PART A

TYPE OF PROBLEMS THAT EXIST

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DEFINITION OF THE TYPES OF PROBLEMS THAT EXIST IN STEADY-STATE EXTENDED ATMOSPHERES

by

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ABSTRACT

In section 1 practical details concerning the equivalence of observational and theoretical descriptions of stellar spectra are reviewed, particularly the difficulty of identifying the observed reference level (continuum) with the theoretical continuum in the case when many lines are present. In this connection thought must be given to how integrals over frequency should be normalised and evaluated because the effective continuous absorption coefficient does not remain constant over the range from 0 to ∞. The choice of spectroscopic details by which to determine T_{eff}, log g and abundances requires careful consideration.

In section 2 the factors by which an extended atmosphere ~~are~~ recognized are summarized and the question is posed do all stars have extended atmospheres. Another question requiring an answer is whether the concepts microturbulence and macroturbulence are physically real concepts or whether they are merely fitting parameters to make a simple LTE theory account for the observed spectra of supergiants in which rather wide lines occur and many multiplets show rather steep gradients. In section 3 the types of line sensitive to non-LTE conditions are described. These are resonance lines, lines arising from metastable levels, subordinate lines for which the upper level is sufficiently separated from the continuum and other levels that this upper level is chiefly populated by radiative processes from the ground or other low lying levels and lines which go

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into emission in low density atmospheres as a result of optical-pumping (fluorescent) processes. Such lines should not be used for abundance determinations by means of LTE theory though this is frequently done.

Theoretical considerations are discussed in Section 4 where first the problem of the two-level atom is sketched and then the problem is generalised to a many-level atom. The parameter λ which gives the probability that a photon is lost from the line by de-excitation processes other than spontaneous emission is defined and it is pointed out that non-LTE physics has the effect of adding a scattering term to the expression for the source function. One example is given of the effect of changing the line source function from the Planck function to a form suitable for isotropic coherent scattering. The line becomes deeper and wider for the same number of atoms. Interpretational problems in stellar spectra are discussed in section 5. It is noted that many lines in main-sequence early type spectra show the effects of departures from LTE. These effects are shown to a conspicuous degree by the spectra of shell stars. The example of He I 5876 in 10 Lacertae, O9V, is discussed and the implication for interpreting the He I lines in all B type main-sequence stars are touched upon. Helium-weak and helium-strong spectra probably indicate variations in density of the outer atmosphere rather than true abundance differences. The spectra of supergiants are also considered and it is pointed out that the Ia supergiants of type B may be hydrogen-poor.

Finally in section 6 the problem of choosing simplified physical representations of line forming when non-LTE physics must be used is discussed. Some relevant points concerning the observed spectral lines used for spectral classification are illustrated by means of partial energy-level diagrams.

Key words: interpretation of stellar spectra, extended atmospheres, line formation.

I. INTRODUCTION

This colloquium is concerned with finding a physical and mathematical description of how the stellar spectrum is formed in an atmosphere that is "extended" but in a steady state. We first need to decide what are the important physical characteristics implied by the word "extended." Secondly a definition of "steady state" should be agreed upon, and thirdly some description should be given of what is meant by spectrum. The spectrum of a star comprises strong absorption lines, weak absorption lines, and continuous spectrum as well as emission lines; each of these characteristic parts of a stellar spectrum has properties determined by the interaction of each atom, ion, or molecule with the radiation field to produce the stellar spectrum.

The ideas we use and the words by which these ideas are transmitted are in many cases rather vague and contradictory. Something of value will have been accomplished at this colloquium if we are able to make our ideas and the words used to describe them more precise. Even more will have been accomplished if we are able to see how existing methods of analysis can be used, and to define rather sharply the physical as well as the mathematical aspects of our problems. In what follows I am going to speak as a stellar spectroscopist anxious to obtain information about the physical conditions in stellar atmospheres from a quantitative study of the parts of the stellar spectrum available to me. The standard way of proceeding is to compare observed spectroscopic observations with theoretical predictions resulting from an explicitly defined physical and mathematical model. If agreement is found then one says the parameters of the model are characteristic of that part of the stellar atmosphere involved in forming the part of the spectrum under consideration. If no agreement is found then the model must be changed.

One must consider carefully the meaning of the word "agreement" used in this context. Clearly the variation with wavelength of any predicted intensity distribution which is the same as the observed spectral distribution within the uncertainties of the measurements must be considered to offer a possible solution to our problem. A choice between several (at first glance possible) models can be made by using a well thought-out selection of spectral details for the comparison.

With stars one observes the total radiation field from the part of the star facing the observer; thus the variation of an intensity distribution with wavelength corresponds to the variation of the theoretical quantity F_ν with wavelength. Furthermore because of practical difficulties the observed intensity distributions are not given in absolute energy units but are expressed as _relative_ intensities. In the case of line profiles, the observed intensities are expressed as fractions of the intensity that would have been available if no line was present. The reference level of intensity is known as the continuum; it is found by drawing a smooth curve through the intensity level of parts of the spectrum where no absorption lines appear to be. When one interprets the observed variation with wavelength of the intensity in the continuous spectrum, which means comparing intensities over a range of several thousand angstroms, the intensity at some particular wavelength is chosen as reference point.

It is important to keep these practical definitions in mind when comparing theoretical and observed spectra because often what seems to be an obvious theoretical level of reference is not what is in practice used. For instance the simple concept of a continuous spectrum and a few superimposed absorption lines is useful only when the lines are too few in number to obscure the trend of the continuous spectrum with wavelength. This is so over much of the normally observed spectral range for O and B stars. In these cases it appears to be straightforward to compare observed and predicted spectra of early type stars. However, if one considers the far ultraviolet spectral range for OB stars, $912Å < \lambda < 1900Å$, the spectra are full of strong lines and the definition of a theoretical continuum that corresponds to the observed datum line selected as the continuous spectrum may be difficult. This problem is familiar to those who attempt to interpret the spectra of stars of types F and later in the spectral region $\lambda <$ 5000Å.

Because the observed quantity is a <u>relative</u> variation of intensity with wavelength, one theoretical parameter of interest is $(\ell_\nu + \kappa_\nu)/\kappa_o$ where ℓ_ν represents the total absorption coefficient due to all possible lines and κ_ν represents the absorption coefficient due to all possible sources of continuous absorption at frequency ν. Here κ_o is the total absorption coefficient due to continuous sources of opacity at the reference frequency ν_o. In all theoretical studies of line formation done to date it has been assumed that one may replace κ_ν by κ_o, whatever the value $\nu - \nu_o$. With non-coherent processes the appropriate expressions for the emergent intensity contain integrals over ν from 0 to ∞ with $(\ell_\nu + \kappa_\nu)$ as a varying parameter. When these expressions are evaluated (typical expressions are the kernel functions of Avrett and of Hummer) use is made of the fact that the variation with ν of ℓ_ν is given

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by a function which is normalised over the range 0 to ∞. However in this range κ_ν is also normalised and the shape function varies from zero to a maximum value to zero again. With non-coherent processes, which indeed are what occur in stellar atmospheres, the transport of radiation at different frequencies rather far from the line center depends quite sensitively on what sort of exchanges may take place in the wings of the lines. It may be a minor point, but it seems to me that we might consider whether in some of the lines it is reasonable to replace $\ell_\nu + \kappa_\nu$ by $\ell_\nu + \kappa_o$, that is, to assume that we can write the total absorption as $\ell_o(\phi_\nu + \beta)$ where β is a constant which is κ_o/ℓ_o and ϕ_ν is a normalised shape function. The rate of change of κ_ν with ν might be closely the same as that of ℓ_ν over an important part of the range and furthermore both quantities may have comparable values over part of the range. Whether such considerations are important depends upon the ratio of ℓ_ν to κ_ν at the line center. If this ratio is large, ℓ_ν and κ_ν will not attain comparable values until $\nu - \nu_o$ is large. Then the difference between κ_ν and κ_o may be significant and an asymmetry might occur between the two sides of the line profile as it is defined observationally. These considerations should be held in mind when deciding upon the appropriate quadrature formula for the frequency integrals, a point which will certainly come to discussion later in the conference.

The parameters that are sought to describe the stellar atmosphere are factors such as the temperature, pressure, and density as functions of depth in the atmosphere and the composition of the atmosphere. The parameters that describe the star are effective temperature and surface gravity. In principle these can be found by studying spectral features that are sensitive to the temperature distribution in the stellar atmosphere and to the density distribution so long as the temperature distribution is determined by the total radiative flux passing through the atmosphere and the pressure structure is determined by the need to maintain hydrostatic equilibrium against the acceleration of gravity. It is by no means obvious which spectral features are most suited for these purposes at each spectral type though the well known spectroscopic type and luminosity criteria serve as an empirical starting selection.

A star with an extended atmosphere usually refers to a supergiant or a shell star. However this concept requires more detailed consideration which will be given below. So far as the words "steady

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state" are concerned, the intent is to restrict the discussion to stars having atmospheres in which spectroscopic changes do not occur, or occur so slowly that the process of change does not need to be considered. We have arbitrarily excluded from discussion rapidly changing, evolving situations. Furthermore in the case of pulsating variables we are not seeking at this time to find the cause of the observed regular changes in light and spectrum so much as to find out what sets of physical variables correspond to the various recurring sets of spectroscopic phenomena. This problem has a known solution if one may assume that the physical variables and the observable spectroscopic details are bound together as though thermodynamic equilibrium existed at each place in the atmosphere. However, the values of temperature, pressure and density found in this way are not consistent with the hypothesis of local thermodynamic equilibrium. Furthermore the motion of the atmosphere may have a significant effect on the line shapes and strengths.

