GENERALIZED VERSUS CLASSICAL MAXIMUM ENTROPY FOR IMAGING

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Abstract. The problem of image reconstruction from incomplete and noisy complex Fourier spectrum is considered. The maximum entropy method (MEM) is of great interest as the most effective nonlinear reconstruction method having superresolution effect. Because objects of radio astronomical observations are incoherent radio sources described by real non-negative distributions, application of the classical MEM is quite reasonable. But it is established that the MEM gives acceptable reconstruction quality mostly in the case of point-like sources and in general it does not ensure satisfactory reconstruction of continous, graytone objects, which can considerably restrict applications of the MEM in astronomy. The generalized maximum entropy method (GMEM) was originally proposed for reconstruction of distributions described by complex functions (Bajkova, 1992) and was considered as having the same properties of the classical MEM. More careful analysis of the GMEM and classical MEM for real non-negative objects allowed to establish that the GMEM ensures much more exact reconstruction, especially in the case of continous objects. Explanation and demonstration of this interesting and very important phenomenon is the purpose of the present paper.

Key words: IMAGE PROCESSING – IMAGE RECONSTRUCTION – MAXIMUM ENTROPY

1. Classical maximum entropy

The classical MEM in Shannon formulation assumes solving the following optimization problem

$$\min \sum_m \sum_l x_{ml} \ln(x_{ml}), \tag{1}$$

$$\sum_m \sum_l x_{ml}\ a_{ml}^{nk} = A_{nk}, \tag{2}$$

$$\sum_m \sum_l x_{ml}\ b_{ml}^{nk} = B_{nk}, \tag{3}$$

$$x_{ml} \geq 0, \tag{4}$$

where a_{ml}^{nk} and b_{ml}^{nk} are constant; A_{nk} and B_{nk} are real and imaginary parts of measured spectrum samples respectively.

The solution which can be found using Lagrange technique is as follows

$$x_{ml} = \exp\ (-\sum_n \sum_k \alpha_{nk}\ a_{ml}^{nk} + \beta_{nk}\ b_{ml}^{nk} - 1), \tag{5}$$

where α_{nk} and β_{nk} are dual factors corresponding to equations (2) and (3) respectively.

Dual factors are to be found by solving the following optimization problem

$$\max \sum_m \sum_l x_{ml} + \sum_n \sum_k \alpha_{nk} A_{nk} + \beta_{nk} B_{nk} \qquad (6).$$

J.G. Robertson and W.J. Tango (eds.), Very High Angular Resolution Imaging, 215–217.

2. Generalized maximum entropy for complex functions

The generalized maximum entropy method assumes solution of the following optimization problem (only the case of complex distribution with non-negative real and imaginary parts is considered) (Bajkova, 1992)

$$\min \sum_m \sum_l x_{ml} \ln(x_{ml}) + z_{ml} \ln(z_{ml}), \tag{7}$$

$$\sum_m \sum_l (x_{ml}\ a_{ml}^{nk} - z_{ml}\ b_{ml}^{nk}) = A_{nk}, \tag{8}$$

$$\sum_m \sum_l (x_{ml}\ b_{ml}^{nk} + z_{ml}\ a_{ml}^{nk}) = B_{nk}, \tag{9}$$

$$x_{ml},\ z_{ml} \geq 0, \tag{10}$$

where x_{ml} is a real part, z_{ml} is an imaginary part of sought distribution.

The solution of the problem (7)-(10) looks as follows

$$x_{ml} = \exp(-\sum_n \sum_k (\alpha_{nk} a_{ml}^{nk} + \beta_{nk} b_{ml}^{nk}) - 1), \tag{11}$$

$$z_{ml} = \exp(\sum_n \sum_k (\alpha_{nk} b_{ml}^{nk} - \beta_{nk} a_{ml}^{nk}) - 1). \tag{12}$$

Dual factors α_{nk} and β_{nk} are to be found by solving the following problem

$$\max \sum_m \sum_l (x_{ml} + z_{ml}) + \sum_n \sum_k (\alpha_{nk} A_{nk} + \beta_{nk} B_{nk}). \tag{13}$$

3. Comparison of the GMEM and classical MEM solutions

Comparison of the classical MEM (1) - (4) and the GMEM (7) - (10) is a question of great interest for the problem of image reconstruction from incomplete and noisy complex spectrum data. Using the GMEM for reconstruction of real non-negative distributions we seek unknown sequences in more general complex form.

