ON THE PROBABILITY OF COLLISION WITH EARTH, AND ORBITAL LIFE-TIME OF BODIES OF ASTEROIDAL AND COMETARY ORIGIN

Kramer E.N., Musiy V.I., Timchenko-Ostroverkhova E.A., Shestaka I.S.
Odessa Astronomical Observatory, Odessa, U.S.S.R.

The radius for gravitational capture of meteoroids by the Earth is

$$\tau = \frac{R+h}{a_0}\left(1-\frac{v_\infty^2}{v_g^2}\right)^{\frac{1}{2}}, \tag{1}$$

where R is the Earth's radius, h is the height where marked deceleration of meteor bodies begins, a_0 is the average distance of the Earth from the Sun, v_∞ and v_g are preatmospheric and geocentric (undisturbed) velocities of the body.

The condition of approach of a meteoroid orbit within the capture distance is

$$a(1-e) \leq a_0 + \tau \ , \ a(1+e) \geq a_0 - \tau \ . \tag{2}$$

Secular perturbations result in slow rotation of the line of apsides. For condition (2) the orbit's approach depends only on the argument of perihelion ω, the complete variation cycle of which is about 10^4-10^5 years. As this value is much less than the orbital lifetime of meteor bodies, the values ω may be considered random. In this case the a priori probability of the orbit's approach can be derived from the equation (Öpik 1951)

$$P_1 = \frac{4\Delta\omega}{2\pi} \tag{3}$$

where $\Delta\omega$ is one of the intervals of critical values of the argument of perihelion favourable for the transit of the meteor body at the distance R+h from the Earth's centre.

Formula (3) allows for the approach of the meteor body to the Earth's orbit at two nodes with the two symmetrical radius vectors of the meteor orbit.

I. Halliday and B. A. McIntosh (eds.), Solid Particles in the Solar System, 199–204.

https://doi.org/10.1017/S0074180900066729 Published online by Cambridge University Press

For

$$a(1-e) < 0.98 \ , \ a(1+e) > 1.02, \ i > 2^\circ$$

the critical interval

$$\Delta\omega = 2\tau(\tan^2\alpha + \operatorname{cosec}^2 i)^{\frac{1}{2}} \tag{4}$$

and consequently the probability of approach is

$$P_1 = \frac{4\tau}{\pi}(\tan^2\alpha + \operatorname{cosec}^2 i)^{\frac{1}{2}} \tag{5}$$

Here α is the angle between the heliocentric Earth radius vector and that of the geocentric velocity of the meteor body

$$\tan\alpha = a(1-e^2)\{e^2-[a(1-e^2)-1]^2\}^{\frac{1}{2}} \ . \tag{6}$$

During the time of orbital approach the average conventional penetration probability of an individual particle into the capture field depends upon elements of its orbit and can be given by equation

$$P_2 = \frac{\tau}{4a^{3/2}}\left[\frac{3-\frac{1}{a}-2\sqrt{a(1-e^2)}\ \cos i}{2-\frac{1}{a}-a(1-e^2)\cos^2 i}\right]^{\frac{1}{2}} \ . \tag{7}$$

The total probability of Earth collision with a cosmic body calculated per time unit (year) is

$$P = P_1 P_2 = \frac{\tau}{\pi a^{3/2}\sin i}\left[\frac{3-\frac{1}{a}-2\sqrt{a(1-e^2)}\ \cos i}{2-\frac{1}{a}-a(1-e^2)}\right]^{\frac{1}{2}} \tag{8}$$

The average number of particles captured by the Earth per year is given by

$$\frac{dN(i,e,a;t)}{dt} = -N(i,e,a;t=0)\cdot P(i,e,a)\cdot\exp[-P(i,e,a)t] \ , \tag{9}$$

where $N(i,e,a;t=0)$ is the initial distribution of a number of particles according to the orbits' elements. Due to reciprocal independence of elements

$$N(i,e,a;t=0) = N_0 p(i)p(e)p(a) \ .$$

Here N_0 is the total initial number of meteor particles moving around the Sun in all possible orbits, $p(i)$, $p(e)$, $p(a)$ being particular probability densities.

https://doi.org/10.1017/S0074180900066729 Published online by Cambridge University Press

Random orbit element values i,e,a (or q) are limited in the terminal intervals. In this case their probability densities are fairly well represented by a β-distribution.

$$p(x,\gamma,\eta) = \begin{cases} \dfrac{\Gamma(\gamma+\eta)}{\Gamma(\gamma)\Gamma(\eta)} \cdot x^{\gamma-1}(1-x)^{\eta-1}, & \text{when } 0\leq x\leq 1,\ \gamma>0,\ \eta>0, \\ 0 \text{ in other cases.} & \end{cases} \tag{10}$$

In Table 1 parameters of two models of a meteor complex (cometary and asteroidal) are given.