II. FACTORS BY WHICH AN EXTENDED ATMOSPHERE IS RECOGNISED

The concept "an extended atmosphere" grew up empirically as a result of comparing details in the spectral type, thus effective temperature, and interpreting the differences in spectrum in terms of LTE theories applied to a single layer of gas open to interstellar space and irradiated from below by the flux emerging from the rather dense layer known as the photosphere. The gas pressure in supergiant reversing layers was found to be approximately 100 times less than that in main sequence reversing layers. Simple geometric considerations indicate that the reversing layer of a supergiant has a considerably greater size than that of a main sequence star of the same effective temperature. Hence the origin of the term "extended."

Many of the extra absorption lines, which appear in the spectra of some B type main sequence stars and lead to the classification of these stars as shell stars, are very like lines in the spectra of A or late B type supergiants. From this observed fact has come the idea that the atmospheres of shell stars have even greater extension and lower density than the atmospheres of supergiants.

The conclusion that the pressure is lower in a supergiant atmosphere than in a main sequence atmosphere comes from the facts that in supergiant spectra the Stark-broadened wings are not strikingly present for H or He I lines and the forbidden He I lines do not appear. Also the lines in the second spectra of the metals are stronger in supergiant spectra relative to the lines from the first spectra of the metals than is the case for main sequence stars of about the same temperature. Further differences between absorption lines in supergiant spectra and those in main sequence stars are that in supergiants the lines have rather wide, steep-sided profiles and that when the spectra are compared multiplet by multiplet it is seen that the supergiant lines, in spite of their greater equivalent width appear to lie on the Doppler part of the curve of growth rather than on the transition part where many of the lines in main sequence spectra lie. This effect is known as the gradient effect; it was interpreted by Struve and Elvey[1] in terms of microturbulence. When Struve[2] noticed that the profiles of the strong lines in supergiant spectra were wider than could be accounted for by the velocity fields deduced from curves of growth and the hypothesis of microturbulence, the concept "macro-turbulence" was introduced to account for the extra width. These attempts to reconcile a simple LTE theory of line formation with what is observed in the case of stars having extended atmospheres have lead to the introduction of physical concepts that are difficult to justify in detail. It is time to ask whether the observed phenomena--steep gradients and rather wide lines in supergiant spectra--should not be interpreted in terms of an improved and more realistic theory of line formation.

In the case of shell stars, absorption lines of the second spectra of the metals very like those in supergiant spectra may be found but the hydrogen lines have much deeper cores, in some cases going nearly to zero intensity, and emission wings are seen for the first few members of the Balmer series. The strong lines of Fe II also often have emission wings. In some stars the relative intensities of the He I lines from the levels 2^3S, 2^1S, 2^3P and 2^1P are not the same as for main sequence stars, the lines from 2^3S and 2^1S being stronger than normally expected according to the observed strengths of lines from 2^3P and 2^1P. Struve and Wurm[3] named such changes in relative line strengths dilution effects and they showed that the relative populations would change in a low density gas in which the radiation

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density was reduced from its thermal equilibrium value in such a way that the atoms tend to accumulate in the metastable levels. Consequently whenever the lines arising from metastable levels are seen to be unusually strong in a stellar spectrum, one speaks of an extended atmosphere. The theory of Struve and Wurm is a simple version of the type of theory one must consider when it is not appropriate to assume LTE.

In summary, the qualitative characteristics of a stellar spectrum that lead to the inference of the presence of an extended atmosphere are (1) broad, steep-sided absorption lines, (2) little or no Stark broadening of the H and He I lines and the absence of [He I] lines, (3) a steep gradient in multiplets of strong lines, and (4) exceptionally great relative strength of absorption lines arising from metastable levels. In an extended atmosphere the density is sufficiently low that one cannot assume that collisional processes will establish LTE level populations.

A phenomenological description of these ideas coupled with pertinent references to the details seen in stellar spectra has been given by Struve.[4] He asked nearly the same questions as those facing us now.

1. Why do some stars possess tenuous outer atmospheres or shells while other stars, apparently of identical physical characteristics, do not have such shells?

2. What is the origin of a shell and how is it supported in apparent violation of the laws of mechanics?

3. How can we account for the remarkable tendency of nearly all shells to vary either periodically or, more often, in an irregular manner?

4. Why do some shells expand while others are stationary?

The first question should be modified to enquire whether all stars have extended atmospheres and to ask what factors make these extended atmospheres visible in the normally accessible spectral region. It is essential to define accurately criteria that are sensitive to the presence of a tenuous outer atmosphere. The low density of particles in an extended atmosphere surrounding a star and the departure of the radiation field from the black-body distribution appropriate to the electron temperature make it necessary to abandon the hypothesis of LTE. We would like to have a calibration of the strength and shape of suitable lines in terms of the density,

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electron temperature, and size of the extended atmosphere. Another relevant factor will be the velocity field in the line of sight. An extended atmosphere can be detected only if spectral lines of adequate sensitivity to the conditions in an extended atmosphere happen to fall in the spectral region under study. Observation has already shown that extended atmospheres of widely different properties exist around many stars. The corona of the sun is an example of an extended atmosphere that can be detected only by very special observations, whereas the extended atmospheres of Wolf-Rayet stars can be detected by simple low-dispersion observations. All stars may possess extended atmospheres in the general sense; we should like to find out what spectroscopic phenomena at each spectral type give the clearest evidence of the presence of the extended atmosphere and of its physical properties. The complementary question (what spectroscopic details in a stellar spectrum can be interpreted reliably in terms of simple LTE theories of spectrum formation?) is also not without interest. Such features can be used with the existing methods of analysis to obtain an estimate of the physical conditions and the abundances of the elements in the parts of the stellar atmosphere where the density is high enough that LTE is a reasonable hypothesis.

III. THE TYPES OF SPECTRAL LINE SENSITIVE TO NON-LTE CONDITIONS

Spectral classification and the recognition of stars with extended atmospheres is based on the apparent relative strengths of selected absorption lines. These relative strengths are chiefly determined by the central intensities of the lines; the width of the line and the exact shape of the profile are less important factors. The central intensity of a strong line is determined by the value of the source function at the edge of the atmosphere. In the case of a two-level atom one can write

$$S_\nu \propto n_2 n_1 \qquad (1)$$

where n_2 is the population of the upper level and n_1 is the population of the lower level. In the case of a normal atmosphere the balance of radiative and

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collisional processes results in certain values of n_2/n_1 through the atmosphere, thus in a certain central intensity for the line. If in an extended atmosphere, owing to the lower density and the changed radiation field, the ratio n_2/n_1 is reduced, a deeper line will result and the line will be said to have become stronger. If the ratio n_2/n_1 is increased strongly, the line may appear in emission. Lines that go into emission are those for which the upper level can be populated by absorption of line radiation generated in an intrinsically strong line such as Lyman α of H or He II 303. In the literature of the 1930's and 1940's this process was called fluorescence. Its occurrence is a sure indication that non-LTE effects are important in at least part of the stellar atmosphere. (In this discussion it is implicitly assumed that the continuous absorption is small with respect to that in the line center and that it remains essentially unchanged by changes in the density of the atmosphere.)

However, it is not sufficient to focus attention on the changes in population of only one level; the change in source function is brought about by the relative change of n_2 to n_1. The populations of levels 1 and 2 can vary at different rates under different circumstances. The type of variation depends explicitly upon the spectroscopic description of levels 1 and 2 and the sizes of the collision and radiative cross sections between these levels and all other levels in the atom or ion including the continuum and doubly excited states lying above the primary ionization limit. All such doubly excited states do not autoionize with high probability.

The types of line which will be sensitive to non-LTE conditions are (1) resonance lines, (2) lines arising from metastable levels, (3) subordinate lines for which the upper level is sufficiently separated from the continuum and other levels that this upper level is chiefly populated by radiative processes from the ground or other low lying levels, and (4) lines that go into emission in low density atmospheres as a result of the optical pumping (fluorescent) processes mentioned above. Resonance lines are sensitive to non-LTE conditions because the spontaneous transition probabilities from neighbouring levels into the ground level are large for permitted dipole transitions whereas often the energy differences from the ground level to neighbouring levels are sufficiently great that the probability of collision-induced transitions is small when electrons having energies corresponding to temperatures of 10,000 to 20,000 degrees are considered. Electron temperatures of this size are expected in the atmospheres of A and B stars. Somewhat similar arguments hold for metastable levels, although it may happen, as with He I, that the lowest metastable level is within easy "collision distance" of the continuum. In that case, when the density is not too low, the population of the metastable level may approach its thermal equilibrium value for the local electron temperature because the level is strongly coupled through collisions to the continuum.

