KINEMATIC AND DYNAMICAL MODELS OF SMALL STRUCTURES IN RADIO SOURCES

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ABSTRACT.

Models of the small-scale structures in radio sources are limited by the resolution, dynamic range, and sensitivity of the observations upon which they are based. However, evidence from variability, brightness, and proper motion observations strongly support the contention that these objects involve directed outflow or ejection of material at relativistic speeds. The evidence for relativistic motion will be discussed, and a variety of models involving both bulk and pattern motion will be analyzed in terms of spatial and temporal characteristics which may distinguish between models. The possibilities and hazards of generalizing nonrelativistic jet models to the relativistic case will also be briefly discussed, especially regarding possible observational signatures of magnetized versus unmagnetized jets.

INTRODUCTION

Theoretical modeling of small-scale structure in radio sources is made difficult by the limited number of constraints obtainable from the available data. The only possible imaging technique at present is VLBI, which is plagued by sensitivity and calibration limitations, and only provides a minimal level of resolution for serious theoretical work. Likewise, studies of short-period variability over all frequency regimes give limited information on the size and velocities of the emitting regions, but do not really place them accurately within the structure of the compact source. Confirmation of theory, apart from the generation of theories, requires considerable cleverness and caution. It is my intent to discuss the present observational and theoretical climate for modeling of small-scale structure, given the assumption that there are indeed relativistic velocities involved. Evidence from VLBI observations will be stressed, and shall be evaluated and discussed in terms of both theoretical models and observational format from which these results may best be derived.

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Kinematic models will be used to demonstrate the variety of possible results given the basic theory, and some discussion of dynamics shall be made in terms of possible qualitative results obtainable from present observations.

OBSERVATIONS

The structures of radio sources observed using VLBI typically fall into four classes: very compact, core-jet, compact double, and steep spectrum. The very compact sources are almost point sources, and hence provide little data apart from size. Unfortunately, this resolution is insufficient to determine whether or not the source exceeds the 10^{12} K brightness limit for synchrotron emission, although interstellar scintillation at radio wavelengths and X-ray variability suggest that this limit is, indeed, exceeded. Core-jet sources, on the other hand, have a definite core with some type of structure extending along some more-or-less well-defined axis. The compact doubles exhibit two more-or-less equal components with little or no change in their separation. The steep-spectrum objects are often rather complex, not necessarily possessing a strong axis of symmetry. These cores also vary in strength, with strong cores typically associated with core-jet sources at the centers of quasars, and the weaker cores associated with a variety of types associated with radio galaxies; even powerful radio galaxies such as Cygnus A tend to have weak compact cores.

The physical resolution scales of these observations vary with frequency and distance, from roughly 0.01 pc for the nearest Seyfert detected using VLBI to about 10 pc for the farthest quasar. Since the predicted size scale for the central engines of radio jets is of the order of a few A.U., this does not even approach the minimum characteristic length scale associated with models of these regions. Thus, the observed structures should be characteristic of the intrinsic power and composition of the jets, as well as the surrounding medium, and depend on the initial conditions at the central engine only insofar as they influence the gross properties of the jet. In other words, the properties of the radio images and the properties of the objects which generate them can only be linked indirectly, by theoretical modeling combined with short timescale variability studies. Conversely, the jets can be studied as if injected by a vaguely-specified ''nozzle'', as objects apart from the region in which they were generated.

A further observational constraint is the limited number of pixels per structure in VLBI images. In particular, very little internal detail of the individual components is possible, and, in the case of jetlike structure, no structural detail transverse to the jet can be resolved. Therefore, there is no reliable information as to the shapes of the structures composing the VLBI images, and hence it is difficult to distinguish between jets, plasmoids, or other types of structures. In fact, the only truly reliable parameters in VLBI maps are the relative flux distribution, and in particular the positions and relative brightnesses of isolated conponents, and the overall structure of continuous emission regions. The relative flux distribution can be tied to some absolute scale by careful calibration, but this is difficult and is generally unreliable by a few percent even if extreme care is taken.

The class of objects which I shall discuss today shall be the core-jet sources. Within the limits discussed above, we can measure three properties to reasonable precision: absolute flux (i.e., variability) at a variety of wavelengths but limited resolution; overall symmetry; and positions of relatively strong isolated components.

