{b}[t]$

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TABLE II.—*continued*

Symbol	Gulkana Glacier	South Cascade Glacier	Name of term	Directly measured or computed	Explanation
Terms relating annual and stratigraphic systems					
$\bar{b}_1(\text{ls})$	0.37	0	Final late snow balance	Measured	Snow accumulated on summer surface (ss_1), measured at t_1 with pits or cores
$\bar{b}_1(\text{i})$	0	−0.33	Final ice balance	Measured	Old firn and ice melt in the ablation area after t_1 and before melt begins the next spring, measured by ablation stakes at t_1 and during period when ice is covered by snow
Annual (fixed-date) system					
$\bar{b}_\text{x}$	1.05	2.39	Maximum balance	Computed	Maximum value of the balance in relation to balance at t_0; amplitude of mass change during hydrologic year. Occurs at same time as $\bar{b}_\text{w}$. $\bar{b}_\text{x} = \bar{b}_\text{w} - \bar{b}_0$
$\bar{b}_\text{a}(\text{f})$	0.55	Not definable	Annual firnification	Measured	The new firn formed on the glacier during the hydrologic year, measured at t_1 in pits or cores. Not definable if snow melt continues after t_1
$\bar{b}_\text{a}(\text{i})$	−0.76	−1.03	Annual ice balance	Measured	Old firn and ice melt in the ablation area during the hydrologic year, measured by stakes at t_0 and t_1
$\bar{b}_\text{a}(\text{fi})$	−0.21	Not definable	Firn and ice annual balance	Computed	The change in mass of firn and ice during the hydrologic year from t_0 to t_1, also the mass between two consecutive summer surfaces at t_1. $\bar{b}_\text{a}(\text{fi}) = \bar{b}_\text{a}(\text{f}) + \bar{b}_\text{a}(\text{i})$
$\bar{b}_\text{a}$	0.06	−0.94	Annual balance	Measured or computed	Change in snow, firn, and ice storage between t_0 and t_1; approximately the difference between precipitation as snow and melt-water run-off for one hydrologic year. Can be measured directly at t_0 and t_1. $\bar{b}_\text{a} = \bar{b}_\text{a}(\text{fi}) - \bar{b}_0(\text{s}) + \bar{b}_1(\text{ls})$ if $\bar{b}_\text{a}$ (fi) is defined.

REFERENCES

American Commission on Stratigraphic Nomenclature. 1961. Code of stratigraphic nomenclature. *Bulletin of the American Association of Petroleum Geologists*, Vol. 45, No. 5, p. 645–65.

Anonymous. 1969. Mass-balance terms. *Journal of Glaciology*, Vol. 8, No. 52, p. 3–7.

Benson, C. S. 1962. Stratigraphic studies in the snow and firn of the Greenland ice sheet. *U.S. Snow, Ice and Permafrost Research Establishment. Research Report* 70.

Meier, M. F., 1962. Proposed definitions for glacier mass budget terms. *Journal of Glaciology*, Vol. 4, No. 33, p. 252–63.

[International Hydrological Decade.] 1970. *Combined heat, ice and water balances at selected glacier basins: a guide for compilation and assemblage of data for glacier mass balance measurements*. Paris, UNESCO/IASH. (Technical Papers in Hydrology, 5.)

https://doi.org/10.3189/S0022143000022449 Published online by Cambridge University Press