Lines sensitive to non-LTE conditions include many of the lines that are used for classifying stars of types A, B, O and Wolf-Rayet. Some examples are the Balmer lines of hydrogen; the He I lines from 2^3S, 2^1S, 2^3P and 2^1P; the He II lines from n = 3 and n = 4; C III 4647, 50, 51 and 5696; C IV 5801, 12; N III 4634, 40, 41; N IV 3478, 82, 84 and 4057; N V 4603, 19; O 17771, 74, 75; Ca II H and K; Mg II 4481; Si II 4128, 30 and 4200; Si III 4552, 68, 74 and Si IV 4088 and 4116. In the cooler stars and in shell stars, strong lines arising from the metastable levels of the ground configuration of Cr II, Mn II, Fe II, and Ni II influence strongly our classification of stars. In each case both the lower and the upper level of the line are sufficiently isolated that radiative processes as well as collisional processes are important in establishing the population ratio n_2/n_1. In a few cases (for example Hα, He II 4686, C III 5696, and N III 4634, 40, 41) particular radiative processes are known to generate emission lines by causing an overpopulation of the upper level of the line. In other cases the ratio n_2/n_1 decreases in extended atmospheres relative to its value in normal atmospheres with the result that deeper (stronger) than normal absorption lines are seen. Most of these changes are due to the decreased density. An increased density in the atmosphere (more collisions) forces the population ratios towards the LTE values. The intensities of none of the above lines should be interpreted in terms of abundance without first making some investigation of the effects of non-LTE and how the relevant level populations depend on the density, temperature, and the available geometric path length.

That the level populations of H and He are sensitive to non-LTE conditions in those parts of the atmosphere where the strong lines are formed, that is in regions where the electron density is less than 10^{13}, has been shown by Strom and Kalkofen,[5,6] Kalkofen and Strom,[7] Kalkofen,[8] by Mihalas[9,10] and by Mihalas and Stone.[11] Most of these studies are concerned with the continuous spectrum of early type stars. The effects on the line spectrum have not been investigated in detail. Hearn (in press) has studied some effects of non-LTE conditions on the strengths of the He I lines. All of these investigations lead to the inference that the strengths of the classification lines are very sensitive to the density in the atmosphere. The sensitivity to temperature changes is much less. These facts are used empirically for recognising supergiants and shell stars. The calculations referred to were made with models believed to represent main sequence stars. It is clear that non-LTE effects are important for understanding the meaning of the spectral classification of main sequence A, B, and O stars as well as that of supergiants.

IV. THEORETICAL CONSIDERATIONS

In the case of an atom with two bound levels only and no continuum, the equation of transfer within the line can be written for plane parallel layers as

$$\frac{\mu dI_\nu(\tau,\mu)}{\phi_\nu d\tau} = I_\nu(\tau,\mu) - S\ (\tau) \qquad (2)$$

with

$$S(\tau) = [1 - \lambda(\tau)] \int_0^\infty \phi_\nu(\tau) J_\nu(\tau) d\nu + \lambda(\tau)\ B_\nu(\tau). \qquad (3)$$

Here τ is the optical depth at the center of the line, $\phi_\nu(\tau)$ is a normalised shape function for the line, $B_\nu(\tau)$ is the Planck function and μ, I_ν and J_ν have their usual meaning. The quantity $\lambda(\tau)$ is the relative probability that a line photon is lost to the line as a result of collisional de-excitation,

$$\lambda(\tau) = n_e C_{21} / \ (A_{21} + n_e C_{21}), \qquad (4)$$

where C_{21} is the cross section for collisional de-excitation, n_e is the electron density at level and A_{21} is the Einstein probability for spontaneous emission. Since both n_e and C_{21} depend upon the electron temperature at level τ, λ is a function of τ.

In the case of many-level atoms, re-emission at frequencies within the line may occur because of spontaneous emission and from the energy field associated with the electrons. This possibility can be taken into account formally by writing the source function in line frequencies as

$$S(\tau) = [1 - \lambda(\tau)] \int_0^\infty \phi_\nu(\tau) \; J_\nu(\tau) d\nu + \lambda^* \; B_\nu(\tau), \qquad (5)$$

where

$$\lambda(\tau) = (n_e C_{21} + \sum_k A_{2k} + D)/(A_{21} + n_e C_{21} + \sum_k A_{2k} + D) \qquad (6)$$

and λ^* is suitably defined. In equation (6) D represents all possible transitions from level 2 that are generated by the radiation field and all collisional transitions except from level 2 to level 1. No general rules can be given for defining λ^*; it represents photons that are emitted at line frequencies as a result of the original line photons going into the energy of the electron gas on one of the alternative routes out of level 2. The index k in equation (6) refers to all levels below level 2 into which the atom can decay be spontaneous emission except level 1. For the lines used to classify stellar spectra k is a small integer, usually less than four.

Other terms can be added to equations (2) and (5) to take account of sources of continuous absorption and blending lines. Once the source function is known, the emergent flux from the plane parallel layers can be found as the Φ - transform for the case $\tau = 0$.

The well known difficulties of this transfer problem arise because equation (2) is an integro-differential equation owing to the occurrence of the quantity

$$J_\nu(\tau) = \frac{1}{2} \int_{-1}^{+1} I_\nu(\tau,\mu) \; d\mu \qquad (7)$$

in equation (5) and because λ and λ^* depend on the radiation field as does τ which depends on the population of the lower level on the line, $n_1(z)$. We have

$$\tau(z) = -\int_z^\infty n_1(z)\ \alpha_0\ dz \qquad (8)$$

where α_0 is the atomic absorption coefficient at the center of the line. The level populations are related to the radiation field and to each other by the demand that a steady state exist. The quantity λ may be defined to be independent of the radiation field by neglecting transitions out of level 2 generated by the radiation field. In the case of some extended atmospheres, photo-excitation processes and photoionizations may be neglected. Then the problem becomes simpler.

One advantage of writing the source function in the form of equation (5) is that one sees that there is a "scattering term"--the first term--and a term from all other processes. When collisional deexcitation is much more important for the upper level than spontaneous emission, $\lambda \to 1$ and the source function becomes identical with the Planck function. Then the equation of transfer has the simple form which is used in all LTE studies. One reason why LTE calculations usually are unsuitable for resonance lines, for lines arising from metastable levels, and for subordinate lines from low-lying levels is that for these lines λ may be significantly different from unity over much of the atmosphere owing to the low density in the line-forming regions and the diluted radiation field.

Finally it should be noted that if the scattering term in the source function dominates, i.e., λ and λ^* are both small, a deeper and stronger line will result for the same number of atoms than is found when λ and λ^* are large. A simple example is shown in Figure 1 where the calculated profile of Si III 4552 is displayed for the case that the reemission in line frequencies is according to Kirchhoff's law (thin line) and when it is given by coherent, isotropic scattering (thick line). The model atmosphere used (Guillaume[13]) is a line-blanketed model of approximate type B1.5V, having $T_{eff} = 23\ 255°K$ and $\log g = 4.0$. The line absorption coefficient corresponds to thermal Doppler broadening and a damping coefficient 10 times the classical damping constant; the fractional abundance by weight of silicon is 1.206×10^{-3}. This example is merely an illustration of the very significant changes in

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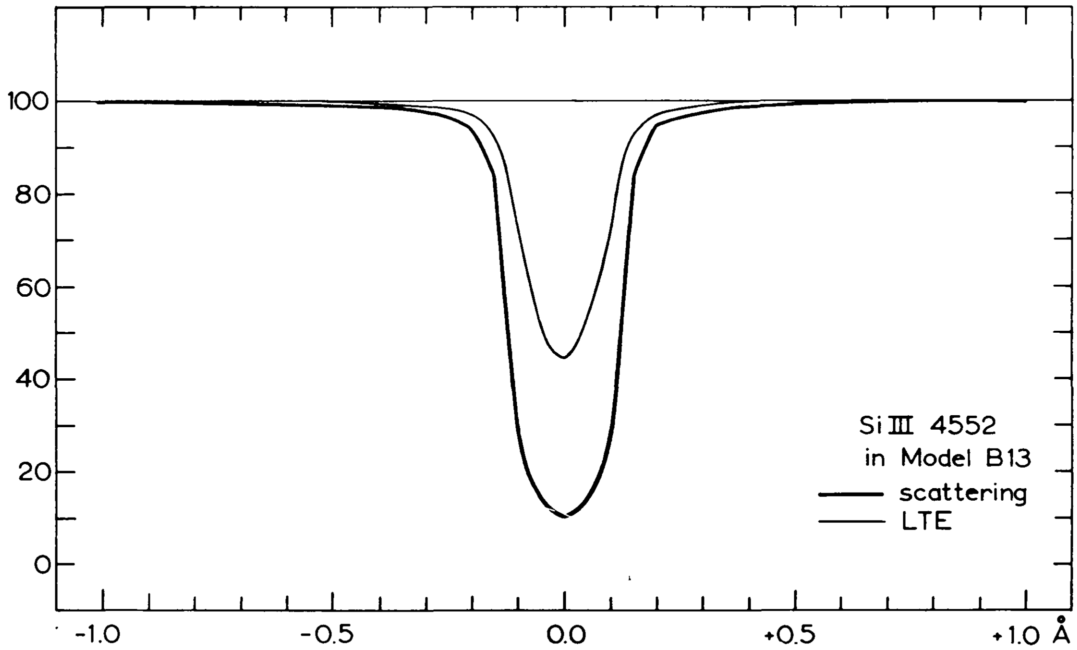

Figure 1. Predicted profiles of Si III 4552 in a line-blanketed model of type B1.5V for the cases (a) that the line is formed by coherent, isotropic scattering (thick line) and (b) that the line is formed in LTE (thin line).

line shape and depth which can occur as the source function varies owing to changes in λ and λ^*. In the example the level populations were calculated according to the Saha and Boltzmann laws. An optical depth of 1.0 in the center of the line has been reached when τ, the characteristic optical depth in which the model is defined, is 0.01. The electron density at the level $\tau = 0.01$ is 5.0×10^{13}.

