and in such cases it is natural to invoke the concept of "satisfiability to some extent or to some degree." As a result the question whether some assertion follows from some other assertion might not have a crisp answer "yes"/"no." It is likely that in general the "relation of following" or more technically speaking, the consequence relation turnstile ($\vdash$) may be itself many-valued or graded. Thus, we have adopted graded satisfiability as well as graded consequence in the thesis.

- It has been imperative to link with fuzzy topological systems (and fuzzy topological spaces as a result), a many-valued logic similar to the link between classical topological systems and geometric logic. This goal has been achieved here with the introduction of a general fuzzy geometric logic. In our case, of course, a further generalization has been made by taking the consequence relation as many-valued also and this in turn gives rise to a generalization of the algebraic structure frame to graded frame.

The thesis is organised as follows. The first chapter includes introduction and almost all possible ground notions which are essential to make the thesis self contained.

In Chapter 2 the notion of fuzzy topological system is introduced and the categorical relationship with fuzzy topology and frames is discussed in detail. Also this chapter contains some methodology for making new fuzzy topological systems from old ones.

Chapter 3 provides a generalization of fuzzy topological system which shall be called $\mathscr{L}$-topological system and categorical relationships with appropriate topological space and frame. Furthermore, two ways of constructing subspaces and subsystems of an $\mathscr{L}$-topological space and an $\mathscr{L}$- topological system are provided.

Chapter 4 deals with the concept of variable basis fuzzy topological space on fuzzy sets and contains a new notion of *variable basis fuzzy topological systems* whose underlying sets are fuzzy sets. In this chapter the categorical relationship between space and system is established.

Chapter 5 contains a different proof of one kind of generalized stone duality, which was done directly by Y. Maruyama, introducing a notion of $\bar{n}$-fuzzy Boolean system.

The last two chapters, Chapters 6 and 7 deal with the ultimate objective. Chapter 6 deals with the question "From which logic can fuzzy topology be studied?" To answer this, the notion of fuzzy geometric logic is introduced. In addition, further generalized notions such as fuzzy geometric logic with graded consequence, fuzzy topological spaces with graded inclusion, graded frame, and graded fuzzy topological systems come into the picture.

Chapter 7 provides categorical relationships among fuzzy topological spaces with graded inclusion, graded frames, and graded fuzzy topological systems.

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Abstract

This thesis investigates combinatorial properties of ultrafilters and their model-theoretic significance. Motivated by recent results on Keisler's order, we develop new tools for the study of Boolean ultrapowers, deepening our understanding of the interplay between set theory and model theory.

The main contributions can be summarized as follows. In Chapter 2, we undertake a systematic study of regular ultrafilters on Boolean algebras. In particular, we analyse two different notions of regularity which have appeared in the literature and compare their model-theoretic properties. We then apply our analysis to the study of cofinal types of ultrafilters; as an application, we answer a question of Brown and Dobrinen by giving two examples of complete Boolean algebras on which all ultrafilters have maximum cofinal type.

In conclusion, we discuss the existence of nonregular ultrafilters and prove that, consistently, every decomposable ultrafilter on a complete Boolean algebra is regular.

Chapter 3 centres around the study of Keisler's order. We prove that good ultrafilters on Boolean algebras are precisely the ones which capture the maximum class in Keisler's order, solving a problem posed by Benda in 1974. We also show that, given a regular ultrafilter on a complete Boolean algebra satisfying a distributivity condition, the saturation of the Boolean ultrapower of a model of a complete theory does not depend on the choice of the particular model, but only on the theory itself. Motivated by this fact, we apply and expand the framework of 'separation of variables', recently developed by Malliaris and Shelah, to obtain a new characterization of Keisler's order via Boolean ultrapowers.

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