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Ship waves and Kelvin

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Ship wakes are fascinating. They can be observed by the human eye and appear to have a V shape when the ship is advancing at constant speed along a straight trajectory. Under idealized conditions, Kelvin found that the angle between the two branches of the V is ∼39°. However, in a number of cases, this angle appears to be smaller. This phenomenon has been studied by various authors, and several explanations have been suggested. The most elegant one, which is based on the amplitude of the ship waves rather than their phase, has recently been revisited by Darmon, Benzaquen & Raphaël (*J. Fluid Mech.*, vol. 738, 2014, R3).

Key words: surface gravity waves, wakes, waves/free-surface flows

1. Introduction

Like many of us, Kelvin was fascinated by the V-shaped wake created by a ship advancing through water. Being an enthusiastic yachtsman and having wide-ranging scientific interests, Kelvin studied this wake in more detail. The wave pattern consists of transverse and divergent waves located between two branches of a V. Kelvin found that the angle between these two branches is ∼39° in deep water. This angle is known in the community as the Kelvin angle. For those not familiar with the subject, this somewhat mysterious value is in fact an exact number: $2\sin^{-1}(1/3)$. The full demonstration is not trivial. Lighthill (1978) in his book *Waves in Fluids* suggested a simplified explanation based on stationary phase arguments that is often used in the classroom. I have to say that even this simplified demonstration remains a challenge, and as a fluid mechanics professor the lecture devoted to the Kelvin angle always requires extra concentration!

With the advance of computers, scientists started to compute ship waves numerically and a more complex picture emerged. Compared to some other topics in water waves, there are relatively few papers on ship wakes, some of them being hidden in conference proceedings or in journals with limited electronic access. It is therefore not surprising that some great papers are essentially unknown to most of the community. For example, the late Ernie Tuck spent a considerable amount of his research activities understanding and computing ship wakes (see the interesting web site

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www.maths.adelaide.edu.au/ernie.tuck/ dedicated to him). Other examples are Francis Noblesse, who worked for many years at the David W. Taylor Naval Ship Research and Development Center – we will come back to his work below – and Xiao-Bo Chen, who as an employee of a certification company has written more than 20 papers related to ship waves.

The recent paper by Darmon, Benzaquen & Raphaël (2014) presents a simple analysis of ship waves that focuses on their amplitude rather than their phase. They write down an expression for the surface displacement far away from the ship and then compute it numerically for various values of the Froude number F_b ranging from 0.3 to 3. The Froude number F_b is defined here as $F_b = V/\sqrt{gb}$, where V is the speed of the moving object (I use the terminology *moving object* on purpose as the ship is modelled by an axisymmetric pressure field), g is the acceleration due to gravity and b is the size of that pressure field. Note that F_b is the Froude number used by the ship hydrodynamics community as opposed to the better-known Froude number $F_h = c/\sqrt{gh}$ used by the water wave community, with c the wave celerity and h the water depth. It is important to emphasize this point, as I have noticed that the Froude number is not defined in some papers dealing with ship waves, even in some review papers such as that of Reed & Milgram (2002). After showing plots of ship waves, Darmon *et al.* (2014) demonstrate analytically that the angle corresponding to the maximum amplitude of the waves decreases as F_b^{-1} for large Froude numbers, thus explaining why the V-shaped wake sometimes appears narrower in some observations of surface ship wakes.

2. Overview

From now on, we abbreviate the Darmon *et al.* (2014) paper by DBR. DBR model the ship, or more precisely the moving object, by an axisymmetric pressure field of size b. The work presented by the authors is a straightforward application of well-known analysis, notably § 2, which is very classical and leads to a well-known fact: namely that a ship wave pattern depends on the *phase* (which is the basis of Kelvin's method of stationary phase and (4.3) and (4.4) in DBR) as well as the *amplitude*, namely the function $f(\theta)$ in (4.4). The tools used by the authors are the same as those used by Havelock (1908) more than 100 years ago.

It is well known that a fully submerged body (for instance, a submarine close to the free surface), a displacement ship at moderate Froude numbers, and a high-speed boat generate wakes that look quite different: the differences stem from the fact that the amplitude function $f(\theta)$ is very different for these cases. This is illustrated in DBR for a *special* (very simple) case of a Gaussian pressure distribution applied at the free surface.

Having said this, it is true that there are not that many studies that consider the characteristics of the Kelvin wake for various Froude numbers. The elegant demonstration given in DBR that the angle of highest wave amplitude decreases as F_b^{-1} has not been shown before to my knowledge.