V. INTERPRETATION PROBLEMS IN STELLAR SPECTRA

Main-sequence Stars

Model atmospheres in radiative and hydrostatic equilibrium can be constructed that give a continuous spectrum very like what is observed for B and early A type main sequence stars. The most satisfactory models include absorption in the strongest lines as a source of opacity as well as absorption in the continua of H^-, H, He and He^+ and electron scattering. What may be termed the classical method for computing model atmospheres adopts the hypothesis of LTE in order to simplify predicting the spectrum. More

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elegant methods of finding the continuous spectrum that allows for non-LTE have been developed by Kalkofen, Mihalas and Strom and by Feautrier[14] but so far as B stars are concerned, the difference of the predicted continuous spectrum in the normally observed spectral range from that of a classical main sequence model is too little to be detected using the presently available observational material (cf. Mihalas[15]). The classical models have been used, with an LTE theory of line formation, to predict the equivalent widths and profiles of absorption lines in the spectrum and to find abundances of the elements. The method is to choose those abundances that give a best overall fit between the predicted equivalent widths and the observed values. The technique is familiar and no detailed comments on this procedure are necessary here.

This type of analysis is a somewhat more sophisticated way to find abundances than simple curve-of-growth analysis in that account is taken of the variation of pressure and temperature with depth in the atmosphere. However, because the hypothesis of LTE is used, the chief free parameter that remains by which to obtain a fit between observed and predicted equivalent widths is the abundance. No attempt is made to compare computed and observed profiles. A further fitting parameter that is used with lines on the flat part of the curve of growth is microturbulence.

Sets of abundances in a number of B and early A type stars have been found by model-atmosphere and curve-of-growth studies. The observed fact that about ten percent of the main sequence stars between types A5 and B3 have absorption line spectra that differ from each other and from the spectra of most stars having the same distribution of energy in the continuous spectrum over the range 3500 to 6700Å leads to differing sets of abundances. Ingenious attempts have been made to explain these anomalous abundances in terms of nucleogenesis in the center of the star and/or spallation on the surface of the star. However, the whole chain of reasoning is not very satisfactory and it is time to consider seriously whether we do not have anomalous atmospheres rather than anomalous abundances; an idea put forward by the author in 1964 (Underhill[16]). By anomalous atmospheres it is meant that the process of line formation, in the case of the lines that lead to deviating abundances, is not an LTE process as has been assumed. That classical model atmospheres and the LTE theory of line formation are not truly

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satisfactory for explaining many details in the observed spectra of normal sharp-lined main sequence stars can be demonstrated by comparing observed and predicted line profiles (Underhill[17,18]). It is easy to show that the radiation in the profiles of all strong and moderately strong lines in O and B type spectra comes from such high levels in the atmosphere that the density is too low to ensure that LTE be established as a result of collisions for the lines that are studied.

It is chastening to note that in the peculiar main sequence stars the so-called anomalous abundances have usually been estimated from lines of the types listed in Section 3 as being particularly sensitive to departures from LTE. The H, He I, O I, and Si III spectra as well as the second spectra of the metals and of the rare earths contain metastable levels. The occurrence of these levels affects the populations of all the lower levels of the relevant atom or ion while closely coupled to the metastable levels. The abundances of Li, Be, Ca, Sr, and Ba are usually estimated from resonance lines. The ionization balance of P II $\rightleftarrows$ P III is particularly sensitive to collisions with He I atoms in the metastable 2^3S state (Underhill[19]), while Ba II can be ionized efficiently by Lα quanta. Furthermore, LTE theories of line formation and the use of the best available gf values for Si II lines do not give a satisfactory understanding of the strengths of the Si II lines in what are usually called normal stars (Underhill[20]). The reason for the observed discrepancies in the Si II spectrum is not understood at present. Clearly many problems remain in the interpretation of the lines from apparently sharp-lined main sequence stars. Any abundance anomalies of less than a factor 100 should be scrutinized carefully to see if they do not reflect departures from LTE rather than true abundance anomalies.

Mention has been made of normal stars. These are stars with spectra like the spectra of most of the spectral classification standard stars. It is going too far to assume without proof that normal also means that the stronger lines, that is the classification lines, can be interpreted correctly by means of the hypothesis of LTE.

The ad hoc fitting parameter microturbulence may well be an indication that in the case of the stronger lines $\lambda < 1$ and consequently a scattering term must be added to the source function. Such a term gives stronger lines for a given number of atoms than does a purely LTE source function. Con-

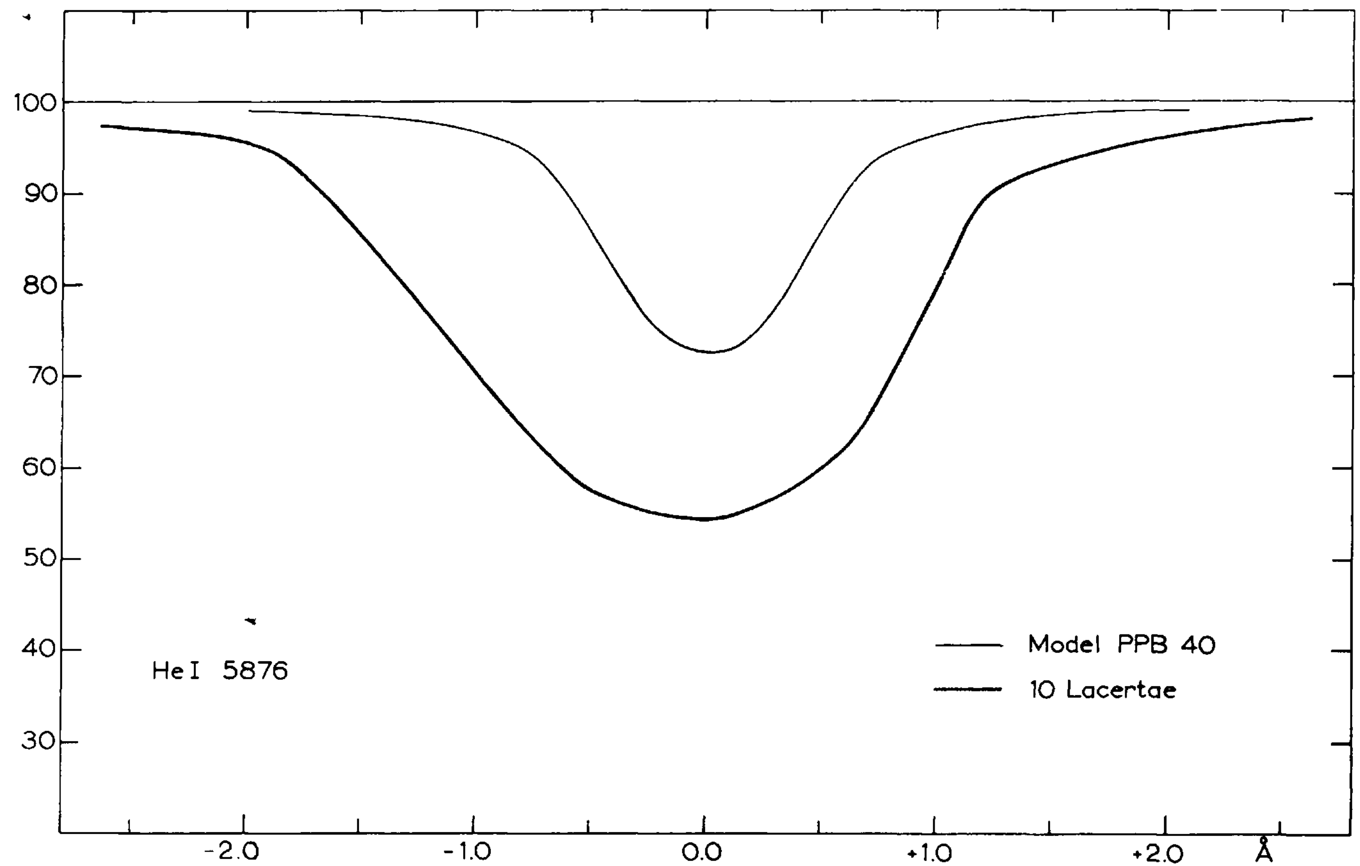

Figure 2. Predicted profile of He I 5876 in a line-blanketed O type model compared with the observed profile for 10 Lacertae, O9V.

cern regarding the interpretation of microturbulence has been expressed by Menzel[21] and by Jefferies and Thomas.[22]

An extreme example of what sort of discrepancies occur between observed profiles and profiles predicted assuming LTE is shown in Figure 2 where the predicted profile for He I 5876 obtained with a line-blanketed model which should represent 10 Lacertae, O9V, well (Underhill[17]) is compared with the observed profile. This observed profile is from only one IIIM spectrogram, 30 Å/mm at 5876Å, obtained at the Dominion Astrophysical Observatory, thus it is somewhat uncertain, but the uncertainties are not greater than ± 5 percent, particularly in the center of the line. Observational error is not the source of the discrepancy. The predicted profile has been found assuming that the line profile is given by the Voigt function with thermal Doppler broadening and a damping constant 100 times the classical damping constant. Such a value for the damping constant is close to what a more exact representation of the Stark effect of the $2^3P^0 - 3^3D$ line of He I will give. The predicted equivalent width is 0.344Å which is comparable with the value predicted by Mihalas[23] using the correct Stark broadening theory and a model of similar pressure-temperature structure. The equivalent width of the observed profile in Figure 2 is 1.14Å; Traving[24] has found 1.23Å and Mihalas[25] gives 0.90Å. No reasonable changes in the model nor adjustments of the line absorption coefficient will compensate for this discrepancy. It is necessary to use a theory of line formation that takes into account the fact that $\lambda \neq 1$ for He I 5876 in the outer layers of an O type star. The electron density is less than 1.7×10^{14} in the layers of model PPB40 which are important for forming the parts of the profile between ± 1.0Å.

