DYNAMOS OF THE SUN AND STARS, AND ASSOCIATED CONVECTION ZONE DYNAMICS

Peter A. Gilman
High Altitude Observatory
National Center for Atmospheric Research

1. Introduction

I see my assignment in this talk as being to focus on the interaction between the magnetic fields produced by dynamo action and the dynamics of the fluid flow which drives this dynamo, and to make some connections to the solar and stellar dynamo problems. To do this really requires we start with a fluid dynamical model that satisfies relevant laws of fluid dynamics, and in which the flow can actually respond to the induced magnetic field. Thus the so-called kinematic dynamo models are not enough for our purposes, and we must address the full MHD dynamo problem in a self-consistent way. For example, we do not allow ourselves the license to vary independently the convection and differential rotation, as is commonly done in kinematic dynamo calculations, because the laws of physics do not allow that. We have been attempting to do self consistent MHD dynamo modeling at Boulder for the past several years, starting from a nonlinear fluid dynamical model for convection in a rotating spherical shell. This model we believe is physically complete in itself, with a minimum of ad hoc assumptions. It is much simpler than the real sun, but contains a lot of the physics we consider most relevant to the solar and stellar dynamo problem. I like to view the model more as an analog to the solar or a stellar convection zone, rather than as an approximation--much as a laboratory rat or monkey is used as an analog to a human being in many medical experiments. Each exists and is physically complete, and much, though not all, of their biochemistries are the same or are closely related.

Our model is for fully three-dimensional, nonaxisymmetric, nonlinear convection in a deep shell initially in uniform rotation and heated uniformly from inside. (Details of the computational approach are given in Gilman, 1975, 1977.) It has a central gravity, and constant viscosity and thermal conductivity (which we are forced to compare to eddy viscosities and thermal diffusivities on the sun). Stress free

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velocity boundary conditions are applied at top and bottom boundaries along with constant heat flux bottom and constant temperature top. The fluid is currently incompressible but a compressible version is now undergoing testing and should be operational this fall. (And we do have some results for linear compressible convection in a deep rotating spherical shell, see, eg. Gilman and Glatzmaier (1981), Glatzmaier and Gilman (1981a,b,c, 1982).

My plan in this talk is first to comment on the observational constraints provided by the sun which seem most relevant to the MHD dynamo problem, and then summarize the hydrodynamic results bearing on those constraints. Then I will discuss what kind of dynamo behavior we actually get with the model and contrast that with the observations. Finally, I will generalize and extrapolate some to a more general picture of rotating convectively driven spherical shell dynamos, and comment on possible implications of these generalizations for stellar dynamos.

2. Observational constraints

You all know the principal observational characteristics of solar cycles--the migration of spot zones toward the equator, Hale's sunspot polarity law, field reversals at the poles near cycle maximum, amplitude variations from cycle to cycle, and between north and south hemispheres, etc. In addition, the surface differential rotation of the sun is well known. As Robert Howard has told you, there also appear to be small amplitude ($\pm \sim 5$m/sec) torsional oscillations on this mean rotation profile, linked in phase to the solar cycle, but extending to much higher latitude. The search for giant cell convective velocities has so far revealed only fleeting glimpses, and an upper limit of roughly 10m/sec per longitudinal wave number (LaBonte, Howard and Gilman, 1981, and earlier references cited therein). There is some indication of other rotation rate changes with time both within a cycle and from cycle to cycle, of a few percent. For example, Eddy et al (1978) reported from sunspot measurements a declining rotation rate in the first half of the 20th century, followed by a leveling off and possible increase. There is a suggestion in these data of an inverse correlation between amplitude of the cycle envelope, and low-latitude rotation, which may be a significant clue regarding the long-term workings of the solar dynamo.

The most important missing element in our knowledge of solar rotation for the dynamo problem is its variation with depth. The faster rotation rate of spots compared to the plasma has long been interpreted as evidence of an increase of angular velocity with depth (Foukal, 1972). Angular velocity increases with depth near the surface were suggested by Deubner et al. (1979) from frequency shifts in 5 minute oscillations, but the errors are large and more recent measurements at Kitt Peak (unpublished) do not confirm this result. More startling are new observations by Hill and colleagues (to be published; see also Gough, (1982)) indicating interior rotation rates several times the surface rate. It is not clear yet how much of the proposed increase occurs within the convection zone itself, but such a result, if confirmed, will be very important for solar differential rotation and dynamo theory.

This is because, as many of you know, kinematic dynamo theory applied to the sun favors angular velocity increasing with depth to give migration of the zone of sunspot formation toward the equator with time, while global convective models, at least those in which the strong influence of rotation upon convection in the deep layers of the convection zone is taken into account, favor angular velocity constant on cylinders and decreasing with depth.

As you also know, there is now evidence of luminosity changes associated with sunspot passage across the solar disk (e.g. Willson, et. al., 1981) which imply the magnetic field is modifying the energy output of the sun and causing intermittent storage of energy in the convection zone. Dynamo models will ultimately have to address this question, though none appears capable of doing so now, at least not on sunspot spatial scales.

3. Results from spherical shell convection model relevant to the dynamo problem

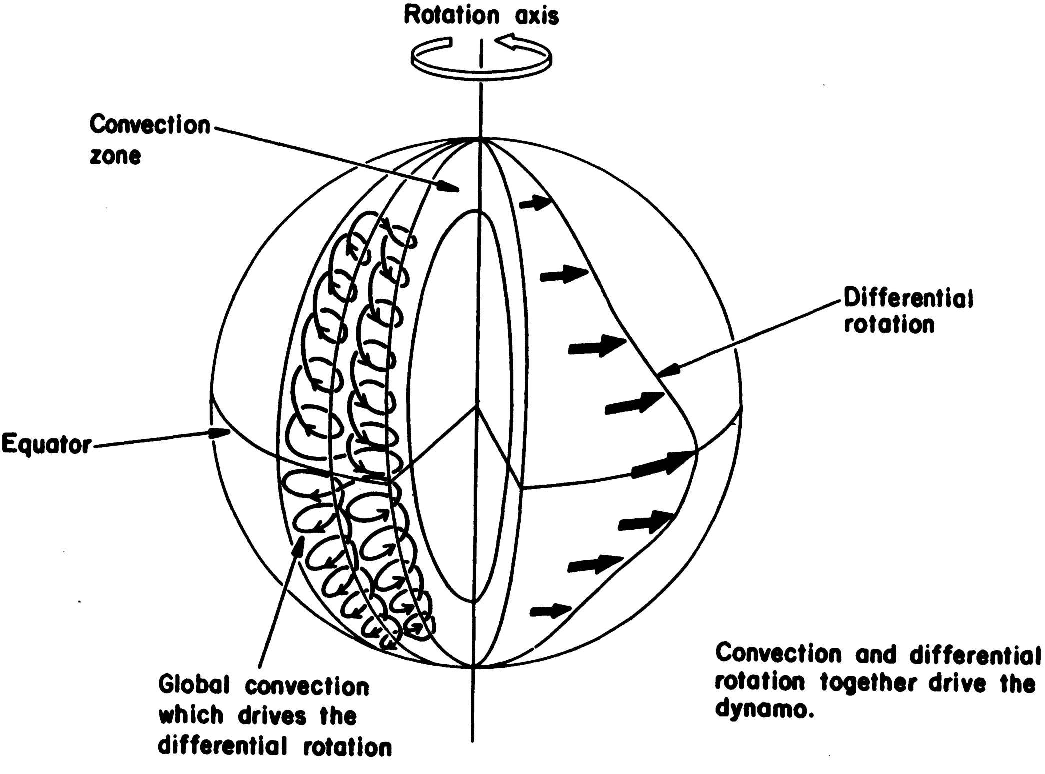

Figure 1: Schematic drawing of global convection and differential rotation from model calculations.

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We have now performed a large number of numerical experiments for nonlinear convection of an incompressible fluid in a rotating spherical shell, varying the convection zone depth, heating rate, viscosity and rotation rate. One general conclusion from all these calculations is that in order for the convection to sustain a large amplitude, broad equatorial acceleration such as the sun has, two conditions must be met: First, the convection zone must be relatively deep, roughly one-third of the radius or more. Second, the influence of rotation upon the convection must be strong so that the turn over time for the convection is not short compared to the rotation time. This condition should be met in the deep part of the solar convection zone. In shallow convection zones the equatorial acceleration has small extent in latitude, and if the influence of rotation upon the convection is weak, the differential rotation is weak, and is in the form of an equatorial deceleration even slightly above critical for convection to occur.