Table 1

Cometary orbits				Asteroidal orbits			
Element	x	γ	η	Element	x	γ	η
$0\leq i\leq\pi$	i/π	1.01	1.1	$0\leq i\leq\frac{\pi}{2}$	$2i/\pi$	1.01	10
$0\leq q\leq 5$	$q/5$	1	1	$1.5\leq a\leq 3$	$\frac{a-1.5}{1.5}$	1	1
$0\leq e\leq 1$	e	10	1.01	$0\leq e\leq 1$	e	1	10

Parameters γ and η of a cometary model are chosen so that the distribution of inclination i should be approximately uniform in the range from 0° to 180° and orbits concentrated with eccentricities close to unity. The distribution of perihelion distances q is assumed to be uniform within 0 to 5 AU.

The choice of parameters γ and η of an asteroid model provides uniform distribution of values of major semiaxis a in the range of 1.5 to 3 AU and has orbits concentrated toward small eccentricities and inclinations. This model provides the possibility of the penetration of bodies of asteroidal origin into the Earth's orbit.

The integrated influx of cometary and asteroidal bodies is calculated by equations respectively

$$\left(\frac{dN}{dt}\right)_c = N_c \int_0^{\pi} di \int_0^1 de \int_{(1-e)/(1+e)}^{1/(1-e)} p_c(i)p_c(e)p_c(q)P_c \exp(-P_c t)dq \,, \tag{11}$$

https://doi.org/10.1017/S0074180900066729 Published online by Cambridge University Press

$$\left(\frac{dN}{dt}\right)_a = N_a \int_0^{\pi/2} di \int_0^1 de \int_{1/(1+e)}^{1/(1-e)} p_a(i) p_a(e) p_a(a) P_a \exp(-P_a t) da \quad , \quad (12)$$

where N_c and N_a are general initial (at the moment t=0) quantities of cometary and asteroidal bodies, index "C" is referred to cometary, "a" to asteroidal orbits.

Let us limit the fields of orbital element variations i,e,a (or q) for the meteor bodies to be within Jupiter's orbit to neglect the capturing and dissipating effect of this planet. Then the orbital distribution of elements will depend mostly upon the Earth's effect.

With t=0 the portion of orbits captured by the Earth yearly is

$$\frac{\left(\frac{dN}{dt}\right)_c}{N_c} = \int_0^{\pi} di \int_0^1 de \int_{(1-e)/(1+e)}^{1} p_c(i) p_c(e) p_c(q) P_c dq \qquad (13)$$

for the cometary model and

$$\frac{\left(\frac{dN}{dt}\right)_a}{N_a} = \int_0^{\pi/2} di \int_0^1 de \int_{1/(1+e)}^{1/(1-e)} p_a(i) p_a(e) p_a(a) P_a da \qquad (14)$$

for the asteroidal model.

Let us assume, for instance, that $N_c = N_a$, i.e. that the total number of cometary and asteroidal bodies present within Jupiter's orbit at the present time is the same. In this case numerical integration of equations (13) and (14) implies that the number of bodies of cometary origin penetrating into the atmosphere yearly is nearly ten times more than that originating in the asteroidal belt.

As the meteor body moves in the Earth's atmosphere, its initial mass M_∞ decreases as a result of ablation. The terminal mass is approximately

$$M_{term} \approx M_\infty \exp\left[-\frac{\sigma V_\infty^2}{2}\right] ,$$

where σ is the ablation coefficient.

Practically it implies that meteor bodies approaching the Earth with velocities $V_g > 20$ km s^{-1} ($V_\infty > 22$ km s^{-1}) are entirely destroyed in the atmosphere. Orbital elements of such bodies must satisfy

$$\frac{1}{a} + 2\sqrt{a(1-e^2)}\ \cos i > 2.56\ . \tag{15}$$

Calculations show that nearly 80% of bodies in cometary model orbits obey inequality (15). Consequently, fireballs of cometary origin do not drop meteorites. Further, one should consider the exceptionally fragile structure of cometary bodies and their crumbling into comparatively small fragments. Consequently, even those bodies which enter the atmosphere with velocities $V_\infty < 22$ km s^{-1} may not reach the Earth's surface either because of small initial mass or because of their further fragmentation in the Earth's atmosphere.