Another indication that changes of the density in the atmosphere (thus changes of the significance of non-LTE physics for the strength of the He I absorption lines in B type main sequence stars) rather than gross changes in the helium abundance may be the explanation of "helium-weak" and "helium-strong" B stars of the same colour is afforded by the spectroscopic changes of HD 125823. Bidelman[26] noted that when the He I lines changed strength the rest of the spectrum remained essentially unchanged. This was confirmed by Thackeray.[27] The changes are more precisely described by Jaschek, Jaschek, Morgan, and Slettebak[28] who find that the variations in the spectrum of HD 125823 can be represented as an

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oscillation between MK types B2V and B7IV. On high dispersion spectrograms the Si II and Si III lines are sensibly unchanged in intensity throughout the variations.

Another observational confirmation of the different sensitivity of lines seen in late B type spectra to non-LTE physics is the variation observed by Struve and Swings[29] and by Struve[30] of the strengths of the Ti II, Mn II, Fe II and Ni II lines in the spectrum of Pleione. Most of these lines are used with unjustified confidence in LTE theories of line formation to deduce anomalous abundances in main sequence stars. The spectra of shell stars show to an enhanced degree the line intensity changes that have been mentioned above as indicating that non-LTE physics should be used. In addition the geometrical arrangement of the extended atmosphere affects the profiles that are observed.

Supergiants

The chief differences between the spectra of supergiants and those of main sequence stars having about the same intensity distribution in the continuous spectrum are that in supergiants (1) the Stark broadening of the H and He lines is greatly reduced, (2) all other lines are stronger and wider, and (3) multiplets generally show a greater gradient. Since the electron pressure is lower in the atmospheres of supergiants than in the atmospheres of main sequence stars, there is even less reason to believe that the hypothesis of LTE will be adequate for a theory of line formation.

Similar discrepancies between LTE theory and observation occur as have been noted for the main sequence stars. Simple LTE analysis of the equivalent widths of lines in the spectra of B type supergiants (for example Unsöld,[31] Voigt[32]) has always lead to a somewhat larger abundance of helium relative to hydrogen than is found for the main sequence B stars. This result is at least partly due to the fact that the observable lines of the He I spectrum are not in LTE in supergiant atmospheres and departures from LTE tend to make the He I lines deeper.

The hydrogen lines in B type supergiants do not look like the lines predicted using LTE theory and classical model atmospheres. Some example of the discrepancies that occur are shown in Figure 3 which displays profiles taken from the observations of van Helden,[33] Lamers,[34] and Smit[35] of Hγ in β

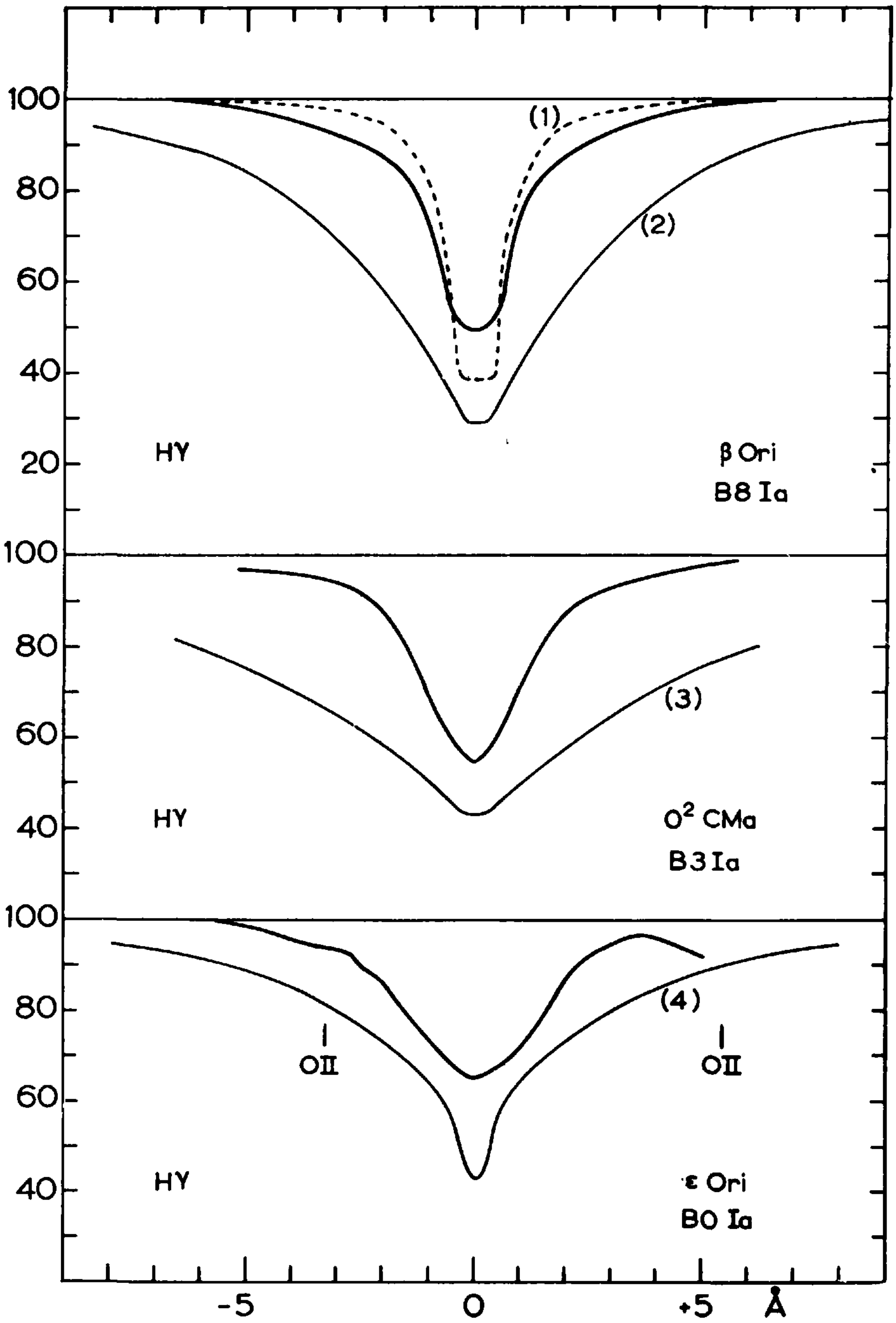

Figure 3. Observed Hλ profiles in B type supergiants compared with Hλ profiles predicted by Mihalas. Profile (1) is from a model with T_{eff} = 10,080°, log g = 1.0; profile (2) from a model with T_{eff} = 10,080°, log g = 2.0; profile (3) from a model with T_{eff} = 14,000°, log g = 3.5; profile (4) from a model with T_{eff} = 24,000°, log g = 3.5. The wavelength unit indicated is one angstrom.

Orionis, B8Ia, o^2 Canis Majoris, B3Ia, and ε Orionis, B0Ia, compared with profiles computed by Mihalas.[23] Mihalas used the best available LTE theory and a normal hydrogen-helium composition. The theoretical profiles are from unblanketed models with the following characteristics:

Profile	T_{eff} (°K)	log g
1	10 080	1.0
2	10 080	2.0
3	14 000	3.5
4	24 000	3.5

These models have the lowest values of log g of each group computed at each effective temperature. It is generally considered that log g in B type supergiant atmospheres is of the order of 2.0 although at effective temperatures near 24,000°K the condition of hydrostatic equilibrium cannot be met for log g smaller than about 3.0. If the effective temperatures of the selected supergiants are higher than the values listed above, say by 2000°, the predicted profiles for the same value of log g will be a few percent (< 5) less deep.

The observed profiles of Hγ do not have wings like the predicted Stark-broadened wings. From consideration of the fit in the wings alone one would postulate that the electron densities in the atmosphere are like those given by a model with log g of the order of 1.5 or less. However, the Balmer series of hydrogen breaks off at n = 25 in β Orionis, at n = 24 o^2 Canis Majoris and at n = 21 in ε Orionis. These values indicate electron densities of the order of 5.8×10^{12} to 2.3×10^{13}. The Mihalas model with T_{eff} = 10,080°K and log g = 2.0 has electron densities of this order in its outer layers while the model with log g = 1.0 has electron densities between 10^{10} and 10^{12}. Consequently one concludes that log g in the atmospheres of the B type supergiants of luminosity class Ia is probably not smaller than 2.0. The apparent fit of computed profile (1) with the observed profile for β Orionis must be fortuitous.

It is clear that the observed Hγ profiles are

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less deep than the predicted LTE profiles over their whole range. (An extrapolation to lower log g must be made in the case of the two high temperature models.). The computed equivalent widths are too large. Another indication that the profiles do not have the Stark wings which are expected is that the break off of the Balmer series is sharp in each of these supergiants. The wings of the last members of the Balmer series do not merge noticeably. Struve and Chun[36] have noticed that the He I lines in the spectrum of 55 Cygni, B3Ia, give evidence of Stark broadening indicating that the electron density is not particularly low in the atmosphere of this supergiant.