A schematic picture of the global convection and differential rotation is shown in Figure 1. Convective rolls with a north-south axis are preferred due to rotation (if the rotational influence is very strong, the roll axis will tend to line up with the rotation axis, rather than bend with latitude as seen here). The horizontal velocity vectors in these rolls are tilted with respect to the east-west direction (due to the coriolis force) in such a way that fast moving particles are also moving toward the equator. This results in a Reynolds stress and net angular momentum flux toward the equator from high latitudes, which is how the equatorial acceleration is produced. The strong influence of rotation upon the convection leads to a strong preference for north-south rolls with this property, allowing a large amplitude differential rotation to build up. Also, with strong influence of rotation, fluid particles moving radially do not conserve angular momentum but are instead subject to longitudinal pressure torques that keep the particle paths from deviating far from the radial direction -- this is a manifestation of the near "heliostrophic" balance between pressure gradient and coriolis forces, which allows the convection to continue to release potential energy and transport heat efficiently. As a consequence radial angular momentum transport is either outward, or weakly inward. In either case, a surface equatorial acceleration is maintained.

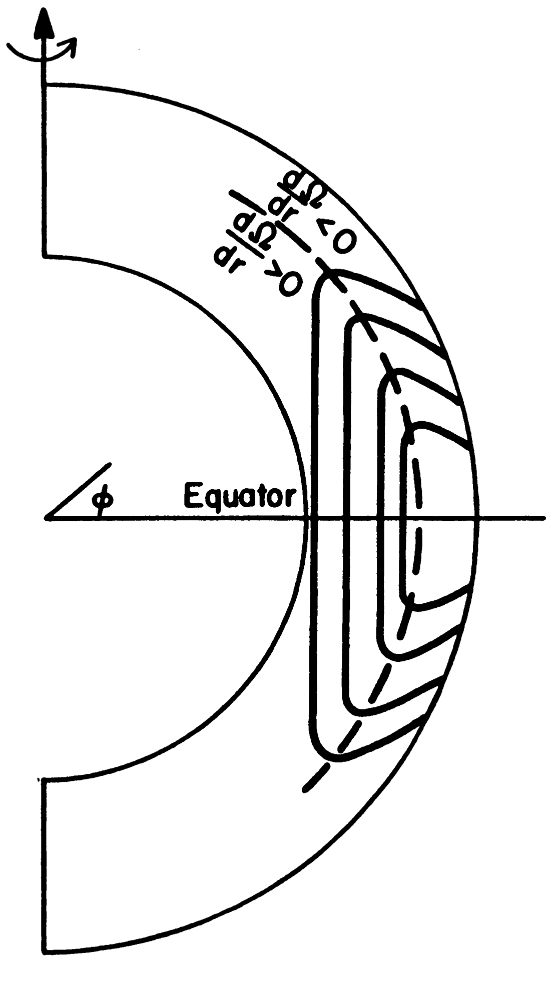
In addition, also due to the constraint of rotation, the angular velocity tends to be constant on cylinders (the Taylor - Proudman theorem), and therefore decreases with depth. This is also a quite general result, modified only when some additional force is large enough to compete with pressure gradients and coriolis forces. It does not depend, for example, on the detailed properties of turbulent transport, unless such transport is large enough to upset the heliostrophic balance. In our model calculations, this can be made to occur by increasing the buoyancy force by increasing the heating rate. Then inward radial angular momentum transport due to departures from heliostrophic balance begins to dominate. But the first consequence of this is to wipe out the surface equatorial acceleration, and then to change the radial gradient so angular velocity increases inward. Because of the loss of equatorial acceleration, the solution ceases to be of interest for the sun, but may be for other stars (see e.g. Gilman, 1980).

The relative amplitudes of differential rotation and the convection which drives it are functions of, among other things, the fluid viscosity. Generally speaking, the lower the viscosity, the larger the differential rotation that can be produced. In an earlier paper (Gilman, 1980) I reported that it appeared that an upper limit to differential rotation energy was perhaps 40% of the total kinetic energy. That estimate was erroneous as we have now found, by reducing the viscosity, examples for which the differential rotation is as much as 80% of the total kinetic energy, the convection only 20%. I mention this because this ratio of energies is very important for determining the kind of dynamo behavior we get, for example, whether magnetic cycles occur or not. Solutions with relatively low amplitude global convection are also easier to fit within the observational upper limits for giant cells.

If we return to Figure 1, we can see the elements needed for dynamo action: differential rotation, and a helical structure to the convection, such that in the northern hemisphere, fluid moving toward the pole has a clockwise rotation, fluid moving toward the equator a counterclockwise rotation. This results in left-handed or negative helicity (scalar product of velocity and vorticity) in the northern hemisphere, right handed or positive in the southern. The shearing due to differential rotation will generate toroidal field from poloidal field, while the transporting and twisting of fields by the helicity will generate new poloidal field. The relative strengths of these two processes clearly depends on the relative strengths of convection and differential rotation.

We can also see from Figure 1 a reason why one would like to do the MHD rather than the kinematic dynamo problem: the helicity arising from the spiral structure of the flow, and the angular momentum transport arising from the tilt of the velocity vectors, are obviously closely linked. A change in helicity due to a change in the convection is likely to be accompanied by a change in the rate of angular momentum transport, and therefore in the differential rotation.

How much of what we have just said concerning convection of an incompressible fluid in a rotating spherical shell is likely to carry over to the compressible case? The answer is that we do not know for certain, because no comparably nonlinear calculations have yet been done. However, from the work of Glatzmaier and myself for linear compressible convection, extended to the initial tendency for nonlinear convection in Glatzmaier and Gilman (1982), we can make preliminary statements. In particular from compressible convective modes strongly influenced by rotation that do extend the full depth of the convection zone, we still get angular momentum transport toward the equator from high enough latitudes to produce a broad equatorial acceleration. The helicity of these modes also looks to be similar to that of the incompressible convection. However, the radial gradient of angular velocity they produce may change sign at mid-depth, with angular velocity constant on cylinders only in the deep part of the convection zone where the influence of rotation upon the convection is strongest. A typical angular velocity profile might look like that of Figure 2. This result does depend upon what is assumed for both the relative magnitude, and depth dependence, of the viscosity and thermometric diffusivity. Its reality and magnitude (and generality) in the fully nonlinear case



DIFFERENTIAL ROTATION $\Omega(r,\phi)$ (Compressible Model - Glatzmaier & Gilman Initial Tendency Calculation)

Figure 2: Schematic profile of differential rotation estimated from first calculations of compressible convection in a rotating spherical shell of Glatzmaier and Gilman (1982) when convective modes extend to full depth of convection zone.

needs to be tested. If such a radial angular velocity gradient reversal is present somewhere in the middle of the solar convection zone, it would have important consequences for solar dynamo models, because it could result in opposite directions of migration in latitude of the toroidal field above and below the level of gradient reversal. We return to this point after discussing our dynamo results from the incompressible model.

4. Dynamo results from an incompressible convection model

a) Early solutions

Our numerical experiments to study the dynamo action of convection in a rotating spherical shell are usually begun by first establishing a finite amplitude statistically stationary state for the convection and the differential rotation it drives, and then adding a small seed magnetic field and following its subsequent growth and evolution as well as the full nonlinear feedbacks on the inducing motions. We concentrated on hydrodynamical solutions for which the differential rotation at the outer boundary was about equal in amplitude and profile to the observed solar differential rotation. The first experiments, described in Gilman and Miller (1981) were for hydrodynamic states in which the convection contained initially at least two-thirds of the total kinetic energy of the flow, the differential rotation no more than one-third. To our dismay, we were not able to find any field reversing dynamos, but only more random dynamo action which maintained a broad magnetic spectrum. The reason was evident from a study of the energetics of the solutions: the differential rotation was not large enough, compared to the convection, to contribute the dominant mechanism for maintenance of the toroidal field. In this case, the helicity of the convection predominated in maintaining all components and scales of field. In the language of turbulent dynamo theory, our system behaved more like an " α^2" dynamo than an " $\alpha - \omega$ " dynamo.

b) Solutions with lower viscosity

The only way around this difficulty was to look for hydrodynamic states in which differential rotation was a much larger fraction of the total kinetic energy of the system. These we found by reducing the viscosity and thermometric diffusivity of the system each by a factor of 10 (still within the range of uncertainty of mixing length estimates, when comparing to the sun). The reason this works is that viscous dissipation is what limits the amplitude of differential rotation in the model, so a smaller viscosity allows a smaller Reynolds stress, and therefore smaller convection amplitude, to maintain the same amplitude differential rotation. Smaller viscosity also implies the influence of rotation upon the convection is stronger. For solar amplitude differential rotation, these new solutions have a kinetic energy spectrum for the convection which falls near the upper limit from observations as calculated in LaBonte, Howard and Gilman (1981).

