SOLUTIONS TO DIFFERENTIAL EQUATIONS VIA FIXED POINT APPROACHES: NEW MATHEMATICAL FOUNDATIONS AND APPLICATIONS

SALEH SALHAN G. ALMUTHAYBIRI🆔

The central aim of this thesis is to construct a fuller and firmer mathematical foundation for the solutions to various classes of nonlinear differential equations than that currently available in the literature. This includes boundary value problems (BVPs) that involve ordinary differential equations, and initial value problems (IVPs) for fractional differential equations. In particular, we establish new conditions that guarantee the existence, uniqueness and approximation of solutions to second-order BVPs, third-order BVPs and fourth-order BVPs for ordinary differential equations. The results enable us, in turn, to shed new light on problems from applied mathematics, engineering and physics, such as: the Emden and Thomas–Fermi equations; the bending of elastic beams through an application of our general theories; and laminar flow in channels with porous walls. We also ensure the existence, uniqueness and approximation of solutions to some IVPs for fractional differential equations. An understanding of the existence, uniqueness and approximation of solutions to these problems is fundamental from both pure and applied points of view. Our methods involve an analysis of nonlinear operators through fixed-point theory in new and interesting ways. Part of the novelty involves generating new conditions under which these operators are contractive, invariant and/or establishing new *a priori* bounds on potential solutions. As such, we draw on Banach fixed-point theorem, Schauder fixed-point theorem, Rus's contraction mapping theorem, and a continuation theorem due to A. Granas and its constructive version known as the continuation method for contractive maps. The ideas in this thesis break new ground at the intersection of pure and applied mathematics. Thus, this work will be of interest to those who

are researching the theoretical aspects of differential equations, and those who are interested in better understanding their applications.

Some of the research from the thesis has been published in [1–6].

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SALEH SALHAN G. ALMUTHAYBIRI, Department of Mathematics,
College of Science and Arts in Uglat Asugour, Qassim University,
Buraydah, Kingdom of Saudi Arabia
e-mail: s.almuthaybiri@qu.edu.sa