The discrepancies in the centers of the hydrogen lines are as interesting as the lack of Stark wings. In each case the line is less deep by 10 to 20 percent of the continuum than what is predicted. In the center of a very strong line the central intensity is proportional to the ratio of the source function at the edge of the atmosphere to the source function at the depth where the continuous spectrum is formed. The predicted profiles displayed in Figure 3 have been computed using LTE theory, thus the computed residual intensity at the center of Hγ represents approximately the ratio of the Planck function at the edge of the atmosphere to the Planck function at the depth where the continuum is formed. The predicted central depths are a representation of the adopted temperature law in the atmosphere. The work of Kalkofen, of Strom and Mihalas using main sequence models indicates that the hydrogen atom will not behave in the atmospheres of supergiants as if it were in LTE, for the density is too low to maintain Saha-Boltzmann populations in all levels. Consequently the source function in the center of Hγ at the outer edge of the atmopshere may be reduced significantly from its LTE value. In any case, the LTE value is the maximum value which can be attained. Let us postulate that the source function in the continuous spectrum, which in an atmosphere of normal composition is dominantly due to absorption in the Paschen continuum of hydrogen, is not significantly changed from its LTE value when more correct non-LTE calculations are made. It follows that acknowledging that LTE is not valid in supergiant atmospheres leads us to expect even deeper Hγ profiles than have been predicted, provided that we do not change the temperature structure of the atmosphere greatly. Thus allowing for departures from LTE will not resolve the discrepancy found for Hγ.

Motion of the atmosphere such as rotation of the star or large scale random motions in the atmosphere will not resolve the problem either, because although one may obtain a shallower profile in this way, the equivalent width is not reduced. The observed discrepancy is not only one of shape of profile, but also one of equivalent width.

Another possible solution is to introduce a rising temperatute in the outermost parts of the model. Such an _ad hoc_ procedure would require rejection of the condition that the transfer of energy through the atmosphere is regulated only by the condition that radiative equilibrium exist. The fact that emission is observed at Hα in the Ia supergiants is, perhaps, an encouragement for proceeding in this manner, but care would be required not to destroy the rather satisfactory interpretation of the colours of B type supergiants provided by models with a temperature that decreases outwards. Furthermore it should be noted that lines of Na I and Fe II are visible in the spectrum of β Orionis while the spectra of o^2 Canis Majoris and ε Orionis do not contain lines suggesting significantly higher levels of excitation than are seen in B3 and B0 main sequence stars respectively. Another objection against adding to the model a high temperature outer layer in which the core of Hγ is formed is that at a short distance from the line center, say 5Å, one would be outside the Hγ line absorption coefficient of this layer and thus would be able to see through to the cooler "normal" atmosphere. According to the LTE predictions one should then see the extended Stark-broadened wings of the Hγ profile formed in the normal reversing layer. Such wings are not seen. The _ad hoc_ solution of adding a hot layer to the model seems unsatisfactory, but it must be admitted that no details have been worked out.

A more promising solution to the problem of the weak hydrogen lines in B type supergiants of luminosity class Ia is to postulate that these atmospheres are hydrogen-poor (Underhill[37]). If the abundance of hydrogen is so small that hydrogen is no longer the chief source of opacity in the region 3650 - 7000Å, then the strengths of the Balmer lines should reflect the hydrogen abundance, _cf_. the calculations of Böhm-Vitense.[38] Strong Stark-broadened wings will not be seen because the low abundance of hydrogen prevents the wings from attaining observable depths at distances from the line center great enough that Stark effect dominates the shape of the line absorption coefficient. The rather steep-sided deep cores may still be due to the effect of departures from LTE.

If the atmospheres of the B type supergiants are hydrogen-poor, but perhaps not quite so poor as the helium stars like HD 124448, the chief source of opacity in the region 3650-7000Å will be absorption from the n = 3 levels of He I. These continuous absorption coefficients vary as ν^{-3}, thus no significant difference in UBV colour is expected between model atmospheres of normal, hydrogen-rich composition and those of hydrogen-poor composition. The temperature-pressure structure may well be different since the opacity in the region 504 - 912Å will differ greatly between hydrogen-rich and hydrogen-poor atmospheres.

The model-atmosphere calculations and line-profile predictions for hydrogen-poor stars with effective temperatures of 12,900°, 9,900° and 7350°K and log g = 2.0 and 4.5 presented by Böhm-Vitense[38] point in the direction of the conclusions sketched above. The value of the suggestion that the Ia supergiants of type B are hydrogen-poor cannot be fully assessed until more models such as those of Böhm-Vitense are available at a greater range of temperature and gravity and until an improved theory for predicting line profiles which takes into account the major effects of departures from LTE is developed. Some models for hydrogen-poor stars in the needed range of T_{eff} and log g have been computed by Klinglesmith[39] but they are not generally available.

The results of Böhm-Vitense indicate that hydrogen profiles of the observed shape can be obtained by reducing the hydrogen abundance in the atmospheres of the Ia supergiants of type B by a factor of at least 1000. If the atmosphere is hydrogen-poor, presumably the whole star is hydrogen-poor. To reach such a state would require mixing the outer parts of the star with the core in which hydrogen had been depleted as the result of nuclear reactions. The hydrogen lines in the spectra of the Ib supergiants of type B are stronger than those of the Ia supergiants. It may be that the Ib supergiants are hydrogen-rich stars at the beginning of their departure from the main sequence while the Ia supergiants are much further evolved.

VI. THE CHOICE OF SIMPLIFIED PHYSICAL REPRESENTATIONS OF LINE FORMING IN NON-LTE

The full problem of line formation in a stellar atmosphere when the hypothesis of LTE cannot be made is too complex to be solved in the most general manner, for it involves the simultaneous solution of coupled transfer equations in all frequencies of interest and the equations of statistical equilibrium. Furthermore the atmosphere must be taken to consist of at least three species of atom: hydrogen, helium and the atom under consideration in several stages of ionization. This last requirement occurs because in some spectra the observed line intensities are affected by energy coincidences between the observed energy levels and radiations such as Lyman α of hydrogen or of He^{+} or because collisions with abundant excited atoms such as metastable helium atoms can cause particularly strong ionization or excitation of certain species. Furthermore hydrogen and helium serve as the chief sources of continuous opacity.

A first simplification may be obtained by postulating that all the levels down to some particular energy below the primary ionization limit have LTE populations with respect to the population of the ion. This step is justified because the higher levels are relatively close together in energy and if the electron density is not very low, Boltzmann populations will be set up as a result of collisions. The chief problem is to decide how far down LTE populations extend. The populations of all levels below the selected level must be calculated using the equations of statistical equilibrium. The number of terms in these equations can be reduced by a judicious selection of the significant processes causing transitions between the various energy states of the atom or ion.

If the spectral lines under study occur between levels that have excitation energies so high that at the local electron densities LTE populations prevail, then adopting the LTE theory of line formation is probably a fairly good approximation. However, most of the significant lines in early type spectra do not fall into this class and the energy-level diagram of the atom or ion must be studied together with estimates of the radiative and collision transition probabilities to see what processes are important in establishing the level populations and the effective value of λ.

Some examples of the distribution of levels in essentially one-electron spectra are shown in Figure 4. The ordinate is excitation potential/ionization potential; the ionization potential of each atom or ion is given. Metastable levels are indicated by an M. More levels occur above the levels which have been drawn. The transitions that give the characteristic absorption lines observed in stellar spectra are indicated. Two types of transitions are observed: (1) resonance lines (Li, Ca^+, Ba^+), and (2) subordinate lines from the second, third, or fourth lowest level (H, C^{+3}, N^{+4}, Si^{+3}, Mg^+).

Clearly resonance lines are particularly sensitive to non-LTE physics since they are formed between levels that are well isolated from all other levels. In the case of Ca^+ and Ba^+ metastable levels occur between the levels of the resonance lines, and the whole set of low lying levels makes an isolated group.

In the case of the subordinate lines, in each case the lower level is sufficiently isolated that non-LTE physics is important in determining its population. Whether the population of the upper level is seriously affected by departures from LTE or not is something that requires detailed investigation in each case. The transitions indicated in Figure 4 are Hα, Li I 6708, C IV 5801, 12, N V 4603, 20, Si IV 4088, 4116, Mg II 4481, Ca II 3933, 68 and Ba II 4554, 4934.

In the case of essentially two-electron spectra somewhat similar patterns emerge. Some partial energy-level diagrams are shown in Figure 5. More levels occur above the levels that are shown. In the case of the light elements the intersystem transitions are rather weak and the sets of energy levels of different multiplicity are pretty well independent of each other. In the heavier elements, for instance Ca^+, the intersystem radiative transitions are fairly strong. In spectra from ions and atoms like those shown in Figure 5 the presence of metastable levels will play a considerable role in establishing the

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Figure 4 (pp. 30-31). Scaled diagrams of the lowest energy levels of some effectively one-electron spectra. The ordinate is excitation potential/ionization potential. The ionization potential is given above each column. The transition that gives the characteristic lines observed for each atom or ion in stellar spectra is indicated. Metastable levels are marked by an M.

→

Figure 5 (pp. 32-33). Scaled diagrams of the lowest energy levels of some effectively two-electron spectra. Ordinate and notation as in Figure 4.

