Objective Homogeneity Relativized[1]

Joseph F. Hanna

Michigan State University

1. Introduction

In his recent book Scientific Explanation and the Causal Structure of the World Wesley Salmon provides a detailed explication of *objective homogeneity*, a concept which is central to his Statistical-Relevance (S-R) model of explanation. One of the purposes of Salmon's explication is to refute Hempel's thesis of the *epistemic relativity of statistical explanation. According to this thesis "the concept of statistical explanation for particular events is essentially relative to a given knowledge situation"* (Hempel 1965, pp. 402-403, quoted in Salmon 1984, p. 48). Salmon introduces (1984, p. 55) the concept that forms the basis for his S-R model as follows: "A reference class A is homogeneous with respect to an attribute B provided there is no set of properties C_i ($1 \leq i \leq k$; $k \geq 2$) in terms of which A can be relevantly partitioned. By a partition of A we understand a set of mutually exclusive subclasses of A which, taken together, contain all members of A. A partition of A by means of C_i is relevant with respect to B if, for some values of i, $P(B|A.C_i) \neq P(B|A)$." In order to clarify the point of his criticism of Hempel's I-S model, Salmon distinguishes (p. 49) between the epistemic homogeneity and the objective homogeneity of a reference class. In his words, "...a reference class is *epistemically homogeneous* with respect to a given attribute--relative to a given knowledge situation--if no way is known to make a relevant partition of it....a reference class is *objectively homogeneous* with respect to a given attribute if there is in fact no way of effecting a relevant partition."

Salmon considers two grounds that might be invoked for denying (as Hempel evidently does) that we could ever be justified in asserting that a given reference class is objectively homogeneous:
(1) "It might be argued that the claim is always false, and that the only cases of objectively homogeneous reference classes are those that are trivially homogeneous because they occur in universal (nonstatistical) generalizations." (1984, pp. 50-51).
(2) "It might be argued that we can never be warranted in asserting the objective homogeneity of the reference class mentioned in any statistical generalization because no such assertion can coherently be made--the very concept of objective homogeneity is not meaningful." (1984, pp. 53-54).

PSA 1986, Volume 1, pp. 422-431

Noting that there are no a-priori arguments against indeterminism and pointing to the evident empirical success of contemporary quantum theory, with its implication that many phenomena are irreducibly stochastic, Salmon dismisses the first of these grounds for denying the possibility of non-trivial objective homogeneity. His primary aim in the text under consideration (1984, Chapter 3) is to refute the second ground by providing a coherent explication of objective homogeneity. While I believe that Salmon has made substantial progress in this endeavor, it seems to me that his account suffers from at least two flaws. The first difficulty is basically technical; the proposed definitions are unnecessarily complicated (and in some respects misleading) and they do not fully capture the underlying intuition which motivates them. Here, I hope only to suggest some minor improvements that will convey more adequately the important concept that Salmon's discussion is intended to explicate.

More significantly, I will argue that in an irreducibly stochastic world, objective homogeneity (while not incoherent) is not so straightforward a concept as Salmon's analysis would suggest. In fact, Salmon faces a dilemma in his attempt to ground his theory of the causal explanation of particular events on this notion. On the one hand, if the universe is deterministic then the concept is trivialized because the only objectively homogeneous reference classes will be those associated with universal generalizations. On the other hand, if the universe is indeterministic (irreducibly stochastic) then for practical purposes the concept of objective homogeneity must be temporally relativized, because the objective probability of particular events evolves over time. Moreover, both epistemic and pragmatic factors play a role in determining which of these objective probabilities are explanatorily relevant. And if one insists on the requirement that explanations be objectively homogeneous, *simpliciter* (i.e., not temporally relativized) then, practically speaking, the concept of objective homogeneity will again be trivialized. The relevant objective probabilities of events which are of practical interest to us will be arbitrarily close to one.

Section 2 is devoted to a reformulation and simplification of Salmon's treatment of objective homogeneity. Due to limitations of space I have not included his original definitions. The reader is urged to consult them.[2] In Section 3, I present a simple example to motivate the temporal relativization of objective homogeneity and then generalize Salmon's definitions to explicate the relativized notion. Salmon's concept of absolute objective homogeneity can then be introduced as a limiting case of the relativized concept.

