

Editorial

Special issue: The Vlasov equation, from space to laboratory plasmas

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Plasmas are high temperature, rarefied dynamical systems made of a very large number of charged particles where the typical collisional time scale is much longer than any dynamical time scale. Equivalently, by comparing spatial instead of temporal scales, we get that the diffusive scale length is typically many orders of magnitude larger than any other characteristic length. As a consequence, a plasma can be considered as collisionless, at least in a first approximation, and the dynamics as Hamiltonian. For instance, in solar wind plasmas the collisional mean free path is of the order of the size of the system, one astronomical unit, and indeed non-Maxwellian distribution functions are today routinely observed by satellite measurements (Christon *et al.* 1989; Collier 1999; Maksimovic *et al.* 2005). The role of the physical processes at play in a collisionless plasmas and their ability to replace collisions to redistribute the energy cascading from the large scales is one of the outstanding and most challenging problems in plasma physics, and was addressed relatively recently by *in situ* measurements in the solar wind (Bale *et al.* 2005). Furthermore, space plasmas appear today more and more as one of the best plasma physics laboratories, where *in situ* measurements span from large scale magnetohydrodynamics dynamics to kinetic processes, as is the case, for instance, for the well-known CLUSTER mission (Goldstein *et al.* 2015).

Plasmas occurring in magnetic-confinement fusion experiments, although to a lesser extent, are also by and large collisionless and their dynamical properties mainly determined by ‘anomalous’ (i.e. collisionless) transport (Connor & Wilson 1994), which dominates over collisional transport.

The possibility to support non-Maxwellian distributions is doubtless one of the most interesting features of collisionless plasmas, and allows, in particular, collisionless long-lived electrostatic and electromagnetic coherent structures (Schamel 2000; Galeotti *et al.* 2005) on a macroscopic scale to be sustained. Many examples are available in space plasmas as, e.g. in the auroral zone (Ergun *et al.* 1998), in the plasma sheet boundary layer of the magnetotail (Matsumoto *et al.* 1998), close to the bow shock (Bale *et al.* 2002), in the solar wind (Mangeny *et al.* 1998). Recently, Alfvén-vortex structures playing a key role in the development of the sub-proton kinetic spectrum have been detected by *in situ* measurements (Lion, Alexandrova

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& Zaslavsky 2016). Furthermore, coherent structures have been observed in large scale kinetic simulations of the Vlasov equation in all fields of plasma physics using both a Lagrangian Particle in Cell (PIC) (Goldman, Oppenheim & Newman 1999) or a Eulerian (so-called Vlasov) approach (Ghizzo *et al.* 1988; Briand, Mangeney & Califano 2007). Starting from the famous electrostatic ‘phase space holes’, also known as Bernstein–Greene–Kruskal (BGK) modes (Bernstein, Greene & Kruskal 2007), many analytical solutions of the Vlasov equation have been found to account for experimental evidence of such structures (Manfredi & Bertrand 2000). In particular, high-resolution space observations have evidenced the occurrence of electrostatic and electromagnetic coherent structures at the electron kinetic scale with corresponding non-Maxwellian distribution functions (Perrone *et al.* 2016).

More recently, laboratory experiments also gave evidence of similar structures of typical width of the order of several Debye lengths. Coherent structures, such as electromagnetic solitons and magnetic vortices, are also found in laser-generated plasmas (Pegoraro *et al.* 2000), and electron vortices are routinely displayed in non-neutral plasma experiments (Romé & Lepreti 2011). Very recently, magnetized vortices known as Alfvén vortex-like structures were detected by *in situ* spacecraft measurements in the terrestrial environment and shown to have a typical width of the order of the inertial ion scale length (Alexandrova 2008). Finally, non-Maxwellian distributions are frequently observed in experiments and simulations of plasma–wall interactions (Valsaque *et al.* 2002).

All the above experimental and computational results highlighted the unimportant role of collisions, even at the kinetic scale, even though it has been recently argued that the effects of collisions can be enhanced by the sharp velocity gradients generated in the particle distribution function by wave–particle interaction processes (Pezzi, Valentini & Veltri 2016). Thus, the theoretical analysis of the generation and evolution of coherent structures and the associated kinetic processes is considered as one of today’s outstanding problems in plasma physics, for instance in the case of plasma turbulence where such structures play a leading role in particular concerning the transition across the ion cyclotron frequency.

