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To Solve Dark Matter Problems Without Dark Matter

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Abstract. We consider solving the dark matter problem via a modified gravitational Lagrangian.

The problems of Dark Matter (DM) have been raised for several decades since the 1930's. From the flat rotation of spiral galaxies (Rubin 1993), one can estimate the mass of galaxies. At least 90% of the total mass of galaxies is composed of unknown dark matter, if the Newtonian gravitational theory is also correct on the scale of galaxies. As we know flat rotation curves are quantitatively related with DM in spiral galaxies. For clusters of galaxies and large superclusters the average mass-to-light ratio is even higher than the one in spiral galaxies. If we consider the Universe formed from an inflation model, the value of the mass density ρ_o is close to critical mass density of the Universe, then the amount of DM in the Universe is at least a factor of five higher than the one in the superclusters. Therefore we can conclude that the larger the scale, the more the DM.

A modern dominant point of view considers DM to be some kind of unknown nonbaryonic matter, e.g. nonzero rest mass neutrino, photino, axion, scalar field and so on, or brown stars. There are more than two dozens candidate types of DM proposed: the mass of these candidates covers a region of 69 order of magnitude. Furthermore even if we knew what DM is, we still have to answer the second question why it distributes in such a way as to flatten the rotation curves. Therefore for the problems of DM there are at least two questions (Xu & Wu, 1998).

Additionally, the main known properties of DM are invisible and dissipationless — they are also properties of the gravitational field. Therefore one may doubt whether there is dark matter at all — maybe the Newtonian gravitational theory cannot be extended to such a large scale. An alternative method for solving this galactic dynamic problem is based on revising the Newtonian gravitational theory on a large scale. Several modified gravitational models (Bekenstein 1988) have been suggested on basis of such possibility, but none of them is successful. In higher-order gravitational theories, we found that some difficulties also appeared in the junction condition from the exterior solution to the interior one, i.e. exterior solution cannot fit a reasonable interior one (Xu & Wu 1998). But the attempt is always meaningful before dark matter is really discovered.

Here we suggest a new general Lagrangian with two adjustable functions. We discuss the meaning of the two adjustable functions and deduce a general field equation. As an example, we take a special case to show the field equations. The linearized field equation has been considered. We obtain a Yukawa-like potential which is useful to explain the flat rotation curves (Xu et al., 1992).

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The new Lagrangian is taken as

$$L = -\frac{c^4}{16\pi G}\Big(f(\tau R) + 2\Lambda(\rho)\Big)\,, \tag{1}$$

where R is the scalar curvature, ρ is the energy density and $\mid \tau \mid$ is the square of the characteristic length of galaxies, f is an adjustable function of τR. When $f(\tau R) = R$ and $\Lambda(\rho) = 0$, it is just the Einstein field equation. When $f(\tau R) = \frac{1}{\tau}\left(R + \tau^2 R^2\right)$, it is a Weyl-type higher-order gravitational theory. In future, we could adjust $f(\tau R)$ to fit the interior solution. $\Lambda(\rho)$ is the other adjustable function of the scalar energy density. When $\Lambda(\rho)$ = constant, it is just the cosmological constant in general relativity the meaning of which is the vacuum energy density. If we consider the coupling between ρ and vacuum energy, the function $\Lambda(\rho)$ is a natural extension of the cosmological constant.

The field equations following from the variational principle, lead to

$$\begin{aligned} H_{\mu\nu} &= (R_{;\mu}R_{;\nu} - g_{\mu\nu}R_{;\sigma}R_{;\rho}g^{\sigma\rho})f'''(\tau R) + (R_{;\mu;\nu} - g_{\mu\nu}\Box R)f''(\tau R) \\ &\quad + R_{;\sigma}R_{\mu\nu}f'(\tau R) - \frac{1}{2}g_{\mu\nu}f(\tau R) = \frac{8\pi G}{c^4}T_{\mu\nu} + g_{\mu\nu}\Lambda(\rho)\,, \end{aligned} \tag{2}$$

where the prime means the derivation with R. Since the scalar density ρ is not the function of $g_{\mu\nu}$ and their derivatives, $\Lambda(\rho)$ is unchanged in the variation with $\delta g_{\mu\nu}$. $\Box$ is the d'Alemberian operator $\Box = g^{\mu\nu}\nabla_\mu\nabla_\nu$, where ∇_μ is the covariant derivative.

The stress-energy tensor conservation law $T^{\mu}_{\nu;\mu} = 0$ has to be revised after consideration of $\Lambda(\rho)$. It reads

$$T^{\mu}_{\nu;\mu} + \frac{c^4}{8\pi G}\Lambda(\rho)_{;\nu} = 0\,. \tag{3}$$

As an example, here we consider the linear approximation of τ only. Since in the DM problem we need to revise the Newtonian gravitational theory at first, the gravitational field is always weak. In case $f(\tau R) = \exp(\tau R)$, we have a Yukawa-like exterior solution in the spherically symmetric case for potential

$$\varphi = \frac{A}{r} + \frac{B\exp(-r/\sqrt(k)}{r} + \frac{C\exp(r/\sqrt(k))}{r} + D\,, \tag{4}$$

where A, B, C and D are constants of integration. An adjustable function $\Lambda(\rho)$ and $f(\tau R)$ might be helpful to fit a reasonable interior solution to an exterior solution to explain the flat rotation curves.

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