2. A Simplified Definition of Objective Homogeneity

The intuition underlying Salmon's approach is that an objectively homogeneous reference class cannot, even in principle, be partitioned into subclasses that are statistically relevant to the occurrence of the attribute in question. However, Salmon faced two sorts of obstacles in his attempts to capture this intuition in a formal definition. The first difficulty is that from a strictly formal point of view, so long as the probability of B in reference class A differs from one (and zero), there will always exist a statistically relevant partition of A.

For, as Salmon points out (p. 56) if we let C_1=B and C_2=$\bar{B}$ then $P(B|A.C_1)$=1 and $P(B|A.C_2)$=0, so a relevant partition has been achieved. Restrictions are obviously needed on the manner in which the elements of a partition can be defined. Salmon (1971) discussed this difficulty in some depth in his early presentation of the S-R model. His solution was to adapt the concept of a *place selection*, which von Mises had introduced in connection with his relative frequency theory of probability, to the context of statistical explanation. A place selection is a procedure which determines whether a member of the main sequence (reference class A) belongs to the subsequence (subclass C_i) without reference to whether the element in question has or lacks the attribute B. Generally speaking, place selections can determine membership either on the basis of the ordinal position of the element in the main sequence (e.g., every third member) or on the basis of attributes (including attribute B) of members of the main sequence that precede the element in question.

Salmon notes that in order to do its job, von Mises notion of place selection must be strengthened by imposing restrictions on the type of procedure that can be used to select a subsequence. Otherwise the concept of homogeneity will again be trivialized. For example, define a real number r ($0 \leq r \leq 1$) as follows: the ith place in the binary expansion or r is 1 if the ith element of A has attribute B, otherwise the ith place is 0. Such a real number will always exist and if it were allowed to determine a place selection the notion of objective homogeneity would be trivialized. The solution is to impose, following Church's method of defining randomness, a condition of "effective calculability" on the procedure determining the place selection. Thus a subclass C_i will only be considered an element of a relevant partition of A provided that there is an effective procedure for determining membership in C_i. This would rule out the subclass C_i associated with the non-computable real number r mentioned above.

The above restriction solves the problem associated with place selections based upon internal properties of the reference sequence A. However, Salmon notes that there is a more serious difficulty associated with external or empirical properties of the sequence. He illustrates this problem (pp. 60-61) with an example of a "dishonest" roulette wheel rigged to a penny tossing device. A penny is tossed randomly and if it results in heads the roulette wheel is made to stop on red, while if it results in tails the roulette wheel is made to stop on black. Assuming that the penny produces a random sequence of heads and tails, the roulette wheel will produce a random sequence of red and black outcomes. Thus, no internal place selection will characterize a subclass of spins of the roulette wheel whose probability of a red outcome is different from the probability of red outcomes in the original reference class A of spins of the wheel. At the same time, the class A is evidently not homogeneous, since knowledge of the prior outcome of the toss of the penny would yield a perfect gambling system (an effective method for predicting the outcome of each spin of the wheel).

In order to overcome the difficulties posed by the possibility of such "external place selections", Salmon generalizes von Mises notion to include *place selections by associated sequences*, where the notion of an associated sequence is defined as follows (p. 61):

https://doi.org/10.1086/psaprocbienmeetp.1986.1.193142 Published online by Cambridge University Press

Definition 1. Let A be a reference class consisting of a sequence of events x_1, x_2,... Any other infinite sequence D consisting of events y_1, y_2,... will be called an *associated sequence* if each event y_i occurs in the absolute past (past light cone) of the corresponding event x_i. It is evident that in the above example the sequence D of tosses of the penny is an associated sequence for the sequence A of spins of the roulette wheel. A *selection by an associated sequence* will determine a subsequence S of reference class A by means of an associated sequence D by stipulating that $x_i \epsilon S$ iff $y_i \epsilon C$, where C is an appropriately determined subclass of the associated sequence D. In the above example, C might be the subsequence of the sequence D of tosses of the penny that result in heads. It is again evident that some restrictions must be placed upon the definition of C if the concept of objective homogeneity is not to be trivialized. For, since the mapping $y_i \leftrightarrow x_i$ is a one-to-one correspondence between D and A, the sequence $C=\{y_i | x_i \epsilon B\}$ would produce a relevant partition of A involving a selection by an associated sequence.