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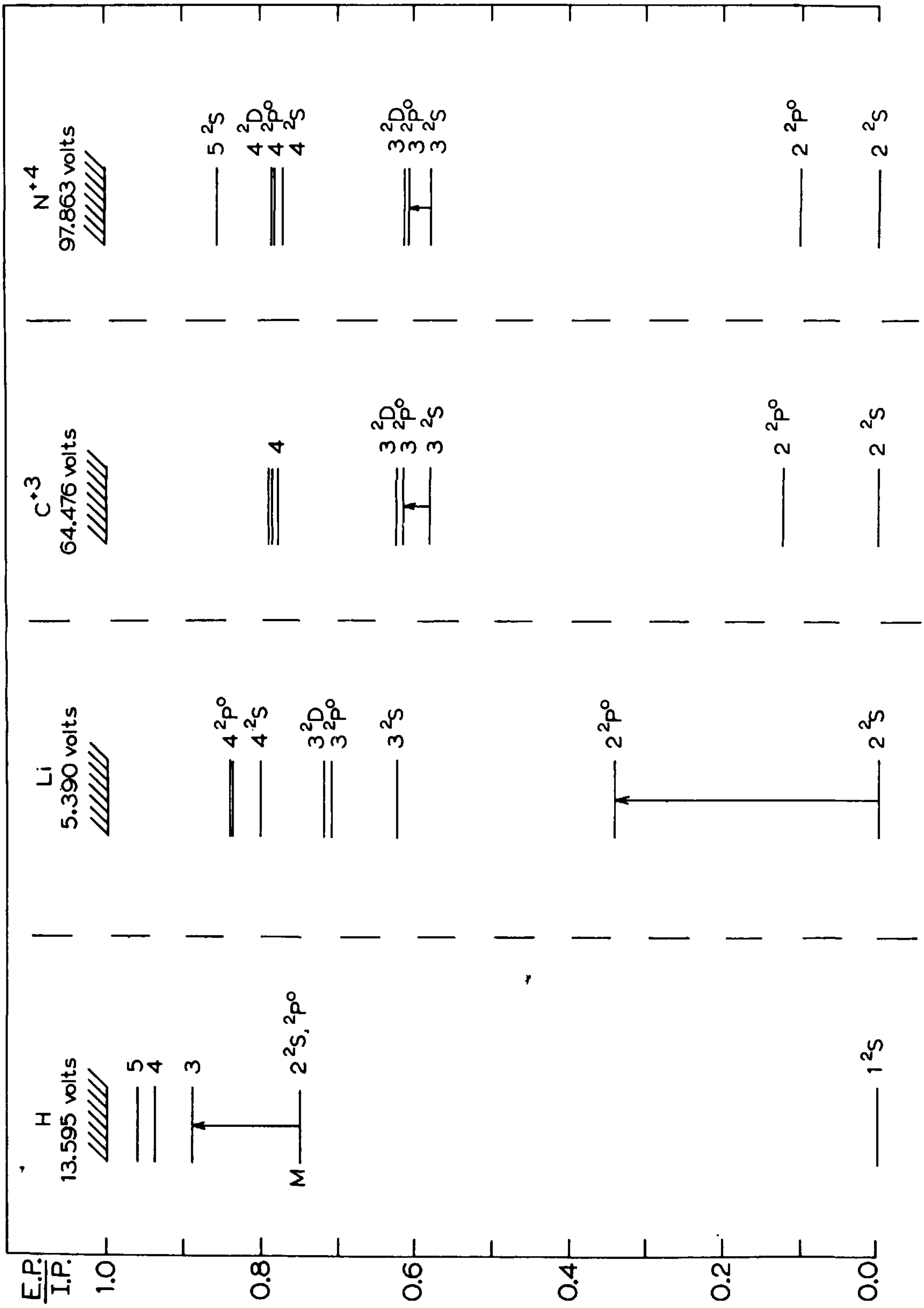
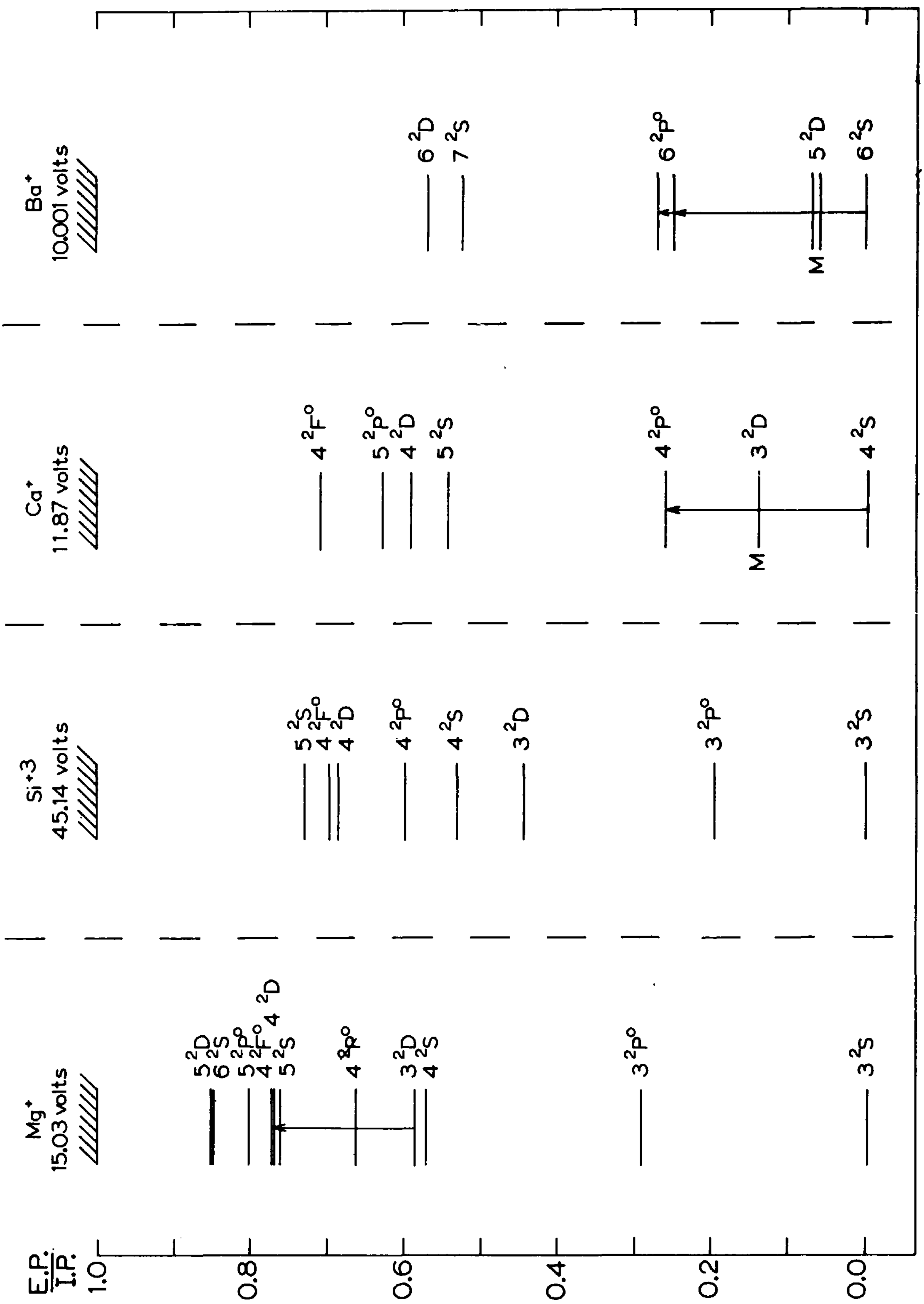
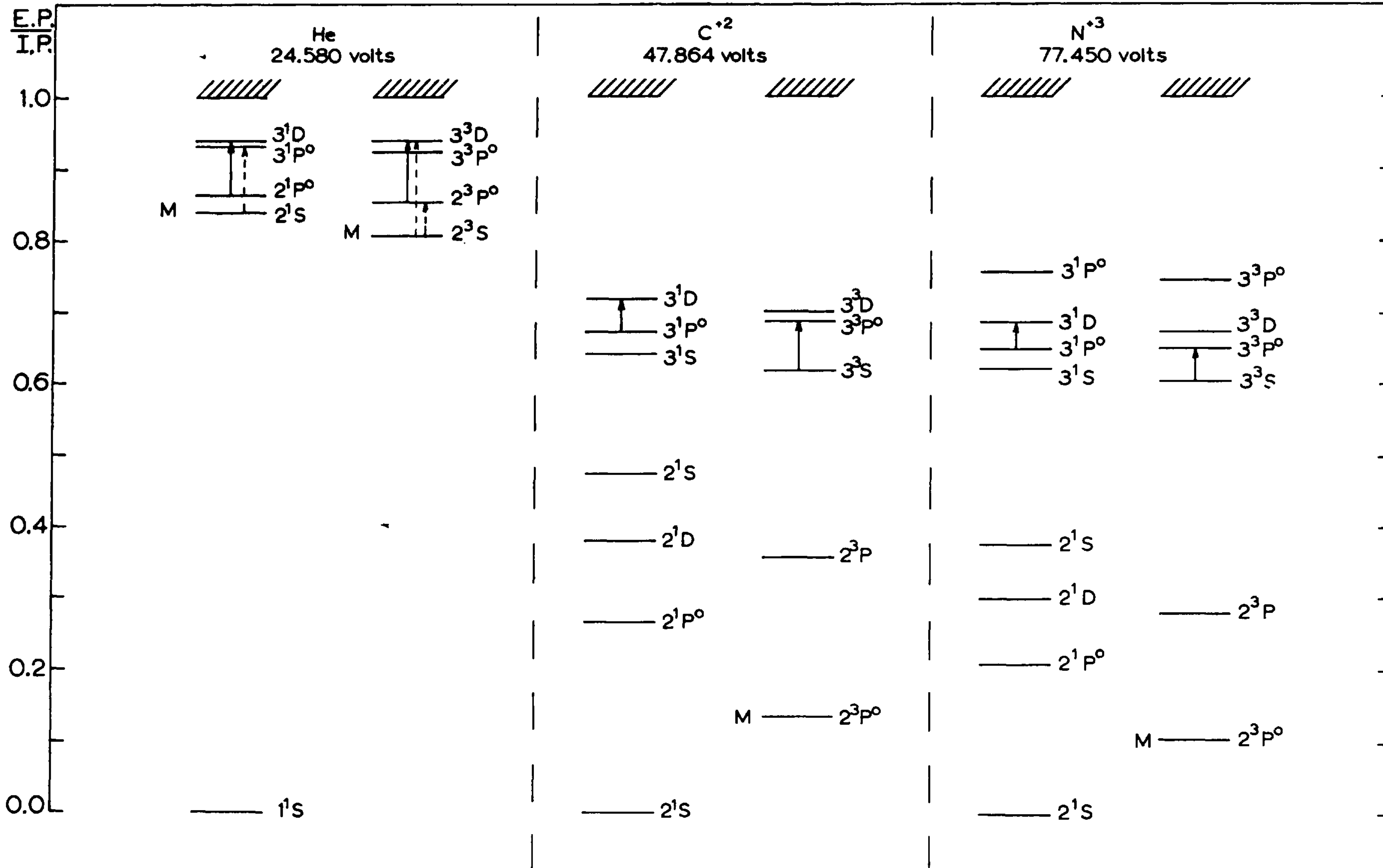