Such fragmentation is rather often observed, particularly in the terminal part of the trajectory, in photographs of bright meteors and fireballs obtained with the instantaneous exposure method (Kramer 1965, Babadzhanov and Kramer 1967, Kramer 1968a, 1968b, Babadzhanov and Kramer 1968, Kramer et al. 1978). All this obviously accounts for the fact that meteor bodies of cometary origin never reach the Earth's surface.

Nearly all the meteor bodies moving in asteroidal-model orbits penetrate the Earth's atmosphere with velocities $V_\infty < 22$ km s^{-1}. With a rather large preatmospheric mass they can fall out on the Earth as meteorites. However, in the general flux of meteor bodies of asteroidal origin those of rather small mass are predominant. On this account and the preponderance of friable meteoroids, presumably there fall as few as three meteorites per several thousands of bright and superbright meteors photographed by fireball networks (McCrosky and Posen 1968, McCrosky and Ceplecha 1968, Ceplecha et al. 1973, McCrosky et al. 1976, Ceplecha 1977) -Pribram, Lost City and Innisfree (Ceplecha 1961, McCrosky et al. 1971, Halliday et al. 1978).

The average orbital lifetime (characteristic time of capturing by the Earth) for fireballs of asteroidal models (with $q \approx 0.95$ AU) is approximately 3×10^8 years. Cometary bodies are nearly of the same lifetime, if their orbits lie inside Jupiter's orbit (i.e. when one can neglect Jupiter's effect).

About 15% of all fireballs photographed by the Prairie Network (McCrosky et al. 1976) have orbital lifetimes from 10^7 to 10^8 years. Nearly the same percentage applies to the fireballs captured by the Earth from 10^9 to 10^{10} years whereas over 50% of all fireballs have orbital lifetimes from 10^8 to 10^9 years.

The existence of orbits with lifetimes of the order of 10^8-10^9 years and more is due to the orbits' transformation (in particular to Poynting-Robertson effect, at least for relatively small bodies). Those whose orbital lifetime is of the order of 10^7 years seem to have originated from recent fragmentation of bigger parental bodies whose orbits are of longer lifetime (Simonenko 1978).

REFERENCES

Babadzhanov, P.B. and Kramer, E.N.: 1967, Doklady Akademii Nauk SSSR, 177, pp.1043-1046.

Babadzhanov, P.B. and Kramer, E.N.: 1968, In "Physics and Dynamics of Meteors", L. Kresak and P.M. Millman (ed.), D. Reidel Publ. Co., Dordrecht-Holland, pp.128-142.

Ceplecha, Z.: 1961, Bull. Astron. Inst. Czech. 12, pp.21-47.

Ceplecha, Z.: 1977, ibid. 28, pp.328-340.

Ceplecha, Z., Jezkova, M., Bocek, J., Kirsten, T. and Kiko, J.: 1973, ibid. 24, pp.13-22.

Halliday, I., Blackwell, A.T. and Griffin, A.A.: 1978, J. Roy. Astron. Soc. Can. 72, pp.15-39.

Kramer, E.N.: 1965, Geomagnetism i Aeronomia 5, pp.276-283.

Kramer, E.N.: 1968a, Meteoritika 28, pp.54-59.

Kramer, E.N.: 1968b, In "Physics and Dynamics of Meteors", L. Kresak and P.M. Millman (ed.), D. Reidel Publ. Co., Dordrecht-Holland, pp.236-238.

Kramer, E.N., Skoblikov, A.S., Skoblikova, L.Ja.: 1978, Astronomiczeskiy Circular, Astronomical Report of USSR Academy of Science, No.999, pp.4-5.

McCrosky, R.E. and Ceplecha, Z.: 1968, Smithsonian Astrophys. Obs. Special Report, No. 288.

McCrosky, R.E. and Posen, A.: 1968, ibid. No. 273.

McCrosky, R.E., Shao, C.Y. and Posen, A.: 1976, Smithsonian Center for Astrophysics Reprint. Series No.665.

Öpik, E.J.: 1951, Proc. Roy. Irish Acad. 54, pp.165-199.

Simonenko, A.N.: 1978, Meteoritika 37, pp.3-24.

Editors' note: Tables referenced in this paper may be obtained from the authors.

https://doi.org/10.1017/S0074180900066729 Published online by Cambridge University Press