Salmon has a lengthy discussion, in which he develops a number of specific examples, that demonstrates the need for restrictions upon the definition of the subclass C of the associated sequence D. In view of the limited space, I will not comment directly upon that discussion or introduce all of the concepts and definitions to which it leads. Rather, I will present a simplified version of Salmon's account and then indicate briefly why I believe it captures the plausible intuitions which he was explicating. The fundamental intuition is that a reference class A which determines a specific probability, say p, for an attribute B is not objectively homogeneous if there exists a gambling system which would (even in principle, and not merely in actual practice) enable one to predict the presence or absence of attribute B with a limiting frequency of success greater than p. The sequence of coin tosses provides the basis for an effective gambling system, because one can place bets on the spin of the roulette wheel on the basis of the earlier outcome of the coin toss.

This intuition is captured, I think, by the following sequence of definitions, of which the first is a simplified version of Salmon's Definitions 2 and 3 (pp. 68-69) and the latter two are identical, respectively, with his Definitions 4 and 5 (pp. 69-70):
Definition 2. Let $D=\{y_1,y_2,...\}$ be an associated sequence of a reference class $A = \{x_1,x_2,...\}$. The class C qualifies as an admissible selective class of the associated sequence D if and only if for any y_i in D, the membership of y_i in C or $\overline{C}$ could, in principle, be ascertained by a computer that receives information from a physical detector responding to y_i, but that receives no information gathered by the detector (or from any other source) in response to x_i or to any events z_i in the absolute future of x_i.
Definition 3. Let the ordered class $y_1,y_2,...$ constitute an associated sequence D with respect to the reference class A. Then a *selection by an associated sequence* S is any selection within A defined by the rule $x_i \epsilon S$ iff $y_i \epsilon C$, where C is an admissible selective class.
Definition 4. A reference class A is *objectively homogeneous* with respect to an attribute B iff the probability of B within A is invariant under all selections by associated sequences.

Definition 2 simplifies Salmon's treatment in two respects: (i) it avoids altogether the notion of an *objectively codefined subclass* (p. 68), and (ii) it avoids both condition 2) of Salmon's Definition 2 and condition 2) of his Definition 3. Salmon mentions that the two conditions of his Definition 2 are likely redundant but includes both "because each of them formulates an important intuition, and in dealing with an issue as subtle as objective homogeneity, a bit of overkill may not be a bad thing." (p. 68). However, in my view there is only a single intuition involved here. Salmon's condition 2 (which I have omitted from Definition 2 above) reads: there is no event $y_i \epsilon D$ whose membership in C or $\overline{C}$ would be altered if x_i or any event z_i in the absolute future of x_i were not to occur. But surely the temporal relation that this condition is intended to preclude is in direct conflict with the stipulation in condition 1) that a computer be able to "effectively decide" membership in C without information from future events. The notion of information invoked in Definition 2 is admittedly vague, but I think it is sufficiently clear to rule out the sorts of examples that motivate Salmon to introduce condition 2.