Variability fractions and time scales vary considerably. Some sources have X-ray variability time scales on the order of hours, and vary by a large fraction of their average flux, whereas some vary slowly if at all. This variety can be contrasted to the overall structure, which tends to be one-sided and, in the cases where large-scale structure is also present, appears to correlate with the sidedness of the large-scale structure. The positions of isolated structures in many sources are observed to vary so as to give the appearance of faster-than-light, or superluminal, motion. The short variability time scales, one-sidedness, and superluminal motion can be explained by relativistic motion.

BASIC THEORY

Consider a fixed point in space and an emitter moving relative to that emitter at some speed and viewing angle to θ, and define $\beta = v/c$, where c is the speed of light. Let the separation between them at time $t = 0$, r_0, be such that $r_0/R << 1$, and the final time of observation t_f such that $ct_f << R$. The light emitted from the initial and final positions of the emitter travels along parallel lines to the observer. The light emitted at time $t = 0$ arrives at the observer's position at time $Rcos\theta/c$; the light emitted at time t_f arrives at the observer's position at time $t_f + (R/c - \beta t_f)cos\theta$. Thus, the difference in arrival time, $\Delta t_{obs} = (1 - \beta\cos\theta)t_f$, can be considerably less than the physical time interval t_f At the same time, the projected physical separation between the two positions is simply $\beta sin\theta ct_f$. The apparent velocity of separation, $v_{app} = \beta csin\theta/(1 - \beta\cos\theta)$, may exceed the speed of light c for $\beta > 1/\sqrt{2}$. The maximum separation velocity $\gamma\beta c$, where $\gamma = (1 - \beta^2)^{-1/2}$, occurs at $\cos\theta = v/c$; thus, almost all sources which exhibit superluminal motion should have their jets aligned close to the line of sight. Given the high percentage of compact core-jet sources which now appear to exhibit superluminal motion, it appears that either superluminal motion is not due to relativistic motion along a well-defined direction, or there is a selection effect which enhances the number of observable sources which have jets which are nearly aligned with the observer.

This enhancement is due to the beaming of the flux from an emitter moving at relativistic velocities. It can be shown that, for an optically thin isolated component, $S(\theta) = S_{90}(1-\beta\cos\theta)^{-(3+\alpha)}$, where S_{90} is the flux observed along viewing angle $\theta = 90°$, and α is the spectral index (Scheuer and Readhead 1979). For an optically thin jet, or an infinitely long line of emitting isolated components, the exponent changes from $3+\alpha$ to $2+\alpha$ because, since the pattern over which we integrate the flux is essentially fixed relative to the observer, the emitting volume in the frame of the observer is stationary relative to the observer, whereas for a moving isolated component it is not. A further demonstration of this is that the emission from an excitation moving through stationary gas --- that is, some disturbance which illuminates gas which is stationary relative to the observer --- goes as $(1-\beta\cos\theta)^{-1}$, because of the difference in travel time between light emitted from the front and back of the moving emission region. In any case, the flux from an optically thin emitter is strongly enhanced in the direction of relativistic motion, which explains the large percentage of superluminal sources and the one-sidedness of the source; the ''counterjet'' can be much fainter than the jet even if intrinsically as bright. Note also that the spectral index for an optically thick source is 2.5, and so the optically thick flux is much more weakly beamed.

The beaming pattern, written above as a function of viewing angle θ, is traditionally presented as a number-flux relation P(x), i.e. as the probability of observing a source with flux ratio $S/S_{90} > x$. This is derived by assuming that the source orientations are randomly distributed relative to the observer, so that $dP \propto \cos\theta d\theta$, and integrating over all values of θ where $S/S_{90} > x$. The results for the standard models are shown in figure 1. These probability distributions may be changed by including evolution effects (such as an emitter which fades with distance from the core); this will be discussed later.