With these new solutions, we have found several examples of cyclic dynamos, together with a variety of electromagnetic feedbacks. A typi-

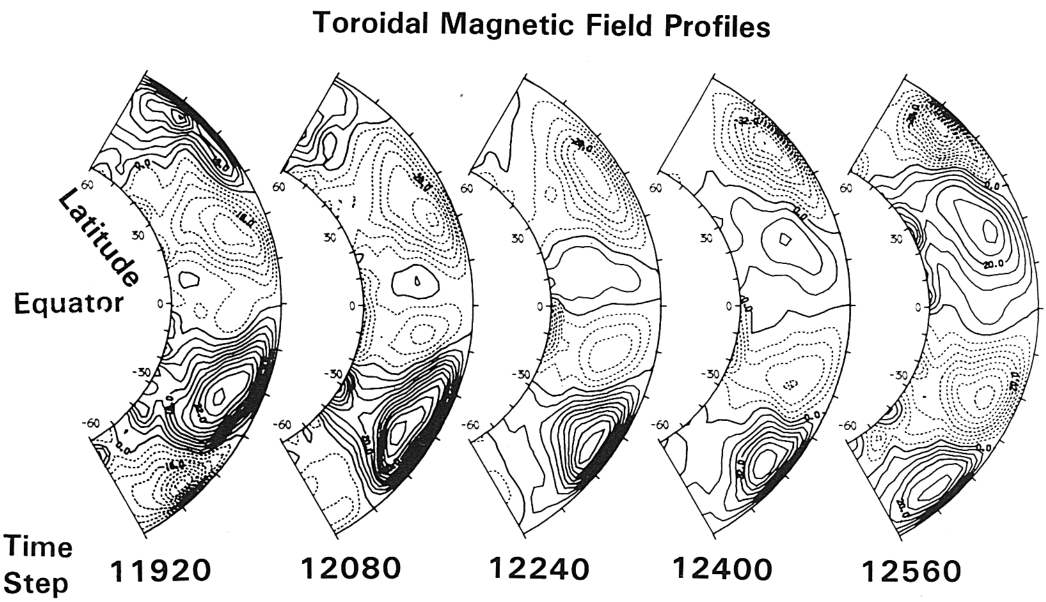


Figure 3: Contours of toroidal magnetic field amplitude for a sequence of time steps from typical solution of the convectively driven dynamo model. Solid contours represent positive field (into page) dashed contours, negative field.

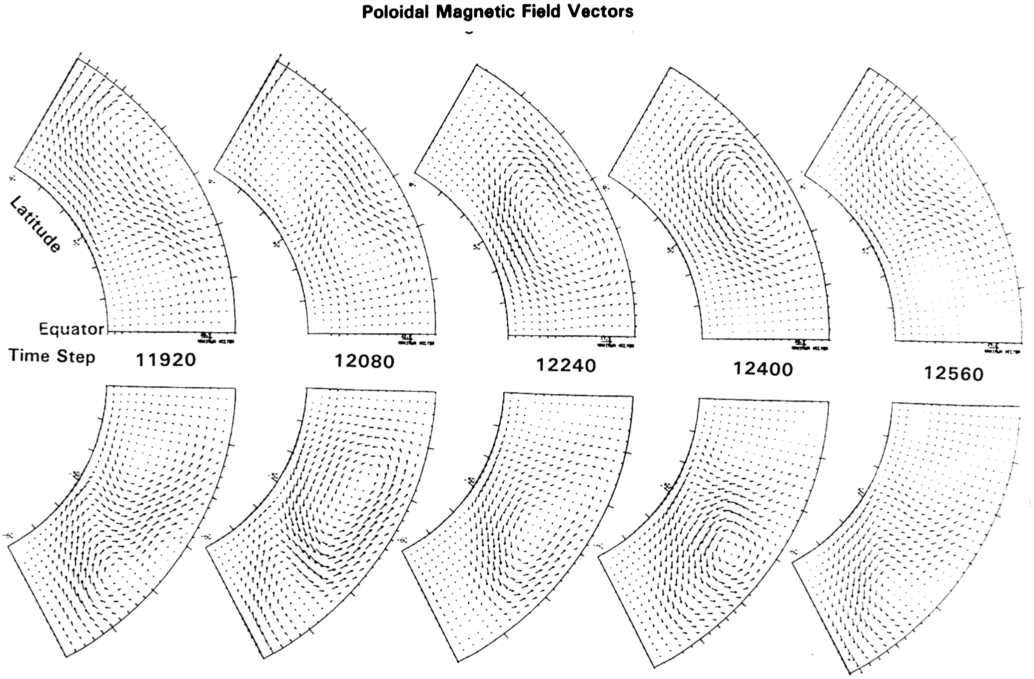


Figure 4: Poloidal field vectors for same time steps as in Figure 3.

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cal example of toroidal and poloidal fields from such a solution is shown in Figures 3 and 4. Solid and dashed contours represent toroidal field amplitudes of opposite signs. (The calculations are cut off at 60 latitude for computational stability, an artifact that needs to be removed in subsequent models). From Figure 3, one can see that a new magnetic cycle in toroidal field begins deep in the convection zone near the equator, then grows in amplitude and expands upward and toward the poles, reaching the outer boundary in mid-latitudes. In Figure 4, the poloidal field migrates along with it, producing the strongest surface poloidal fields near the high latitude end of the toroidal field cycle. In a qualitative sense, the migration is as predicted from $\alpha - \omega$ dynamo theory.

I call your attention to another detail, namely the tendency for the poloidal field vectors in high latitudes in the interior of the convection zone to line up parallel to the rotation axis, and therefore with the contours of differential rotation, since they are cylindrical. This tendency toward "iso-rotation" appears to be important for limiting cyclic dynamo action to lower latitudes, particularly the region outside the tangent cylinder to the inner boundary. That is, a poloidal field parallel to the rotation contours is not sheared out into a new toroidal field. One can see from Figure 3, particularly in the southern hemisphere, a tendency for the toroidal field to avoid polar regions at deep levels. We will comment on this point further in talking about dynamos in stars generally. The reason the effect occurs in the model is that, at high latitudes, the convective flow itself including particularly the axisymmetric meridional circulation, tends to be parallel to the rotation axis, causing the poloidal field lines they induce to have a similar orientation. This is the opposite of low latitudes, where the dominant convective flow is a rotational one about the roll axis, which is parallel to the rotation axis..

c) Energetics of a typical solution

Rather than look at a lot of details of these complex dynamo solutions, I think it is instructive to instead concentrate on their energetics. Figure 5 shows time traces of the various kinetic and magnetic energies of the same solution as shown in Figures 3 and 4, in dimensionless form. The magnetic cycles show up most clearly in the toroidal field energy -- max to min in each cycle is a change of a factor of two or so. By contrast, the poloidal field energy is much noisier. This is because it is maintained by convection, which itself is much noisier than the differential rotation responsible for maintaining the toroidal field. This is obvious from the kinetic energy traces for these quantities seen near the top of the figure.

From the dimensional time scale drawn in the middle of Figure 5, one can see the cycles this model produces are at least an order of magnitude shorter than for the sun, even though differential rotation has solar amplitude. We return to a discussion of this point later.

The toroidal field energy trace also shows an "envelope" to the magnetic cycle, whose typical time scale is several cycles. This envelope amplitude appears to be connected to long term changes in dif-

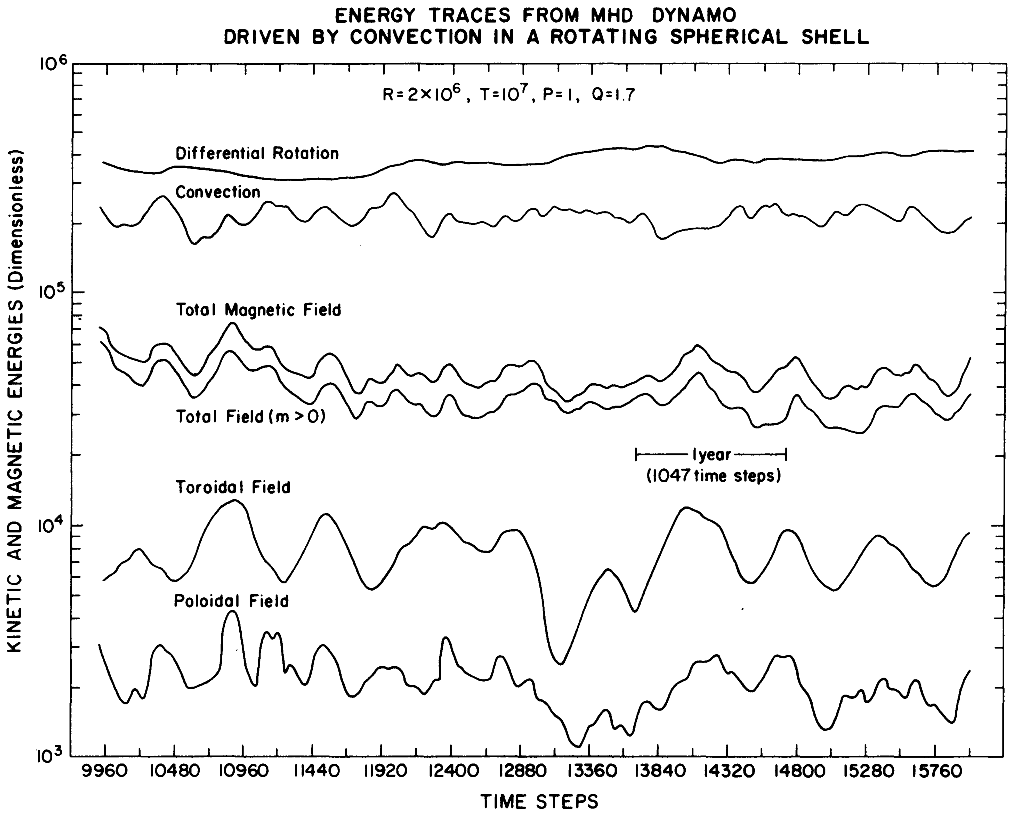

Figure 5: Kinetic and magnetic energies (dimensionless) as a function of time for several thousand time steps from the model solution shown in Figures 3 and 4, illustrating cycle amplitudes and envelope changes. Note the dimensional time scale of 1 year on the figure.

