

## BOOK REVIEWS

HELMBERG, G. M., *Introduction to Spectral Theory in Hilbert Space* (North Holland Publishing Company, Amsterdam, 1969), xiii + 346 pp., 140s.

This book provides a gentle but thorough introduction to the spectral theory of a single linear operator on a Hilbert space. The definitions and elementary properties of inner-product and Hilbert spaces are given in chapter I. This chapter contains a section on normed linear spaces, which includes a brief discussion of the various topological notions required, and sections on the Hilbert spaces  $l^2$  and  $L^2[a, b]$ . Chapter II is concerned with the geometry of Hilbert space. Subspaces, orthogonality and bases are discussed here. The classical bases for various  $L^2$ -spaces (Legendre, Hermite and Laguerre functions) are fully dealt with. The chapter concludes with a proof of the important theorem that there is an isometric isomorphism between any two separable Hilbert spaces over the same field. Chapter III is devoted to the theory of linear operators on a Hilbert space. There are sections on bounded linear operators, bilinear forms, adjoint operators and projection operators. The chapter concludes with a thorough discussion of the Fourier-Plancherel transform on  $L^2(-\infty, +\infty)$ . The first half of chapter IV continues with the theory of linear operators. There are sections on adjoint operators, closed linear operators, differentiation and multiplication operators on  $L^2$ -spaces. In the remainder of the chapter the basic ideas of spectral theory are introduced—invariant subspaces, eigenvalues and spectra. The chapter concludes with a detailed study of the spectrum of a self-adjoint operator. Chapter V is concerned with the spectral theory of a compact linear operator. There is a section on Fredholm integral equations, and the spectral decomposition theorem for a compact self-adjoint operator is proved. In chapter VI, there is proved in turn the spectral theorems for a self-adjoint, unitary and normal operator. The author's development of this theory is close in spirit to the treatment in "Functional Analysis" by Riesz and Sz-Nagy. The final chapter is devoted to the spectral theory of an unbounded self-adjoint operator. There is a section on the Cayley transform, and the spectral theorem for an unbounded self-adjoint operator is proved. The book concludes with two appendices, the first on the graph of a linear operator and the second on the Riemann-Stieltjes and Lebesgue integration theory used in the main text. There are numerous exercises throughout the book, which serve to illustrate the theory, and a comprehensive bibliography. The classical motivating examples for the abstract theory are fully worked out in the text. This is an admirable work on which to base a graduate course on Hilbert space.

H. R. DOWSON

LUKACS, EUGENE, *Stochastic Convergence* (Heath Mathematical Monographs, 1968), viii + 142 pp.

The description of a book as a monograph tends to convey the impression that the author has allowed himself more licence than would have been the case had he set out with the purpose of writing a textbook. In this case the book itself confirms this impression. Very markedly it reflects the personal tastes and interests of the author rather than the requirements of a defined class of readers. Because of this, it is difficult to describe briefly the content and level of the book, and a more detailed account than is normal in a review becomes necessary.

Chapter 1 states, rather than discusses, basic ideas and theorems, knowledge of which is a prerequisite. Most of these are just what one might expect—ideas such