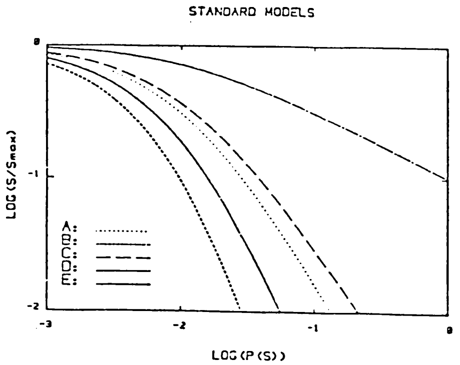

Figure 1

Another effect of relativistic motion can be seen by considering the arrival time of pulses emitted by a moving source at time intervals Δt_0 as measured by a comoving observer. The observed arrival times for the pulses are separated by a time interval $\Delta t_{obs} = \Delta t_0(1 - \beta \cos \theta)$. Thus, the time intervals are shorter for an emitter moving almost in the direction of the observer, and so the variability time scale is shorter. Also, the inferred size of the source, $c/\Delta t_{obs}$, is smaller than the size observed for a stationary source. Since the observed flux is larger than for a stationary source, and the inferred size is smaller than for a stationary unresolved source, then the brightness temperature inferred could be extremely large if it were assumed that the emitting region were stationary.

The point of this is, that the basic sense of the observations is consistent with the basic theory. However, the details are not necessarily consistent with the simple model presented here. The physical sources themselves are certainly much more complicated than this. One way to study the importance of these physical details is to select some simple models, such as relativistic shocks and plasmoids which fade with time, and examine the flux pattern in the frame of the observer for different intrinsic characteristics of the emitter, such as structure, shock strength, and opacity. This requires that the radiative transfer be carried out in full generality.

GENERAL THEORY

The general theory involving careful treatment of the radiative transfer is done by defining each of the relevant reference frames, and then transforming all quantities until everything is transformed into the reference frame of the observer (Lind and Blandford 1985). The three reference frames are (1) the comoving frame of the individual fluid elements, (2) the comoving frame of the emitting pattern, which is assumed to be stationary, and (3) the comoving frame of the observer. The relevant quantities are (1) emissivity and opacity, (2) shape and optical depth, and (3) flux. In detail, the emissivity and opacity are determined for each fluid element in its comoving frame, and then transformed to the comoving frame of the emitting pattern (e.g., the comoving frame of a relativistic shock which is itself moving relativistically relative to the observer. The radiative transfer equations are integrated along lines of sight in the pattern frame to obtain the optical depth and intensity. These are then transformed into the frame of the observer, and then the intensity in the observer's frame is integrated over to obtain the observed flux. All transformations are done assuming a power law for emissivity and opacity, and the flux calculated at a fixed frequency in the frame of the observer.

The results of these calculations for a planar relativistic shock deviate considerably from the case for a uniform optically thin sphere. Even the optically thin flux distribution deviates from that for an optically thin sphere, because of the internal relativistic velocities of the shock model. Furthermore, since optical depth depends strongly on orientation angle, $\tau = \tau_0(1-\beta\cos\theta)^{-(5/2+\alpha)}$, a high optical depth near the line of sight can suppress the flux toward the observer relative to the optically thin case, but may decrease so as to provide the strong optically thin beaming at larger angles. This reduces the observed effects of beaming. Similar results are derived for a variety of conical shocks, oriented both toward and away from the core. Even if we restrict ourselves to optically thin models, the distributions shown in figures 2 and 3, for pattern Lorentz factors of $\gamma = 7$ and $\gamma = 2$ respectively, demonstrate the large variety of possible flux distributions which may be associated with a given pattern Lorentz factor. Even given knowledge of the pattern Lorentz factor as measured by superluminal motion, some estimate of the orientation angle, and an accurate number-flux relation (equivalently, function of flux with angle), it is impossible to estimate the physical velocity of the jet without a detailed model of the jet structure.