ferential rotation amplitude, in the sense that a declining sequence of cycles is associated with a generally rising differential rotation energy (which is experiencing a much smaller percentage change). This is what we would expect if the j x B force is modulating differential rotation amplitude. Relatively quick change in differential rotation, such as the sharp drop following step 13800, is due to a combination of a sharp rise in j x B force due to previous induction when convection was larger, combined with a drop in Reynolds stress due to a subsequent drop in convection amplitude. This event is followed by a new magnetic cycle of considerably weaker amplitude in toroidal field, a result of the weaker differential rotation.

Thus we can see that in this model, cycle envelope changes are a natural outcome of the interplay between the dynamo and its drivers, as well as the more independent fluctuations in convection itself. The differential rotation does not appear to respond much to the change of j x B force in an individual cycle. This is probably because the cycles

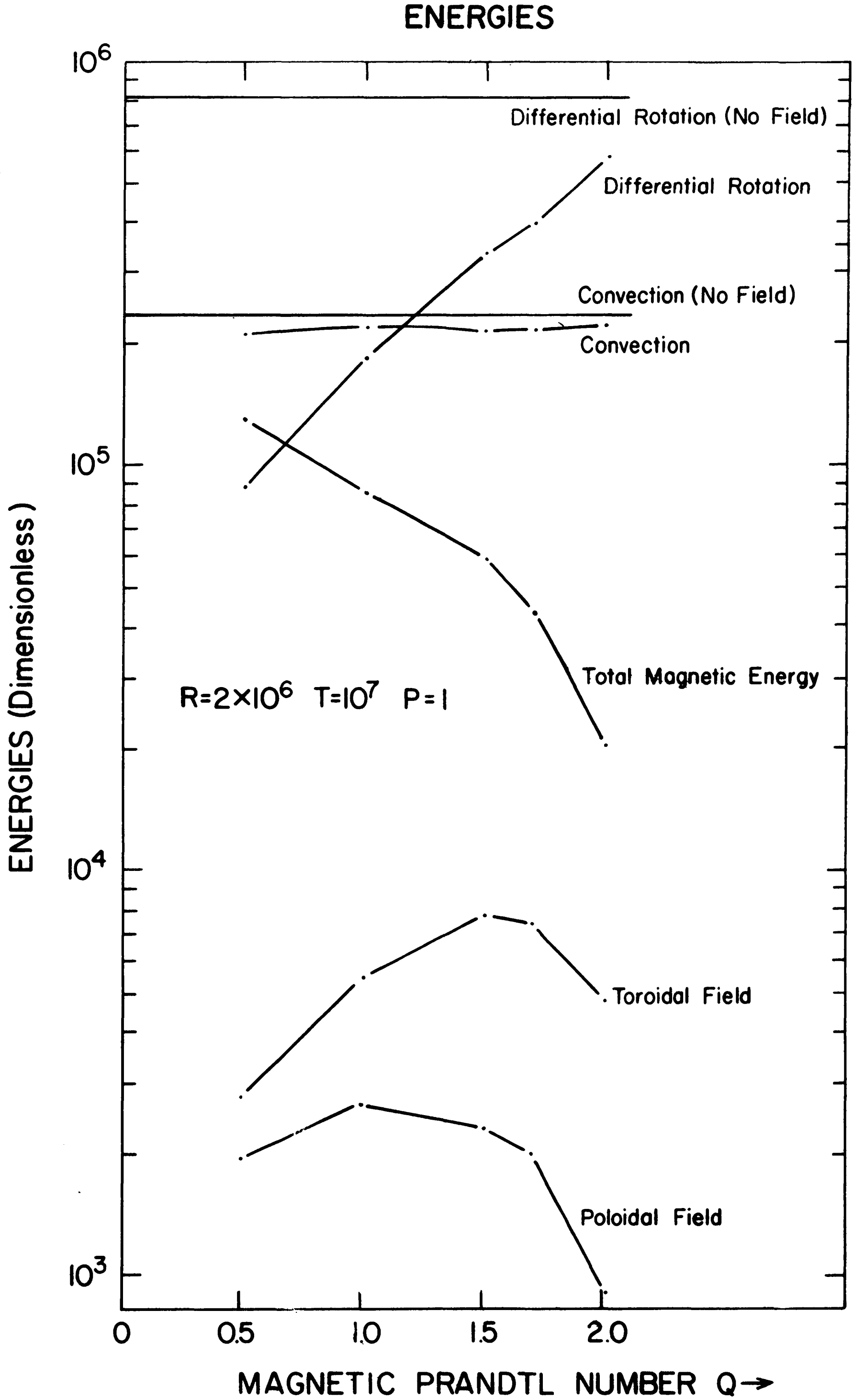

Figure 6: Average kinetic and magnetic energies from various dynamo solutions as a function of magnetic Prandtl number $Q = \eta / \kappa$, in which η is the magnetic diffusivity, and κ the thermometric diffusivity.

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are relatively short, only a few convective turnover times, while the differential rotation changes more on its own internal "spin up" time, which is, for the viscosity chosen, considerably longer. Were the induced magnetic cycles much longer, we would expect to see a single cycle modulation of differential rotation energy as well.

It is worth noting that the differential rotation changes we find correspond to changes in absolute rotation velocity at the equator of only 1 - 2%, so they are in the range discussed from observations.

d) Effects of decreasing magnetic diffusivity: increasing feedbacks

We have found it particularly instructive to study changes in behavior of the dynamo as a function of the assumed magnetic diffusivity η. We measure that quantity in our model by a magnetic Prandtl number $Q = \eta / \kappa$, in which κ is the thermometric diffusivity. (In virtually all our calculations $\kappa = \nu$, the kinematic viscosity, so the ordinary Prandtl number is unity). Then the smaller is Q, the stronger dynamo action we should get, since the magnetic Reynolds number, which is proportional to Q^{-1}, is rising. So by varying Q for the same initial hydrodynamic solution, we can examine how the dynamo changes as we rise above the threshold for dynamo action. In our particular case, this threshold for small amplitude magnetic fields falls between Q = 1.7 and 2.0.

Figure 6 displays average kinetic and magnetic energies for several solutions of the system as a function of Q. Several effects are evident. First, as Q is decreased, the total magnetic energy rises, as we would expect, but the toroidal and poloidal field energies first rise, and then fall off, so that by Q = 0.5 together they constitute only 3% or so of the total magnetic energy of the system, as compared to 30% at Q = 2.0. Through this range of Q, the convection energy hardly changes at all, indicating very little feedback on it; by contrast, the differential rotation has dropped steeply, to the point that it becomes much smaller than the convection which drives it. (Incidentally, the dynamo solution at Q = 2.0 represents a finite amplitude dynamo field not reachable from a small amplitude initial field, which would decay away -- we found it instead by using a Q = 1.7 solution as an initial state.) The change in surface profile of differential rotation is shown in Figure 7, indicating it remains an equatorial acceleration as Q is decreased, but is simply of smaller amplitude (its cylindrical profile with depth is also retained).

What is happening is that as Q is decreased, j x B forces take over from viscous forces in braking the differential rotation, as can be seen in Figure 8, which plots the dimensionless rates at which work is done to buildup (+) or brake (-) the differential rotation energy. These rates are also normalized with respect to the viscous dissipation rate (always -1 unit). At Q = 2.0, viscosity dominates, while by Q = 0.5, the balance is principally between Reynolds stress and Maxwell stress, with viscous dissipation only about 1/3 of the total. (Note that coriolis forces acting on the meridional circulation do not play a significant role at any Q).