From expressions (6) and (13) it is seen that in the cases of the MEM and GMEM we have different dual optimization problems. Therefore it is expected that corresponding solutions for $x_{m,l}$ are also different. Difference between the MEM and GMEM solutions was successfully proved by simulation technique. One of the numerous results, namely, for an object with two continuous gaussian-like components is shown in Fig. 1. As is seen, the classical MEM sharpens the smooth component of the object. In dual Fourier domain this fact relates to over reconstruction of spectrum. As is seen in the pictures, the GMEM allows to reconstruct image much closer to the object than the classical MEM.

This phenomenon can be explained from the fact of appearance in the expression for minimized functional (13) of the additional term for $z_{m,l}$ where dual variables α_{nk} and β_{nk} enter differently as compared with x_{ml} (see (11), (12)). It allows to include additional information (about imaginary part of an object) for finding α_{nk}

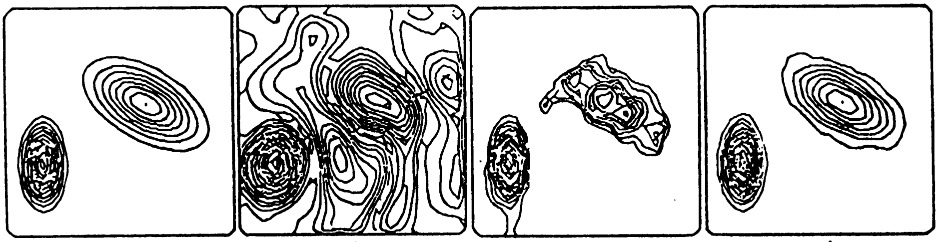

Fig. 1. Comparison of the GMEM and classical MEM solutions. (a) object; (b) "dirty" image; (c) reconstructed one by the classical MEM; (d) reconstructed one by the GMEM.

and β_{nk} into the process of solving optimization problem, which leads to more correct determination of the dual parameters. On the contrary, in the case of the classical MEM the information about the imaginary part of an object is not taken into account explicitly.

There is analogy with Fourier transform here. Indeed, expressions for $x_{m,l}$ and $z_{m,l}$ can be rewritten as:

$$Lx_{ml} = \ln(x_{ml}) + 1 = -\sum_n \sum_k (\alpha_{nk} a_{ml}^{nk} + \beta_{nk} b_{ml}^{nk}),$$

$$Lz_{ml} = \ln(z_{ml}) + 1 = \sum_n \sum_k (\alpha_{nk} b_{ml}^{nk} - \beta_{nk} a_{ml}^{nk}).$$

In the limiting case, when $m, l, n, k = 1, ..., N$, complex two-dimensional sequence (Lx_{ml}, Lz_{ml}) is coupled with complex two-dimensional sequence $(\alpha_{nk}, \beta_{nk})$ via Fourier transform and $(\alpha_{nk}, \beta_{nk})$ is determined uniquely by (Lx_{ml}, Ly_{ml}) and vice versa.

Thus, for image reconstruction from incomplete Fourier spectrum it is more correct to seek solution in complex form (using the GMEM), even in the case of a real object because Fourier transform is one-to-one transform of functions determined generally in complex domain.

4. Conclusions

The maximum entropy method generalized for reconstruction of complex distributions is compared with the well-known for astronomers classical maximum entropy method. It is shown that the generalized MEM having different solution as compared with the classical one, ensures much more exact reconstruction, so that its application to image reconstruction from incomplete and noisy spectrum data is more preferable even in the case of real distributions.

References

Bajkova, A.T.: 1992, 'The generalization of maximum entropy method for reconstruction of complex functions', *Astron. and Astrophys. Tr.* vol. **1**, p.313.