is overwhelming evidence (e.g. Goldbach's conjecture); it is the business of the statistician to facilitate decisions in the light of inadequate evidence. In fact, however, there are grave philosophical difficulties in applying probability theory to the " real world " even when to the non-philosophical eye the evidence points unmistakably in one direction (as the tobacco manufacturers have recently pointed out!); but there are none in applying it to numbertheory. Both numbertheory and probability theory have an axiomatic basis, and so if the hypotheses of a theorem in probability theory apply to a number-theoretical situation then so do the consequences.

(It should perhaps be mentioned that the literature contains also " applications " of probability theory to numbertheory of a quite different (and as usually presented fairly meaningless) nature, often to the theory of primes. One considers the set S of all sequences s: $a_1 < a_2 < \ldots < a_n <$ of positive integers which have certain properties P (usually not explicitly stated) enjoyed by the sequence of prime numbers. In the set S a probability measure m is (usually tactitly) introduced and it is then shown that m-almost all sequences s in S have a certain property Q (e.g. that every sufficiently large integer is the sum of two distinct members of s). It is then concluded that it is " very probable " that the prime numbers have Q. It is held to be a deep mystery when two practitioners of this art come to contradictory conclusions (on the basis of different-unstated-P's and m's). Needless to say, the remarks above do not apply to these arguments!)

The author, who has made important contributions of his own, treats exhaustively one only of several types of application of probability theory to numbertheory, namely to the asymptotic distribution of multiplicative and additive number-theoretical functions. Fairly detailed reviews of the first edition (1959) are available in, e.g. *Mathematical Reviews*; the second edition is similar in scope but contains considerable new material. The following may be taken as a typical special case of what is proved: Let $\omega(n)$ denote the number of prime factors of the integer n. For any real number ξ and positive integer N denote by $F_N(\xi)$ the number of $n<N$ for which

$$\omega(n) < \log\log n + \sqrt{(\log\log n)}.$$

Then

$$\lim_{N\to\infty} F_N(\xi) = \frac{1}{\sqrt{(2\pi)}} \int_{-\infty}^{\xi} \exp\left(-\tfrac{1}{2}u^2\right) du,$$

where the right hand side is the " error-function ". Estimates are given of the rapidity of the approach to the limit. Here $\omega(n)$ is an additive function in the number-theoretical sense. Most of the important number-theoretical functions or their logarithms are additive, and the result quoted above is typical for a class of additive functions. The case of $\omega(n)$ is, however, in some ways particularly simple and more precise results are given for it than can be obtained generally.

The reviewer understands that a translation into English is under preparation.

J. W. S. CASSELS

BOURBAKI, N., *Éléments de mathématique*, *Fascicule XX* (Theorie des ensembles, Chapitre 3—Ensembles ordonnés, cardinaux, nombres entiers) (Hermann, 1963), 152 pp., 36 F.

This is the second edition of Fascicule XX of the well known Bourbaki series. Compared with the first edition, the layout of this book has been altered: the exercises for each section are collected together at the end. Also, the indices of notation and terminology are more convenient. The changes in the text are mainly minor ones which remove obscurities, but part of section 1 from the first edition has been rewritten and expanded and has become section 7 of the second edition.

https://doi.org/10.1017/S0013091500025682 Published online by Cambridge University Press

The effect of the changes is that what was already a good book is now even better. The most difficult part of the book has been subjected to the greatest change: section 7, on projective and inductive limits, is much more readable than its corresponding part in the first edition, but it is still difficult. The exercises play as important a part in section 7 as they do in the rest of Bourbaki's work; that is, valuable results and counter-examples are given as exerises.

R. M. DICKER

Infinitistic Methods. Proceedings of the Symposium on Foundations of Mathematics, Warsaw, 2-9 September 1959 (Pergamon, 1961), 362 pp., £5.

This is a record of the proceedings of the Symposium on the foundations of mathematics which was held in Warsaw in 1959. Of the 27 papers presented at the Symposium, 22 are included in this book, and 12 of these are written in English. The subject of the Symposium was declared to be " infinitistic methods in the foundations of mathematics " and to a large extent that also describes the book. Most of the papers are on mathematical logic and are naturally classified in that way. However, there are four titles that suggest other connexions: " Some properties of inaccessible numbers "; " Locally small categories and the foundations of set theory "; " Les logiques à plusieurs valeurs et l'automatique "; and " A practical infinitistic computer ".

It is clear that this book will be important to logicians, but other mathematicians will probably find little of interest in it. An irritating feature is the occasional appearance of spelling mistakes: the book was edited and produced in Poland.

R. M. DICKER

KNEEBONE, G. T., *Mathematical Logic and the Foundations of Mathematics* (Van Nostrand, 1963), xiv+435 pp., 65s.

This book is intended as an introductory survey, or guide-book, on mathematical logic. It is very readable and can be recommended as a source of knowledge and instruction. It is not a textbook on the mathematical details of the subject; indeed, most of the mathematics is omitted and the reader is expected to refer to other books for complete proofs. This is certainly not a disadvantage. The author is able to write an account of the various topics which is suitable for a wide readership and which serves as an introduction to the existing textbooks on mathematical logic. Furthermore, by adding supplementary notes, the author is able to survey the literature and give references without disrupting his text. Thus this book is useful in several ways: it is sufficiently self-contained and simple to be read by students; it is an introduction to the more mathematical texts; and it surveys the field for those who need to dig deeper. It will be valuable to the beginner as well as to the advanced student; however, the latter should not expect too much of the book—it does not contain everything you need, but it does contain a lot that is worth having.

The book covers a wide range of knowledge; the size and depth of this coverage is indicated in what follows. Part I, on mathematical logic, consists of chapters on traditional logic, the propositional calculus, the calculus of predicates, and some further developments. Part II, on the foundations of mathematics, considers the history and development of formalised mathematics. the limitations of formal systems, and intuitionism. There are also chapters on recursive arithmetic, and the theory of sets. Part III is on the philosophy of mathematics; and there is an appendix on the recent developments in mathematical logic. Part I is a clear exposition of the fundamentals of the subject and the supplementary notes at the ends of the chapters indicate the extent of our knowledge. Part II is treated in much the same way as