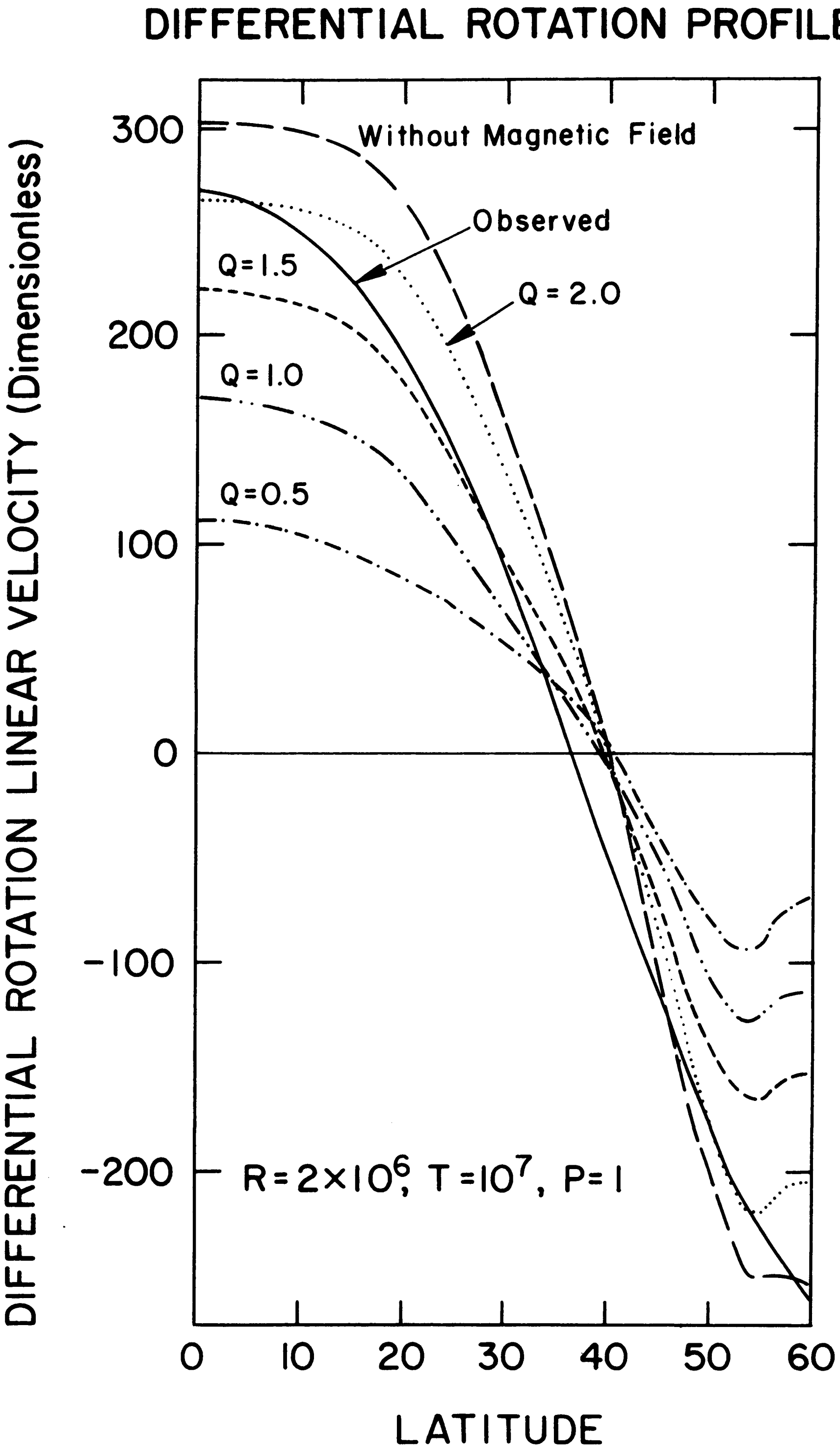

Figure 7: Dimensionless surface differential rotation profiles from the model for several values of Q, illustrating the damping by j x B forces. A typical average differential rotation for the sun is plotted for reference in the same units.

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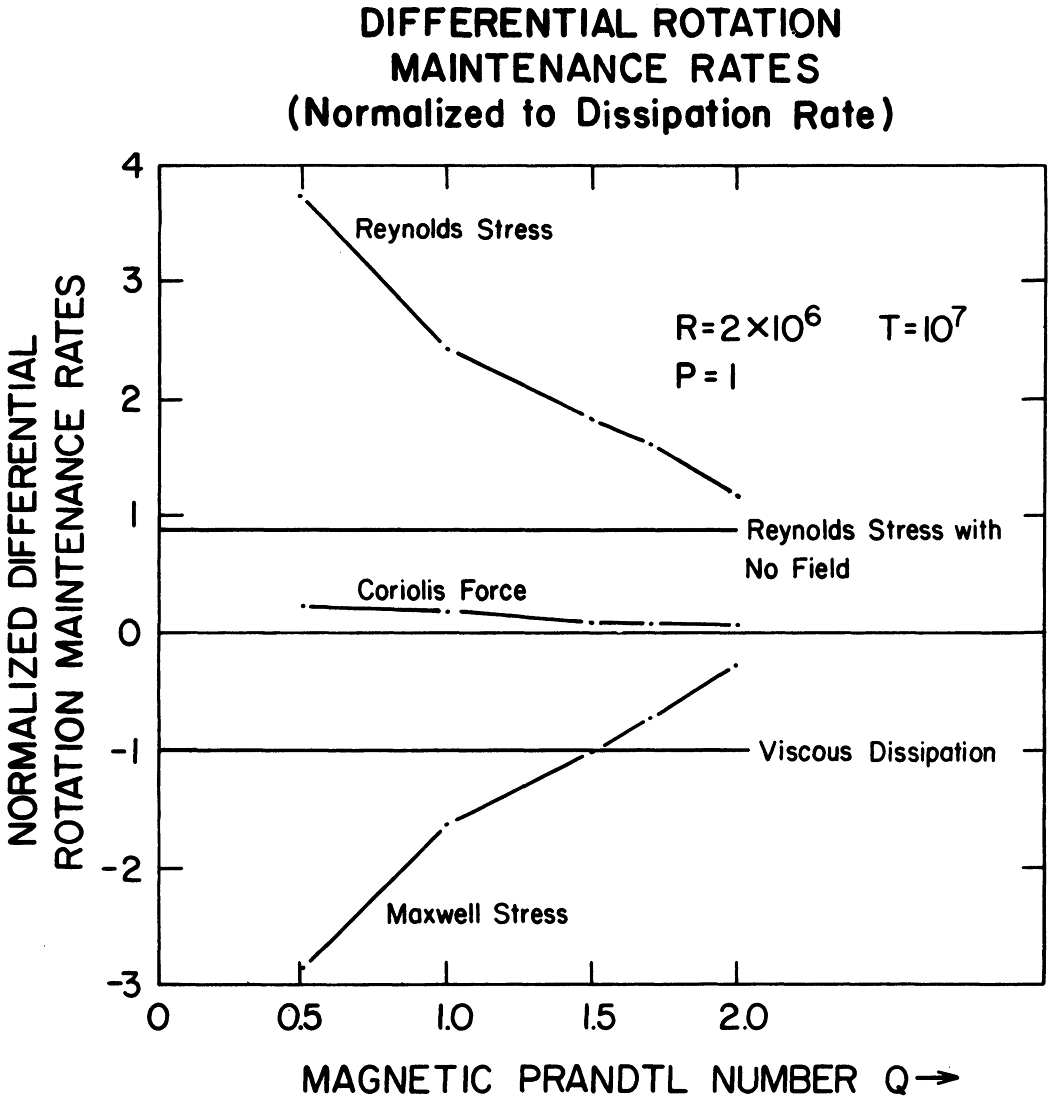

Figure 8: Rates at which the differential rotation is maintained (+) or braked (-), by Reynolds stresses, Maxwell stresses, and coroilis forces acting on the meridional circulation as functions of the magnetic Prandtl number. All rates are integrals over the whole volume of the fluid, and are normalized with respect to the viscous dissipation rate.

While the j x B force on the differential rotation is large enough to take over from viscosity in braking it, it is not so large that it interferes with the basic "heliostrophic" balance between pressure gradient and coriolis forces, so the Taylor-Proudman constraint is still satisfied and the differential rotation remains constant on cylinders. The j x B force affects the differential rotation at higher Q more than it does the convection because the viscous dissipation of differential rotation is relatively small. The profile is much larger scale than is

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that of the convection which has much fine structure. There must be a value of Q below which convection, too, is damped, thus limiting the total magnetic energy, but we have not investigated this regime.