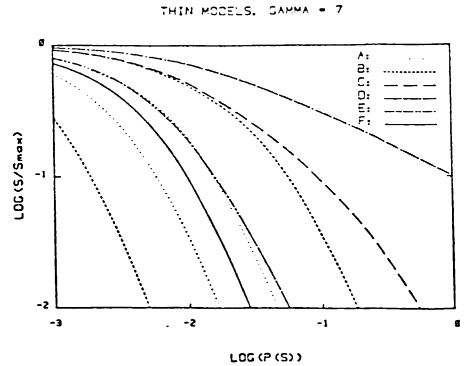

Figure 2

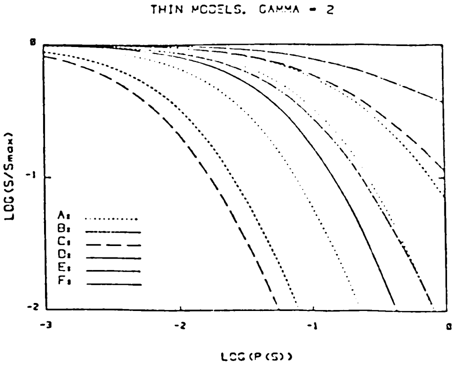

Figure 3

Another effect is best illustrated by examining the flux distribution of an ensemble of optically thin homogeneous spheres ejected along a fixed direction relative to the observer, in which the emission from the spheres is allowed to vary with distance from the core. If only one sphere is ejected, and it never fades, then the standard result for the optically thin sphere is obtained as discussed under the basic theory section. If a train of spheres have a fixed finite lifetime in the comoving frame of the observer Δt_l, and are ejected from the core at a fixed time interval of Δt_e, where $\Delta t_l >> \Delta t_e$, then the observed lifetime of a sphere is $\Delta t_{obs} = \Delta t_l(1-\beta\cos\theta)$, and at any given time the number of emitters observed is $\Delta t_{obs}/\Delta t_e \propto (1-\beta\cos\theta)$. The total flux is simply the number of emitters times their individual fluxes, and so the result for the continuous jet is then obtained.

Now, consider the case where $\Delta t_l \approx \Delta t_e$. There are two possibilities. If only one component is seen at a time, then the probability of seeing a source with a given flux S is then $dP \propto (1-\beta\cos\theta)\cos\theta d\theta$ rather than $dP \propto \cos\theta d\theta$ because of the shortened apparent lifetime of the emitting region. If a number of components are seen at one time, then the mean flux will be the same as that for the continuous jet, but the fractional variability $\Delta S/S = 1/N \propto (1-\beta\cos\theta)^{-1}$. Note that, the more nearly aligned the source axis and the observer, the larger the fraction by which the source varies. Recall also that, the more nearly aligned the source axis and the observer, the shorter the variability time scale. Thus, the most variable sources should also vary the fastest. This is consistent with observation. If the brightnesses of the spheres fall off as some function of distance form the core, then other flux distributions can be obtained. The variety of possible number-flux relations attainable with different evolution/ejection models is at least as great as the variety of number-flux relations attainable with different models for the emitting regions (as seen in figures 2 and 3), and again, seriously diminishes the predictive power of the number-flux relation in modeling the structures of the sources.

A final modification is to assume that the path followed by the emitting regions curves. For a single emitter moving out along this path, the effect is for the emitter to brighten and fade, depending on the angle relative to the observer, and for the observed proper motion to vary as well. For multiple emitters, the relative fluxes between them would vary as they moved along the curve. If all of the emitters were identical, the same spot would brighten and fade although the emitters were moving relativistically. In effect, the curve acts as a screen. A number of other screen models can be constructed, some of which would allow superluminal motion without beaming, some of which would allow beaming without superluminal motion, and some of which would allow both.

As a final refinement, allow a variety of emitters of varying structures to be ejected at varying intervals along slightly different angles or along curved paths. Because of the variations in intensity and optical depth with angle, a variety of different components may dominate the flux at any given viewing angle, and so, given the variety of functions associated with the ensemble of plausible models, the detailed number-flux relation --- or, the distribution of flux with angle --- cannot be determined unless the details of the source are accurately known. In short, the fitting of number-flux relations to theoretical curves in order to determine the velocity of the jet material is at best risky. Furthermore, the use of the proper motion of components to determine the jet velocity is also risky, since the pattern speed of a component need not be closely related to the velocity of the underlying jet. Given these conclusions, one may well wonder what can be done with the available data.