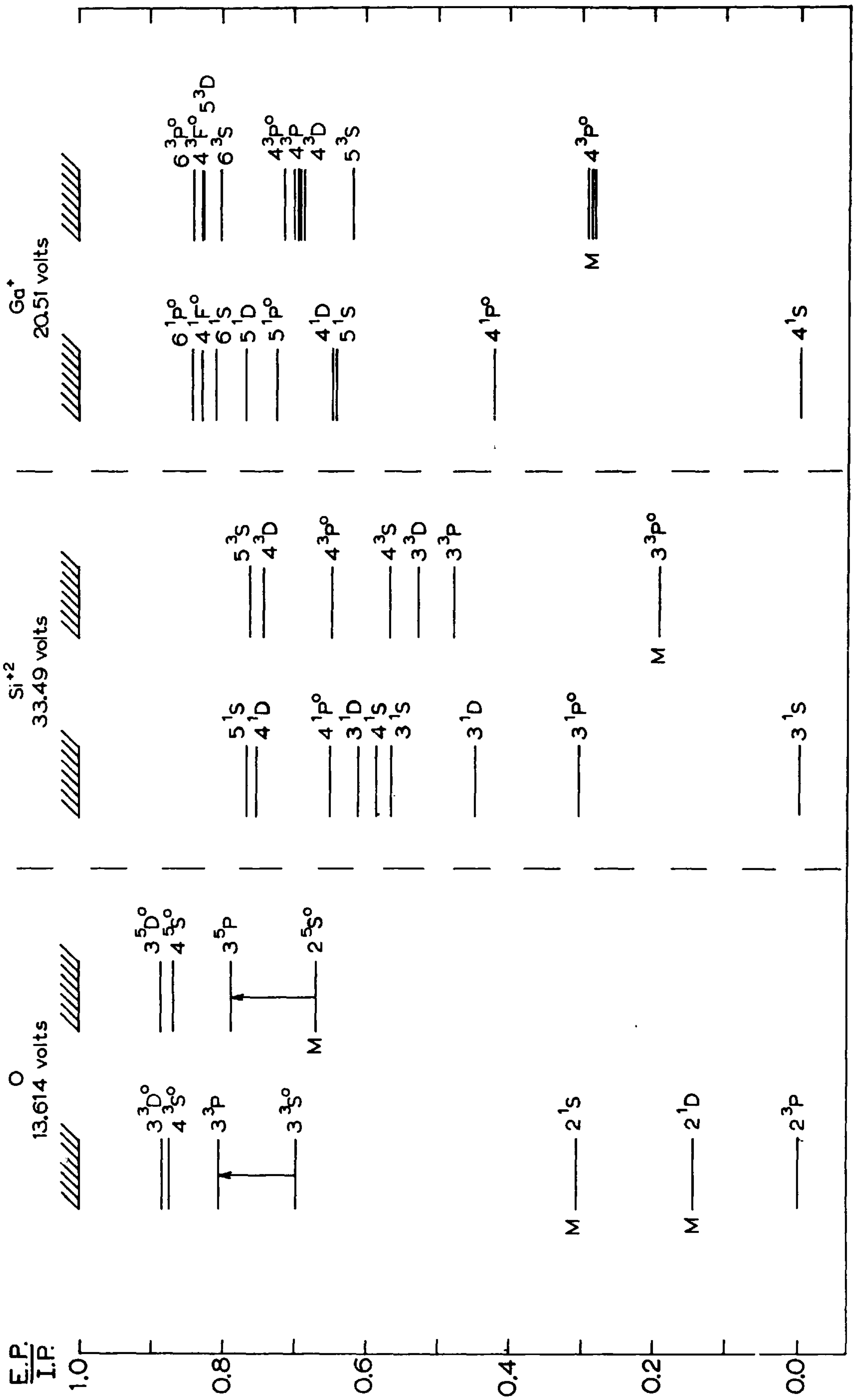
E.P./I.P.
1.0
0.8
0.6
0.4
0.2
0.0
H
13.595 volts
Li
5.390 volts
C+3
64.476 volts
N+4
97.863 volts



E.P./I.P.
1.0
0.8
0.6
0.4
0.2
0.0
Mg+ 15.03 volts
Si+3 45.14 volts
Ca+ 11.87 volts
Ba+ 10.001 volts
M



E.P./I.P.
He
24.580 volts
C+2
47.864 volts
N+3
77.450 volts
1.0
0.8
0.6
0.4
0.2
0.0
M
3¹D
3¹P°
2¹P°
2¹S
1¹S
3³D
3³P°
2³P°
2³S
3¹S
2¹D
2³P
3³S



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level populations. No resonance lines are observed from spectra of this sort. The chief subordinate transitions of He I are shown as well as C III 5696 (3^1P^0 - 3^1D) and 4647, 50, 51 (3^3S - 3^3P^0); N IV 4057 (3^1P^0 - 3^1D) and 3478, 82, 84 (3^3S - 3^3P^0); O I 8446 (3^3S^0 - 3^3P) and 7771, 4, 5 (2^5S^0 - 3^5P); Si III 4552, 68, 74 (4^3S - 4^3P^0) and Ga II 4251 etc. (4^3D - 4^3P^0). The chief absorption lines of Ga II which are observed (Bidelman and Corliss[40]) come from the 4^3D levels.

It is quite clear that the chief lines that are observed in the spectra of He and the ions shown in Figure 5 come from levels sufficiently isolated that one cannot assume that the level populations are in the Saha-Boltzmann ratios and that $\lambda \equiv 1$ for lines like those shown. Adoption of LTE methods of analysis to estimate the populations of the upper and lower levels of triplet lines (quintet in the case of O I) may be justified to some extent because it appears quite possible that the triplet levels are rather tightly bound to the continuum by collisions. The difficulty arises in relating these populations to the total abundance of the element and the populations of the singlet levels (singlet and triplet levels for O I). Because of the rather large energy gaps in the lower part of the energy level diagram, the populations of the lower singlet levels are most likely not in LTE at electron densities of the order of 10^{13} - 10^{14} which are the electron densities found in the atmospheres of main sequence stars.

The 3^1P^0 - 3^1D transitions of C III and N IV appear selectively in emission in Of stars and have flat-topped profiles in Wolf-Rayet stars. The 3^1D level is believed to be selectively populated by a radiative process in the case of C III and by a collisional process in the case of N IV.

No procedure can be given here for how to solve for the desired level populations and line intensities. However the data displayed in Figures 4 and 5 should make it clear that the full non-LTE problem must be solved and that it is not necessary to consider anything like an infinite number of levels or of transitions between these levels.

One question that remains is how significant are transport problems for determining the radiation field that enters into the equations of statistical equilibrium which determine the level populations. One way of advancing with the problem might be to simplify considerably the transfer problem in most radiative transitions and just retain a full transport representation for the line in question and for one or two related lines. If there is a differential field of motion in the atmosphere such that over a geometric length corresponding to unit optical depth in the line the Doppler shift places the relevant line radiation outside the extent of the line absorption coefficient, the transfer problem can be simplified. Then the atmosphere, or at least well defined parts of it, can be treated as if optically thin. Such a manipulation is done, for instance, in order to estimate the line shapes to be expected from a spherically expanding atmosphere. However if the radiation field is completely separated from the populations of the energy levels, the strengths of the observed spectral lines can tell nothing about the physical processes going on or the abundances of the elements.

VII. SUMMARY

The interpretational problems posed by the spectra of main sequence stars, shell stars, and supergiants of types A5 and earlier have been reviewed. It is indicated that the hypothesis of LTE is untenable for the interpretation of most lines that are characteristic for the classification of these stars. A sketch has been given of how an improved theory might be developed. Particular problems that are encountered are listed in the abstract. A significant result of these yet qualitative considerations is the realisation that the atmospheres of the Ia supergiants of type B may be hydrogen-poor. In no other way can one obtain weak hydrogen lines such as are observed and still maintain electron densities of sufficient magnitude to cause the Balmer series to break off between n = 21 and n = 25.

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