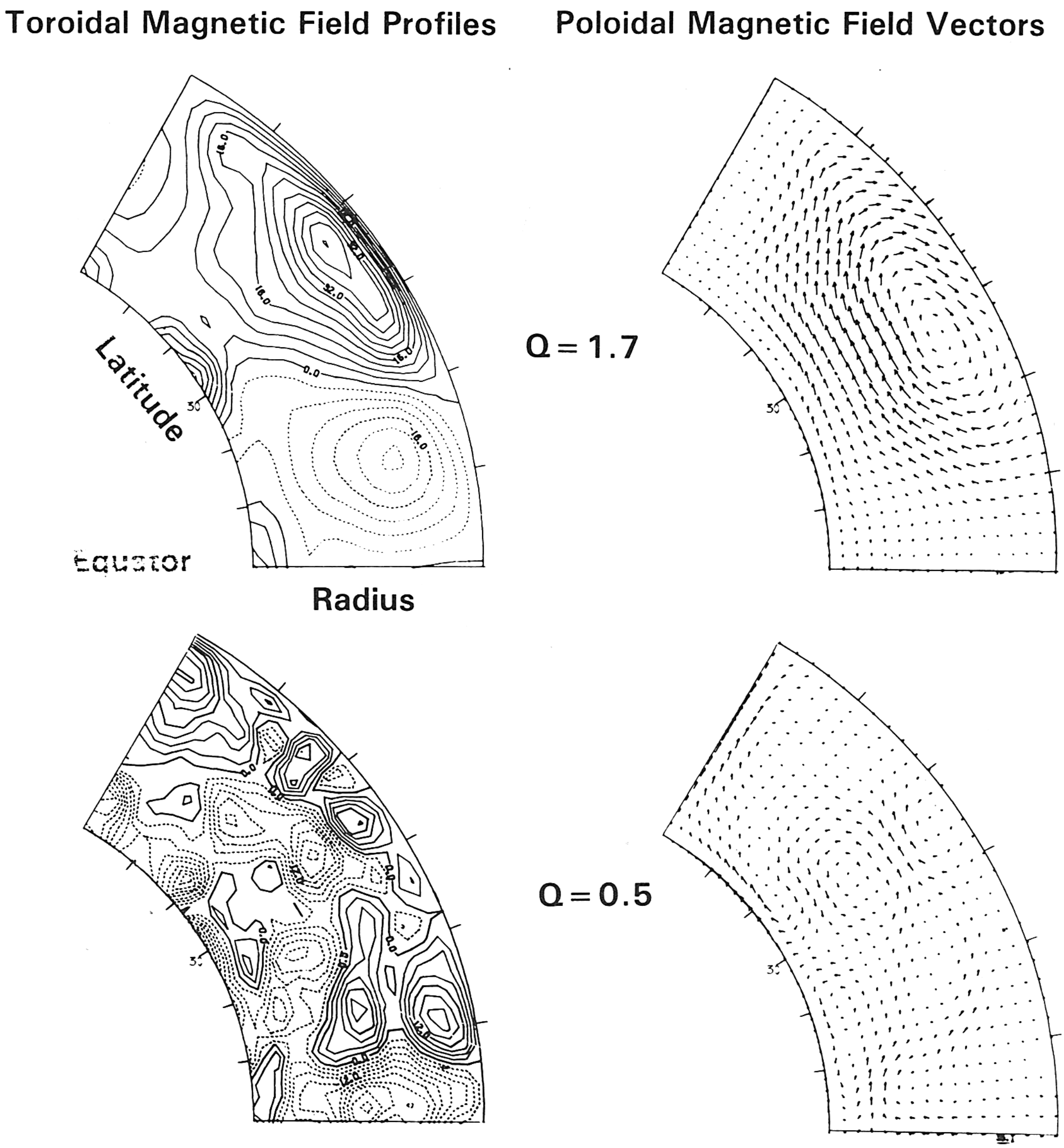

Figure 9: Sample toroidal and poloidal field profiles for a high Q and low Q case, showing break up of organized global field into much more chaotic form as the differential rotation drops.

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As a result of this damping of differential rotation with decreasing Q, less toroidal field is induced from poloidal field, and the dynamo changes its character. Typical toroidal and poloidal field profiles are shown in Figure 9. By Q = 0.5, both poloidal and toroidal fields have lost their global coherence, and have just as much fine structure as the small scale fields at high longitudinal wave number in the model. Furthermore, the magnetic cycle has disappeared. It does not seem to be hidden in the noise of the instantaneous toroidal field profile for Q = 0.5, it is really not present at all. If one looks at the rates of maintenance of toroidal field shown in Figure 10, one can see what has happened. At Q = 2.0, the dominant mechanism of toroidal field maintenance was shearing of the poloidal field by differential rotation. The effect of helicity, or the " α effect", which is the sum of shearing and transport of field by the convection, is actually negative. But by Q = 0.5, the differential rotation has decreased to the point that transport and shearing by convection is now the dominant mechanism in maintaining the toroidal field. We are back closer to the " α^2 dynamo" case, rather like what we found with higher viscosity (although our "mean field" is not smooth and coherent on a global scale). Thus, the dynamo is capable of shutting off its own cycles; and for it to retain a cycle requires the sum of poloidal and toroidal field be a substantial fraction of the total magnetic energy of the system (in these model solutions, nearly 10%). This also illustrates how important a strong differential rotation is to produce cycles in the first place, something long known from kinematic dynamo theory.

It is clear from our results that the dynamo is very sensitive to its feedbacks, because all of these changes occur within a factor of 4 of the threshold magnetic diffusivity for any dynamo action at all.

e) Other feedback effects.

We have already seen that j x B feedbacks can profoundly affect the differential rotation, including its average amplitude and modulations from cycle to cycle. The Q = 2.0 solutions illustrate that a finite amplitude magnetic field can allow a dynamo to be sustained when a weak field would die out. In addition, Gilman and Miller (1981) showed that even a weak magnetic field can completely change the history of convection patterns. Two time dependent solutions for the flow, initially identical but one with a small magnetic field, will diverge away from each other in their amplitude and phase histories of the various convective modes in the spectrum. This precludes long term predictions, and will contribute to a certain randomness in the cycle envelope, and in field amplitudes with time within a given cycle.

One feedback effect we have looked for but not found is magnetically driven torsional oscillations. It turns out the model does have torsional oscillations in it, but these are present even without a magnetic field. They do migrate in the same direction as the toroidal field in the model but a rate which is faster by 40% or so. They have a similar amplitude to that observed on the sun. They appear to be a form of axisymmetric inertial oscillations in which the restoring force is the coriolis force. It is entirely possible magnetically driven tor-

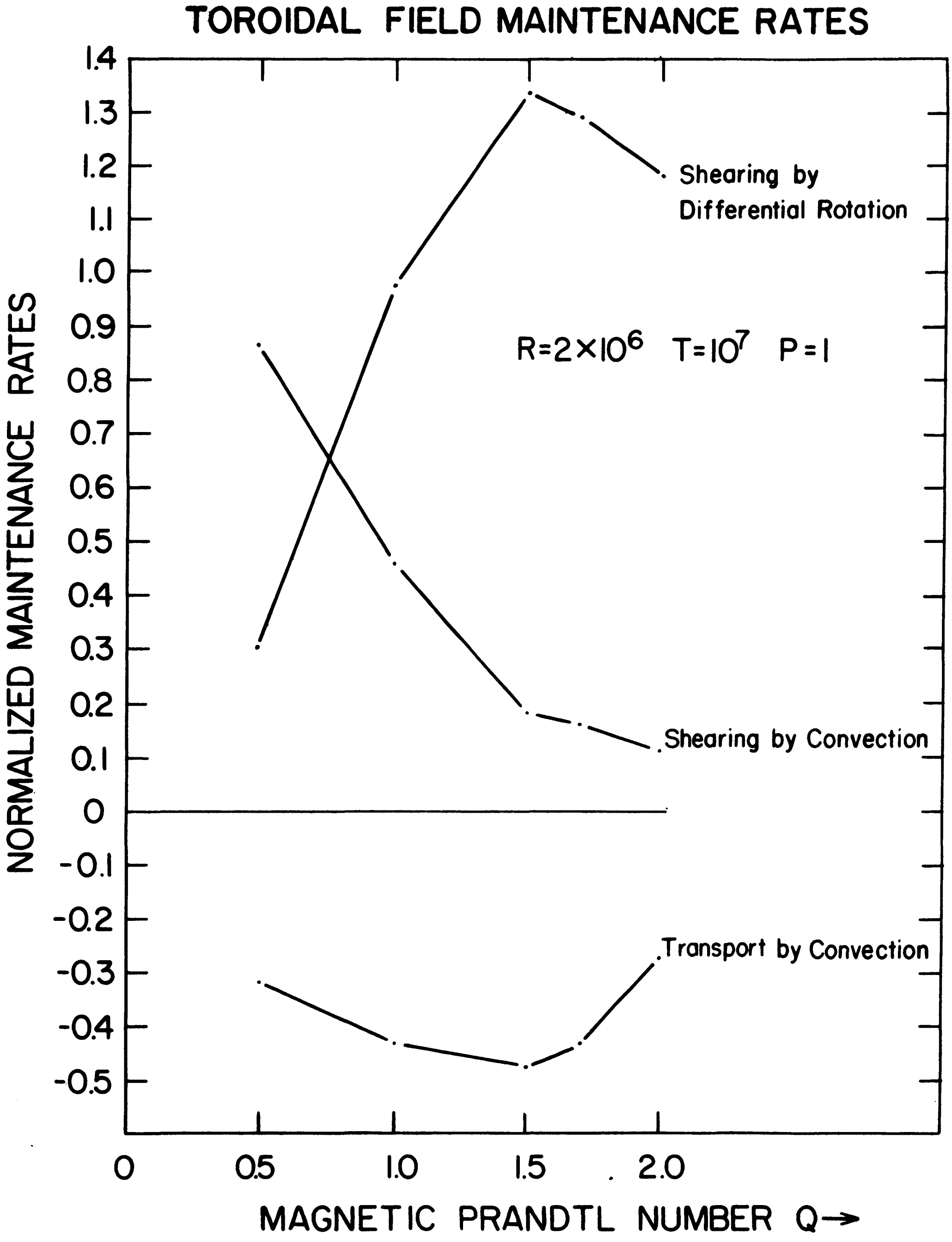

Figure 10: Rates at which toroidal field is maintained by various processes, normalized with respect to the magnetic field dissipation rate, as functions of magnetic Prandtl number.