Interestingly enough Lighthill (1978) on page 274 of his book writes (I have replaced Lighthill's notation l for the ship's length by b):

> A ship generates preferentially waves of those wavelengths which predominate in a Fourier analysis of its disturbance to the water; these tend to be of the same order of magnitude as the ship's length b. If this is *large* compared with V^2/g [in other words, if the Froude number F_b is small – my comment], then the pattern tends to be dominated by waves around the maximum possible wavelength $2\pi V^2/g$, moving forwards behind the ship at *small* angles to the

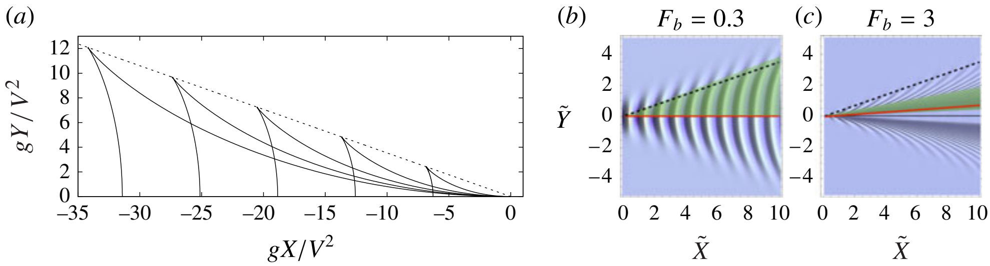

FIGURE 1. Ship wake. (*a*) Sketch showing the transverse and divergent waves inside a wake. (*b*) Wake at small Froude number for a Gaussian pressure field. (*c*) Wake at large Froude number for a Gaussian pressure field. Panels (*b*) and (*c*) are from DBR (their figure 1). $\tilde{X} = gX/2\pi V^2$ and $\tilde{Y} = gY/2\pi V^2$.

> ship's path [this is in perfect agreement with the first subplot in figure 1 of DBR – my comment]. Another exception arises when b is small compared with V^2/g [in other words, if the Froude number F_b is large – my comment], as with speedboats. Then the pattern is dominated by waves much shorter than the maximum wavelength $2\pi V^2/g$. These are waves propagating at large angles to the ship's path; that is, waves with their *crests* at small angles to the path [this is in perfect agreement with the last subplot in figure 1 of DBR – my comment]. The above remarks show clearly that the nature of the ship-wave pattern depends critically on the ratio V^2/gb, often called the *Froude number*. For low Froude numbers, longer waves with crests nearly perpendicular to the ship's path may predominate; for high Froude numbers, much shorter waves with crests at a small angle to the ship's path. On the other hand, for a wide range of intermediate Froude numbers, the entire wedge-shaped pattern may be evident, and the most clearly identifiable crests may be those on the boundary.

Several authors have noticed that the angle can be smaller than 39°. DBR provide one explanation among others. The key feature of their explanation is that it is elegant and clear. The main drawback of their explanation is that it completely neglects the shape of the hull. But I doubt that the same analysis can be made for more complex hull geometries. Barnell & Noblesse (1986) indicate that several theoretical explanations of the features observed in synthetic aperture radar (SAR) images of ship wakes have been proposed. These explanations include interactions between the cross-currents created by a ship in its wake and surface gravity waves and the occurrence of a sharp peak in the amplitude of the divergent waves in the Kelvin wake for a ship form having a large flare angle.

Barnell & Noblesse (1986) define three regions: (i) an inner region adjacent to the track of the ship where only transverse waves can exist, (ii) an outer region where both transverse and divergent waves are present, and (iii) a region at the boundary between the inner and outer regions where short steep divergent waves, as well as transverse waves, can be found. Then they look at the amplitude and the steepness of the waves in the vicinity of the ship track. Their figure 15 shows a peak in wave amplitude that is indeed inside the Kelvin wake. However, Barnell & Noblesse (1986) focus on wave steepness rather than wave amplitude. Steepness is more important when it comes to radar observations. Short sea waves are influential in radar back scattering. The figures 16 and 18 of Barnell & Noblesse (1986) also show peaks in wave steepness inside the Kelvin wake. For a given hull shape, the line along which the steepness of the divergent waves takes a maximum value is independent of the

Froude number F_b but the magnitude of the maximum depends on F_b. The main conclusion of Barnell & Noblesse (1986) is that the results depend strongly on the shape of the hull.

Noblesse & Hendrix (1991) analyse the short divergent waves in the steady wave pattern of a ship on the basis of a linear far-field flow representation and a nonlinear near-field flow approximation. Numerical simulations for the Wigley hull show that short divergent waves that are too steep cannot exist in reality within a significant sector in the vicinity of the ship track. So it appears that nonlinearity also plays a role in the study of ship wakes.

3. Future

The demonstration provided by DBR that the line along which the amplitude of the waves is maximum inside the Kelvin wake depends on the Froude number should now be extended to more realistic hull shapes. Indeed, the Gaussian pressure distribution considered is DBR is not representative of hull shapes. In view of the existing literature that has considered real hull shapes, it would be interesting to see if the analysis of DBR can be generalized and if the results for real hull shape will show the same dependence on the Froude number. Moreover, the analysis of DBR could be repeated for the steepness rather than for the amplitude. It also appears that nonlinearity and possibly surface tension can also play an important role. Several papers have been devoted to the influence of surface tension (or surfactants) on ship waves. Of course, the problem becomes much more complex if the course of the ship is not a straight line or if the speed is not constant. A beautiful example is shown in the figure by the title, showing a ship wake in Jackson Lake, Grand Teton National Park.

The final comment is on the 'sleeping beauty' of existing literature. Even though some tremendous progress has been made in making old literature available, more global data bases should be made available. In the meantime, I stress again that it is important for communities to read papers that are not necessarily in their own direct territory. As I am finishing this short paper, a new paper on ship wakes has just been published: Doyle & McKenzie (2013).

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