In spite of that, fluid models such as magnetohydrodynamics (MHD) have a long history in plasma physics, and are widely used to describe all sorts of systems, laboratory, space and fusion plasmas alike. Although they are known to neglect important effects at the kinetic scale, first of all Landau damping and microinstabilities, fluid models have been successfully used to describe the low-frequency large-scale plasma dynamics in laboratory and space plasmas. And indeed it can be shown that the fluid equations, the few first moments of the Vlasov equation, offer an adequate representation of the plasma motion at such scales. However, the MHD approach rapidly breaks down as soon as nonlinear or geometrically driven interactions start to inject energy at kinetic scale lengths. The injected energy then cascades towards smaller and smaller scales, first to the ion Larmor radius then down to the electron scale, often producing an important feedback also towards the large-scale motions. As a result, the system undergoes a transition through different physical regimes that are not all amenable to hydrodynamic modelling. We must nevertheless mention that a strong effort has been made to include the main ion kinetic effects and even Landau damping in a fluid approach (Dorland & Hammett 1993). Such models, also known as nonlinear Landau fluid models (Sulem & Passot 2015) are very promising and could constitute an excellent compromise between fluid MHD-like approaches and the kinetic Vlasov models or, at least, to make an important bridge between such two approaches.

In such multi-scale, multi-physics scenarios, one has to rely on a kinetic description based on the Vlasov equation for the single-species distribution function (d.f.) f_a ,

$$\frac{\partial f_a}{\partial t} + \mathbf{v} \cdot \frac{\partial f_a}{\partial \mathbf{x}} + \frac{q_a}{m_a} (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \frac{\partial f_a}{\partial \mathbf{v}} = 0, \quad (0.1)$$

where a is the species index. Coupled to the Maxwell equations that provide the self-consistent electric and magnetic fields \mathbf{E} and \mathbf{B} , the resulting Vlasov–Maxwell system represents the fundamental physical model to describe the dynamics of a collisionless plasma in a six-dimensional phase space, thus representing a formidable computational challenge even with the help of the most powerful supercomputers.

The electrostatic model, where the Vlasov equation is coupled to the Poisson equation, still represents formally a very difficult computational task; however, many ‘real’ systems can be investigated in this limit because of the possibility of reducing the phase space dimensionality without altering the main physics. This leads to the possibility of investigating fundamental processes such as those, for instance, in the near Earth environment, parametric Langmuir decay (Henri *et al.* 2010), Langmuir wave dynamics (see Briand 2015 and references therein), auroral bipolar wave structures (Ergun *et al.* 2001), Langmuir turbulence (Henri *et al.* 2011), Langmuir wave packets (Ergun *et al.* 2008).

The Vlasov–Maxwell system is capable of describing at once all the different physical regimes, from the large-scale hydrodynamic to the small-scale kinetic regime, but neglects collisional effects. It is worth adding that kinetic effects, in particular microinstabilities, are often very efficient to redistribute the ‘ordered’ energy, cascading from the large (hydrodynamic) scales towards the small-scale particle motions. This redistribution drives the system towards a kind of ‘isotropization’ even in the absence of collisions. Thus, collisionless kinetic processes can in some sense replace or mimic the role of collisions (Zelenyi & Artemyev 2008). For instance, it was pointed out recently that the effective thermalization observed in a plasma–wall transition can be ascribed, at least in part, to purely collisionless effects (Coulette & Manfredi 2015).

Theoretically, the Vlasov equation is obtained from the Liouville equation for the N -particles distribution in the mean field limit, where each particle interacts with an average field generated by all plasma particles, while two-body and higher-order correlations are completely neglected. A fundamental feature of this (Hamiltonian) model is that the d.f. is subjected to strong topological constraints that reduce the degrees of freedom of the system. For example, the d.f. can be transported and rolled up in complex ways in phase space, but different d.f. isolines never break and reconnect. As a result, transitions in phase space from a laminar-type state (i.e. free streaming) to a vortex-type state (i.e. particle trapping) are forbidden. This situation is similar to that of an ideal MHD plasma, where the magnetic field lines frozen-in condition prevents transitions between magnetic energy states with different magnetic connections. However, the intricate structure of the d.f. can often effectively mimic the formation of a phase-space vortex, although the distribution isolines always remain open. A tiny number of collisions (or of numerical diffusion, in the case of simulations) is then enough to complete the transition to a true vortex-type state. However, this transition violates, at least locally, the Vlasov equation.