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sional oscillations will be produced in the model in solutions whose dynamo period is much longer than we presently find.

f) Symmetry of the magnetic field about the equator.

In our solutions, we have found that, if the initial magnetic field has one dominant symmetry about the equator (for example, toroidal field either symmetric or antisymmetric about the equator), this symmetry is likely to be preserved over many magnetic cycles (several thousand model time steps) and initial growth of the field for either symmetry is virtually the same. However, due to departures from one particular symmetry in the motion field, the second symmetry does come into the magnetic field with time, and there are occasions when the dominant symmetry changes, and becomes locked into the opposite form for several cycles. This phenomenon seems to be easier to produce with the model than is a "maunder minimum", apparently the opposite of the sun.

It is easy to see how a symmetry change can come about if one just allows the convection to be temporarily a little more vigorous in one hemisphere than the other. A stronger poloidal field is then induced in that hemisphere, earlier in the cycle, which the differential rotation then amplifies into a stronger toroidal field also at an earlier phase in the cycle. A small succession of these events leads to the phase of the cycle in one hemisphere getting well ahead of that in the other, leading to the opposite symmetry of field about the equator. It would seem that in the sun, the two hemispheres are more closely linked than in the model.

5. Generalizations and applications

a) Application to the sun

Obviously, our dynamo solutions fail to reproduce the solar dynamo in some important respects. Kinematic dynamo theory so far has done better, but its practitioners have had more free parameters and functions at their disposal. Our problem is how to get the correct direction of migration of toroidal field, and cycle period, while still staying within the constraints of fluid dynamics. In the former problem, compressibility may help greatly. If the schematic differential rotation profile shown in Figure 2, with an angular velocity maximum at mid-depth, proves to be the rule in the compressible case, then we might generate magnetic cycles which start near the equator deep in the convection zone, for which the toroidal field then migrates roughly parallel to the axis of rotation toward the surface and higher latitudes. Upon reaching the reversed gradient of angular velocity there, it then migrates back toward the equator. In the sunspot cycle, all we would then be seeing is this upper branch of the cycle.

This picture offers some real advantages. For example, it would explain why spots first appear in mid latitudes, and the equatorward moving branch in each hemisphere would keep the two hemispheres more closely linked together. A disadvantage would be that it is not clear how the high latitude part of the torsional oscillations would fit in.

Such a hybrid dynamo would prove unnecessary if we were to find that with several density scale heights in the convection zone, the convective modes maintaining the equatorial acceleration also produced an angular velocity increasing inward all the way to the bottom of the convection zone. This would be more consistent with what is emerging from the oscillations measurements, but there is no evidence from what compressible convection calculations we have done so far that it will happen.

Either way this problem is solved, it would provide resolution of the conflict between differential rotation and dynamo models over the sign of the radial gradient of angular velocity.

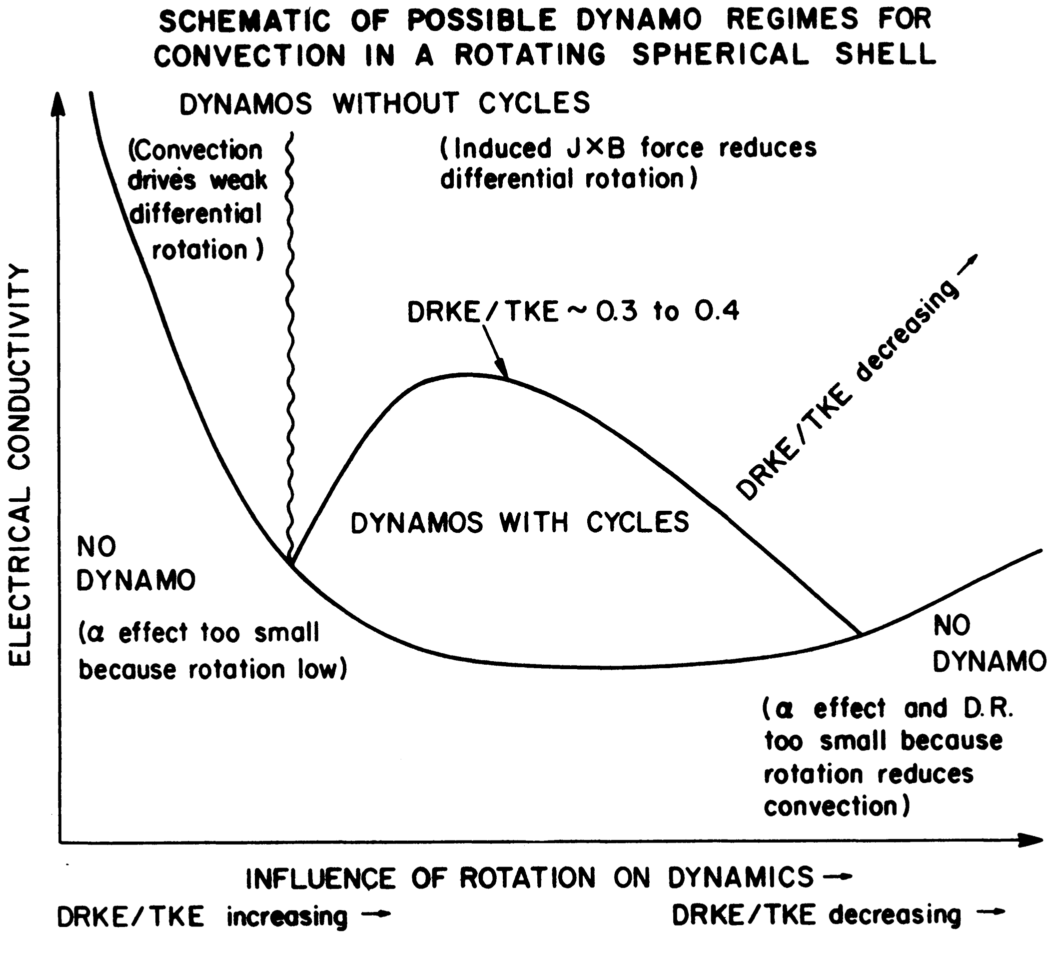

Figure 11: Schematic regime diagram for kinds of dynamos expected from model as functions of the conductivity and influence of rotation upon the dynamics.

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As for the short dynamo period we find, we see at least two possibilities. One is that the global convection driving the differential rotation of the sun is even weaker than in our current model -- for which rms. horizontal velocities are already no more than 10m/sec per wave number. The other is that, due to the highly intermittent magnetic field structure in the solar convection zone, not all of the helicity present in the convection is felt by the field. For example, a field largely confined to the boundaries of convection cells would not feel the helicity of the cells' interior. If this is the case, it will be very difficult to deal with in global dynamo models, because of practical limits on spatial resolution, and will require sophisticated parameterization of the unresolved small scale interaction of velocities and magnetic fields to capture its global effects. The role of magnetic buoyancy also needs to be considered further.

b) Generalizations from our dynamo results.

Based on our experience with the dynamo model so far, as well as other physical arguments, it is possible to construct a plausible, but very schematic, "regime diagram", illustrating what kind of dynamo action to expect. This may help us think about dynamos in other stars. Such a diagram is shown in Figure 11. In Figure 11, the vertical axis is the electrical conductivity for the model (proportional to η^{-1}. The horizontal axis is an unspecified measure of the influence of rotation upon the dynamics. It could, for example, be the ratio of rotation frequency to buoyancy growth rate or convective turnover rate, although, particularly at low and moderate influence of rotation, viscosity also enters in. Below the horizontal axis, we indicate that, without magnetic fields, as the rotational influence grows, the differential rotation energy DRKE grows as a fraction of the total kinetic energy TKE of the system. But eventually, the rotation has such a strong influence on the convection it nearly suppresses it. This drives the differential rotation down at an even faster rate, because it is driven by Reynolds stresses, which are second order in convective velocities.

