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Constraining planetary interiors with the Love number k_2

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Abstract. For the solar sytem giant planets the measurement of the gravitational moments J_2 and J_4 provided valuable information about the interior structure. However, for extrasolar planets the gravitational moments are not accessible. Nevertheless, an additional constraint for extrasolar planets can be obtained from the tidal Love number k_2, which, to first order, is equivalent to J_2. k_2 quantifies the quadrupolic gravity field deformation at the surface of the planet in response to an external perturbing body and depends solely on the planet's internal density distribution. On the other hand, the inverse deduction of the density distribution of the planet from k_2 is non-unique. The Love number k_2 is a potentially observable parameter that can be obtained from tidally induced apsidal precession of close-in planets (Ragozzine & Wolf 2009) or from the orbital parameters of specific two-planet systems in apsidal alignment (Mardling 2007). We find that for a given k_2, a precise value for the core mass cannot be derived. However, a maximum core mass can be inferred which equals the core mass predicted by homogeneous zero metallicity envelope models. Using the example of the extrasolar transiting planet HAT-P-13b we show to what extend planetary models can be constrained by taking into account the tidal Love number k_2.

Keywords. planets and satellites: interiors, planets and satellites: individual (HAT-P-13b), methods: numerical

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1. Love numbers

Love numbers are planetary parameters that quantify the deformation of the gravity field of a planet in response to an external perturbing body. The perturber of mass M orbits the planet at a distance a and causes a tide raising potential (Zharkov & Trubitsyn 1978) of $W(s) = \sum_{n=2}^{\infty} W_n = (GM/a) \sum_{n=2}^{\infty} (s/a)^n P_n(\cos\theta')$, where s is the radial coordinate of the point under consideration inside the planet, θ' the angle between the planetary mass element at s and the center of mass of M at a, and P_n are Legendre polynomials. The tidally induced mass shift causes a change in the planet's potential, which reads at the planet's surface: $V_n^{\rm ind}(R_{\rm p}) = k_n W_n(R_{\rm p})$, giving a definition of the Love numbers k_n (Love 1911). They solely depend on the radial density distribution of the planet. Of special interest is the Love number k_2: it is a measure for the level of central condensation of an object. A planet of constant density represents maximum homogeneity with $k_2 = 1.5$. In conclusion, k_2 can provide additional constraints for the planet's interior. However, k_2 is not a unique function of the planet's core mass. For three-layer models we find a degeneracy with respect to the density discontinuity in the envelope (Kramm *et al.* 2010).

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2. HAT-P-13b

The system HAT-P-13 (Bakos *et al.* 2009) consists of two planets which are assumed to be in apsidal alignment so that the theory described in Mardling (2007) applies and an allowed interval for the Love number k_2 of $0.116 - 0.425$ can be determined, hereafter denoted by Batygin-Bodenheimer-Laughlin (BBL)-Interval (Batygin *et al.* 2009). We constructed models for HAT-P-13b with planet mass $M_\mathrm{p} = 0.853\,\mathrm{M_J}$ and planet radius $R_\mathrm{p} = 1.281\,\mathrm{R_J}$ within a two-layer model consisting of a rocky core and a H/He/metal-envelope (Saumon *et al.* 1995). We varied the temperatures in the outer atmosphere, which was assumed to be fully adiabatic or isothermal to 1 kbar, and the metallicity in the envelope.

For these models we calculated the core mass and the Love number k_2 (Fig. 1). We conclude that for a given k_2 only a *maximum possible* core mass can be inferred. Assuming the BBL-Interval (Batygin *et al.* 2009) is correct, this maximum possible core mass is $\approx 0.32\,M_\mathrm{p}$ ($= 87\,\mathrm{M_\oplus}$). Further constraints could be made if there was more precise knowledge about the envelope's temperature profile and/or metallicity.

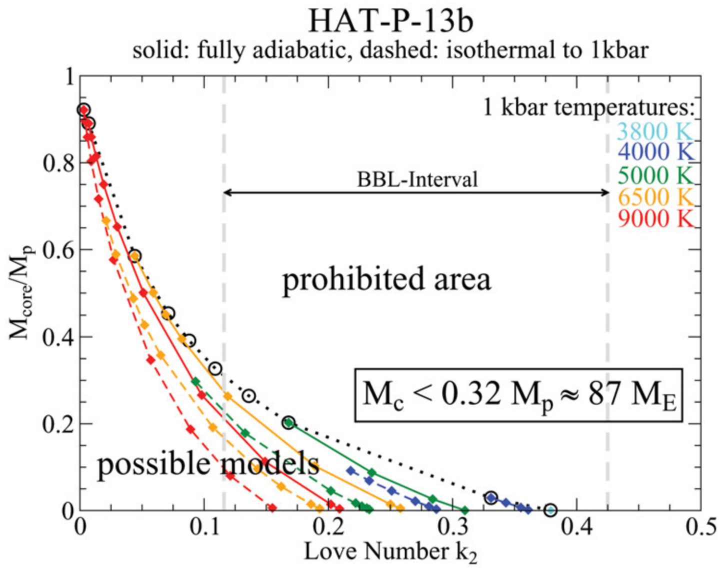

Figure 1. Core masses and Love numbers for several two-layer models of HAT-P-13b. Shown are models with a fully adiabatic (solid) or isothermal to 1 kbar (dashed) envelope for different 1 kbar temperatures (symbol coded, for color version see electronic proceedings). The dotted line consists of models with a fully adiabatic, zero-metallicity envelope. With $k_2 = 0.116 - 0.425$ we deduce a maximum possible core mass of $\approx 87\,\mathrm{M_\oplus}$.

We also performed calculations for the Love numbers of GJ 436b (Nettelmann *et al.* 2010a, Kramm *et al.* 2010), GJ 1214b (Nettelmann *et al.* 2010b) and Saturn (Kramm *et al.* 2010).

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