In order to study the plasma dynamics, the Vlasov equation for the ions and electrons must be solved self-consistently together with the Maxwell equations. Given the nonlinearity of the problem, this is a formidable task and, apart from a few very

special cases such as Landau damping or linear wave propagation, it is practically impossible to obtain analytical solutions. For this reason, the majority of today's work rely on a computational approach, mainly based on Lagrangian or Eulerian numerical codes. Fully six-dimensional problems (three dimensions for space and three dimensions for velocity) can barely be attacked with modern supercomputers, and in most cases one must adopt some kind of reduced model. For phenomena on the ion time scale, it is often useful to adopt the so-called hybrid approach (Valentini *et al.* 2007, 2014; Servidio *et al.* 2015), where the electrons are considered as a (possibly massless) fluid. For high-frequency phenomena, one can instead treat the ions as a fixed neutralizing background and only solve the Vlasov equation for the electrons. In either cases, one avoids the problem of taking into account both the ion and the electron time scales, thus dramatically reducing the computational effort in term of CPU hours and memory requirements. More recently, PIC codes that use stable implicit schemes were introduced aiming at including, as far as possible, the kinetic dynamics of both species (see Lapenta (2012) and references therein).

The Special issue 'The Vlasov equation: from space to laboratory plasmas' takes its origin from a recent conference on the Vlasov equation held in Copanello (Calabria, Italy) in June 2016. This is part of the series of the 'Vlasovia' conferences, organized every three years alternatively in Italy and in France since 2003. Previous editions were held in Nancy (2003), Florence (2006), Marseilles (2009) and again Nancy (2013). Most papers appearing here originate from contributions and discussions presented at the 2016 Vlasovia Conference. Nevertheless, this Special Issue is open to contributions from all interested scientists, provided they are related in some way to the physics and mathematics of the Vlasov equation. The forthcoming articles cover a wide range of plasma physics research: theory of magnetized plasmas, space and laboratory plasmas, nonlinear dynamics and Hamiltonian systems, astrophysics, laser-plasma interactions, computational plasma physics. A few oral contributions were also devoted to other applications of the Vlasov equation, notably to solid-state plasmas (Hurst *et al.* 2014) and the physics of self-gravitating systems (Colombi 2015).

One of the successes of the Vlasov approach lies in the modelling of kinetic plasma turbulence by means of large-scale direct numerical simulations (see Cerri *et al.* (2016) and references therein). Currently, a strong effort is focused on making the relevant models as realistic as possible, in particular for what concerns solar wind applications, where there is today a wealth of high accuracy *in situ* measurements by satellites. These measurements have reached higher and higher resolutions, now approaching the electron scale, such as for instance for the Magnetospheric MultiScale (MMS) satellite, providing us with the unique opportunity to investigate experimentally the kinetic physics of fundamental processes, from the problem of how energy is 'dissipated' in plasmas to the understanding of the electron sub-layer in magnetic reconnection (Burch *et al.* 2016; Egedal *et al.* 2016).

In an effort to couple computational and experimental approaches, the 2016 Vlasovia conference featured a session dedicated to the Turbulence Heating Observer (THOR) spacecraft (Vaivads *et al.* 2016), a candidate for the next M4 space mission of the European Space Agency, currently undergoing the study phase. THOR aims at providing high-resolution measurements of the electromagnetic fields and, in particular, of the particle distribution functions with unprecedented phase-space resolution, allowing for a significant increase in our understanding of the dynamics of the interplanetary medium at kinetic scales.

We finally mention that large-scale simulations of magnetically confined plasmas also rely on a the so-called gyrokinetic Vlasov equation, whereby the fast gyration

of the particles around the magnetic field lines is neglected, thus reducing the phase space to five dimensions. Gyrokinetic codes, both PIC and Vlasov, have reached a remarkable degree of sophistication and fine resolution, and are able to tackle realistic problems of plasma turbulence and transport in tokamaks.

In conclusion, the oral presentations at the 2016 Vlasovia conference and the articles published in this Special Issue testify to the vibrant status of current research on the Vlasov equation, be it experimental, theoretical or computational. For a model that was put forward almost 80 years ago by Vlasov (and even earlier by Jeans), it still holds a very promising future.

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