Figure 11 illustrates the obvious point that, for any amount of influence of rotation upon the dynamics, there will be an electrical conductivity below which there is no dynamo action. At low influence of rotation, this is because while convection has its full amplitude, the helicity or α effect is too small to overcome the dissipation. At high influence, it is because rotation is suppressing convection and differential rotation. At higher conductivities but weak rotational influence, we get dynamos, but they are not field reversing, because the differential rotation is too weak. This is the region to the left of the vertical wavy line. To the right of that line, we get cyclic dynamos in an enclosed pocket inside of which the differential rotation exceeds some substantial fraction of the total kinetic energy of the system. From our model, that fraction appears to be in the range 0.3 to 0.4. Above and to the right of that pocket, we still get dynamos, but the cycles are lost, this time because j x B forces reduce the differential rotation to the point where shearing of the poloidal field by it is not the principal means by which toroidal field is maintained.

The various boundary lines we have drawn may not be sharp or unique, but may depend on the direction from which they are approached in the parameter space -- that is, hysteresis may be present, with memory of the previous state of the dynamos that were used as initial conditions for the next calculation. We have already seen an example of this in our solutions at $Q = 2.0$.

Our calculations so far have essentially been along two vertical lines in this diagram, one to the left and one to the right of the wavy vertical line. To really quantify such a diagram and verify its various parts, we obviously need many more calculations, but these are quite expensive with our large model. Such calculations would be worthwhile and much more practical with a simplified version, if a good one can be found.

c) Applications to stars

Even given uncertainties in the regime diagram for the dynamo model, there are additional uncertainties in extrapolating from it to the sun and other stars. For example, to make the connection, we must identify the electrical conductivity with a turbulent conductivity for a convection zone. But that may be affected by the influence of rotation upon convection, in which case the horizontal and vertical axes would not be independent. This would be a problem mostly on the far right-hand side of the diagram.

But let us suppose that the regime diagram in Figure 11 does apply to stars, at least in some form. Stars with cycles like the sun such as observed in Calcium emission by Wilson (1978) obviously would fall inside the enclosed "dynamos with cycles" region, at intermediate influence of rotation on the dynamics. This would be consistent with the observations of Vaughan, et al. (1981) that stars with cyclic calcium emission all have rotation periods of 20 days or longer. Stars with shorter rotation period have no cycles, and higher emission. On Figure 11, these stars would lie to the right of the upper boundary of the cyclic region. For some distance to the right of this boundary, the magnetic field amplitude ought to grow, beyond which it drops as the convection is more suppressed by rotation. Clearly other stars with strong but non-cyclic emission could simply lie well above the whole "cycles" region on the diagram. For these, even at intermediate rotation, the conductivity was sufficiently high that the induced $j \times B$ force shut down the differential rotation, eliminating the cycle. The calcium emission should be larger than for the cyclic stars, which is what is observed.

With respect to cycle periods, we presumably would have the same problem with applying our model to other stars as we have had for the sun, and the solution may be the same and therefore equally difficult -- better representation of the small-scale interaction between velocities and magnetic fields. With respect to the direction of migration of the toroidal field, we have no information from other stars, but it would obviously be extremely valuable input to dynamo models. I would speculate that, the stronger is the influence of rotation on the dynamics of a particular star that has a magnetic cycle, the more likely the dominant migration will be from low to high latitudes, as in our dynamo calculations. I would think this would be particularly true for cyclic red giants in which the convection zone is geometrically very deep but still contains very few scale heights.

The effects of convection zone depth are hidden in the regime diagram in Figure 11. In general, the shallower the depth (characteristic of early type main sequence stars) the shorter is the turnover time in the convection zone. As, for example, Gilman (1980) showed, even though the rotation rate of these stars is higher than the sun, ultimately the influence of rotation on the dynamics is less, reducing the probability of cyclic dynamo action in these stars, and, for the earliest, eliminating dynamo action altogether (see also Durney and Latour, 1978).

But there is an even more subtle effect here. If the suggestion from our calculations is correct that the seat of cyclic dynamo action is principally in low latitudes outside the tangent cylinder to the inner boundary of the convection zone, then the shallower the convection zone in a star with a cyclic dynamo, the more the variable Calcium emission should be confined to low latitudes. If such a star is being observed pole-on, it might be very difficult to measure cyclic variability, even if present. On the other hand, if its axis is nearly perpendicular to the line of sight, even a convection zone only 13% of the radius would produce activity over a belt ± 30° latitude, or half of the observed area. It would be very instructive if the amplitude of cyclic emission could be correlated with orientation of the star's axis, for a range of convection zone depths.

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DISCUSSION

IONSON: Dr. Golub has been stressing for years that the total magnetic flux is essentially constant throughout the solar cycle. What appears to be cycling is the magnetic flux power spectrum. That is, during solar minimum the flux is found primarily in bright points, during maximum in larger scale active regions. Has this very important empirical constraint been confirmed by contemporary dynamo theories?

GILMAN: Not really, and I am not sure we should expect them to, since such a spectrum shift may well depend strongly on the details of surface dynamics of active regions, which dynamo models do not usually account for.

SCHÜSSLER: First I have a comment on Dr. Ionson's question: In a model of the solar dynamo on the basis of fluxtubes, the anticorrelation of ephemeral active regions and big active regions is achieved if ephemeral active regions are assumed to be the shredded parts of fluxtubes under the influence of convection (cf. *Nature* **288**, p. 150).

Next I have a question to Dr. Gilman: You get a ratio Toroidal Field Energy/Poloidal Field Energy ≈ 3 for your model. Don't you think this is too small for the solar case?

GILMAN: It may be. I would note that whatever is happening on the sun that allows its magnetic fields not to sense the full helicity of global convection could also be the cause of the low poloidal field, since in the dynamo theory helicity is responsible for it. So if we solve the problem of the short dynamo period, we may also have solved the problem of poloidal field amplitude. But that remains to be seen.

WEISS: It is possible to include magnetic buoyancy within the Boussinesq approximation, and it would be interesting to see how this affected your results. In order to explain small-scale magnetic structures at the solar surface (and thus satisfy the observers) one would need to include intermittent magnetic fields and magnetic buoyancy in the theoretical description.

GILMAN: I agree, but this task is not an easy one. I decided to wait to look at magnetic buoyancy effects until the compressible version of our model was running, in which buoyancy would occur naturally.

SPRUIT: You mentioned that concentration of fields at the boundaries of cells will reduce the effect of helicity on the field. This suggests that the amount of dynamo action obtained should depend on the numerical resolution. Is this seen?

GILMAN: This would be true only if the resolution were inadequate to resolve the finest structure the model was capable of producing at the magnetic Reynolds number R_m of the flow. In our model, R_m for the smallest resolvable flow patterns is of order unity, so a large increase in resolution should not change the dynamo action much.

FRISCH: First, I would like to stress that to the best of my knowledge, the Gilman-Miller dynamo is the first example of a numerical nonlinear fluid dynamo where no violence is done to the equation of fluid dynamics. They have clearly been able to capture the first non-trivial magnetic bifurcation. Now, from nonmagnetic convection experiments (and calculations) we have learned that there are generally many bifurcations leading to quite different statistical states. It seems hard to rule out similar possibilities for the magnetic bifurcations. For example I would not rule out the possibility of getting field reversals just by increasing Reynolds numbers (and resolution).

GILMAN: Since differential rotation plays such a prominent role in determining whether there are cycles or not, I would be quite surprised if cycles reintroduced themselves for higher magnetic Reynolds numbers of the convection, while rotation and other parameters are kept fixed. It would have to come from a different cause than stretching of the poloidal field into the toroidal direction by differential rotation.

MULLAN: What happens to the dynamo as the convection zone becomes deeper? In particular, does the dynamo mode change radically (or at all) when the convection occurs throughout the star, rather than in a shell only?

GILMAN: I doubt our results would change much for deeper convection zones, since these calculations are already for a depth of 40% of the outer radius. The convective scale, and therefore the magnetic patterns would get somewhat larger, but by no more than a factor of 2. However, if the convective layer were made much shallower, the dynamo would change, because we would no longer have a broad equatorial acceleration.

GIOVANELLI: The calculations shown give very impressive results on flow patterns. I am not so happy with the magnetic fields, which seem to me to have been parameterized to the extent that results of various types can be obtained by varying the parameters. But this makes it very difficult to produce results which can be compared in detail with what we will observe at the surface. Yet this is what the observer expects from any theory. We really want to find out the physical relationships between what we observe and what causes it. There still seems a long way to go.

GILMAN: There is a long way to go, and I think you are asking too much of the theory in terms of details. To make lasting progress on a problem of this difficulty, we need to stick fairly close to the laws of physics and build, step by step.