First, the existence of superluminal motion does indicate that there is relativistic motion of some sort going on, whether pattern motion or bulk motion. If there is pattern motion, it is either motion of some nonemitting jet or well-collimated beam of light against a screen, or there is some material present with a relativistic equation of state, through which the pattern may travel. Given material with a relativistic equation of state, it will eventually expand relativistically unless confined by the surrounding medium, or is already moving outward as a jet. Since almost all superluminal motion appears to involve motion away from the compact core, it appears that a relativistic jet is the best general underlying model.

Second, the classic superluminal sources eject new components at relatively long time intervals, and the components remain discrete and observable for long enough to be observed for several epochs, as has been done for a variety of superluminal sources. If this data is taken at several frequencies, and the data carefully calibrated, then the total flux and spectral index of a component can be monitored, and can be fit to theoretical models. This need not be done only for discrete components; since components appear in a more or less regular way following outbursts for a large number of these sources, and these outbursts can be well-separated, then the flux variations in the core can often be associated with the formation and propagation of one component, which may then be modeled. For example, Marscher and Gear (1985) constructed a very plausible model for a planar relativistic shock propagating in a relativistic jet of small opening angle, and proceeeded to fit it to an outburst in 3C273. Similarly, Hughes, Aller, and Aller (1985) have constructed models for the variations in flux and polarization caused by energy generation and subsequent cooling of material at a relativistic shock front and fit it to radio variability data for BL Lac, with considerable success. This confirms that at least some of the observed features in VLBI maps are relativistic shocks propagating through a faint underlying relativistic jet. This is further supported by high dynamic range imaging of a number of sources, such as 3C273 (Unwin and Davis 1987) and 3C371 (Lind 1987), which reveal a continuous underlying flux distribution with isolated components superimposed on top of it. The regularity in the falloff of the flux with radius for the above-mentioned map of 3C273, and for 3C120 (Walker, Benson, and Unwin 1987), suggest as well that either classic superluminals are intrinsically simple, or that beaming dominates such that one particular type of feature totally dominates the flux. In the first case, we can hope to model them; in the second, it is questionable.

Third, variability timescales, particularly in the X-ray, appear to indicate that much if not all of the flux from the compact core is indeed beamed. Given the large flux enhancement of an optically thin emitter moving at relativistic speeds over either a stationary or an optically thick emitter, it is not surprising that the compact core flux should come, not from the central core, but from some part of the outflow itself. In this regard, the inhomogeneous jet model of Blandford and Königl (1979) has proven to be in good agreement with the spectra and variability timescales of various compact cores, and suggests that the relativistic jet models are, in essence, correct.

RELATIVISTIC HYDRODYNAMICS

Numerical simulation of nonrelativistic jets has provided a number of interesting general results, but their usefulness for modeling of one-sided jets, both large and small-scale, is questionable because of the differences between relativistic and nonrelativistic fluid mechanics. For example, the factors of $\gamma = (1-\beta^2)^{-1/2}$ in the momentum and energy equations provide for a large change in energy density for a small change in velocity, and can cause large pressure contrasts. On the other hand, there may well be some structural signatures which carry over from nonrelativistic to relativistic jets. An example of such a possible signature is the morphological difference between jets with strong and weak ordered toroidal fields. The weakly magnetized jet is simply terminated by a Mach disk structure, and the shocked jet material flows back into a surrounding cocoon (Lind 1987). The strongly magnetized jet recollimates into a broader, weaker jet which continues on (Clarke, Norman, and Burns 1987). These jets also demonstrate the complexity of ''real jets'', which may contain fixed and traveling planar and oblique shocks, as well as considerable vorticity and turbulence.

CONCLUSIONS

Inhomogeneous models of VLBI jets are at best difficult to construct because of the sparcity of the observations and the strong dependence of the observed flux distribution on model-dependent parameters. However, in many cases, which include the superluminal radio sources, the isolation of individual components allows the use of multifrequency imaging and flux monitoring to determine semi-quantitative models of individual outbursts or components. The precise modeling of compact radio sources requires high dynamic range imaging and frequent observation of variable sources --- attainable once the VLBA is built --- and theoretical modeling of relativistic flows. The latter is difficult numerically, but some qualitative results from present, nonrelativistic simulations may allow us to distinguish between general classes of theoretical models. Theoretical modeling of compact radio sources is indeed difficult and limited, but, with the use of all available data and cautious interpretation, progress can be made!

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