The most serious technical defect in Salmon's treatment is his inclusion of condition 2) in definition 3. It was this condition that necessitated his introduction (pp. 68-69) of the distinction between an *objectively codefined subclass* and an *admissible selective class.* Condition 2 of his Definition 3 (which I have also omitted in the definitions presented above) states that in order for C to qualify as an admissible selective class it must occur within D in a mathematically random fashion. But that surely is not required. Suppose A were an infinite sequence of coin tosses in which heads was always followed by tails and vice verse. Then the class D of immediately preceding tosses (i.e., $y_i = x_i$-1) is an associated sequence and the set C of those y_i which result in the outcome tails is certainly (in an intuitive sense) an admissible selective class, yet C does not occur within D in a mathematically random fashion. Salmon's Example 4, which motivates this second condition, discloses the source of the confusion I think. He imagines "a superstitious crapshooter who invariably says just before rolling the dice, 'Gimme a------,' where the blank is filled by "seven" or the name of whatever he expects to get" (p. 68). The sequence D of utterances of the words 'Gimme a------.' qualifies as an associated sequence for the sequence A of rolls of the dice. The subsequence C of D is defined as follows: let r be a real number whose ith binary digit is 1 if the ith roll of the dice results in 7; otherwise the ith binary digit is 0. Then $y_i \epsilon C$ iff the ith binary digit of r is 1. Clearly such a real number exists, and if it is allowed to define an admissible selective sequence, the notion of objective homogeneity will be trivialized as before. Salmon introduces condition 2 of his Definition 3 in order to eliminate such counter examples. However, the condition is clearly unnecessary. If r is not an effectively computable real number, then in accordance with Definition 2 (as presented above), C is not an admissible selective class, because (irrespective of whether or not information from the events y_i is used) no computer will be able to compute the ith digit of its binary expansion for every i. Conversely, if r is effectively computable then there will exist a computer, conforming to the restrictions of Definition 2, which effectively determines membership in C. But my intuition is that in that case the class A is not, in fact, homogeneous.

https://doi.org/10.1086/psaprocbienmeetp.1986.1.193142 Published online by Cambridge University Press

Although the concept of information invoked above is vague, I think that Salmon's approach to the definition of objective homogeneity is basically on the right track. The underlying intuition is that a reference class A is objectively homogeneous relative to an attribute B provided that there does not exist an effective procedure for defining (or a computer which is able to generate) a subsequence C of A such that $P(B|A.C) \neq P(B|A)$, where the procedure (computer) only makes use of information regarding events in the absolute past of elements of A. If we deify the computer, thinking of it as a device with access to the complete state of the universe in the absolute past of each element of A and having unlimited powers of calculation, then Salmon's account of objective homogeneity generalizes the ontic version of Laplace's conception of explanation (see, 1984, pp. 17-18) to an irreducibly stochastic universe. In an irreducibly stochastic universe Laplace's demon will not be able to predict with certainty future states of the system, but only to assign those future states objective probabilities. The objective probability that a particular event (an element of reference class A) has attribute B is the probability of B in the relevant subclass of an objectively homogeneous partition of A (as determined in accordance with Definition 4).

However, there is a fundamentally important difference between the deterministic version and the stochastic version of Laplace's demon. In a deterministic universe, one can speak of the objective probability of events, *simpliciter*. The probability that any particular element of reference class A has attribute B will be either one or zero, and that probability will have been fixed from the beginning of time, so to speak. However, in an irreducibly stochastic universe, the objective probabilities of particular events will evolve over time. In the next section I will explore the implications of the temporal relativity of objective probabilities for Salmon's conception (see pp. 22-23) of the ideal S-R basis for the explanation of a particular event. The main point is that the concept of an objectively homogeneous reference class needs to be temporally relativized. The original definition will follow as a special case of the generalized concept.

3. The Temporal Relativity of Objective Homogeneity

Consider a quantity of radioactive material that is undergoing decay in the presence of a peculiarly constructed counter. Over a period of three moments the counter records events of radioactive decay and its internal state is determined by the total number of decay events that have occurred since the beginning of the experiment (for the purposes of the present paper, I will think of time as unfolding in discrete moments). Let us suppose that in any given moment the objective probability of decay is given by -- P(no decay events)=1/4, P(one decay event)=1/2, and P(2 decay events)=1/4. The system, call it S, is constructed in such a way that at the end of three moments it "blows up" if and only if at least 4 decay events have occurred in the three intervening moments. Let the reference class A consist of an infinite sequence of such systems (analogous to an infinite sequence of tosses of a penny) and let the attribute B be that the system "blows up". We may ask whether the reference class A is objectively homogeneous relative to the attribute B.

The objective probabilities associated with the system S are given in figure 1.

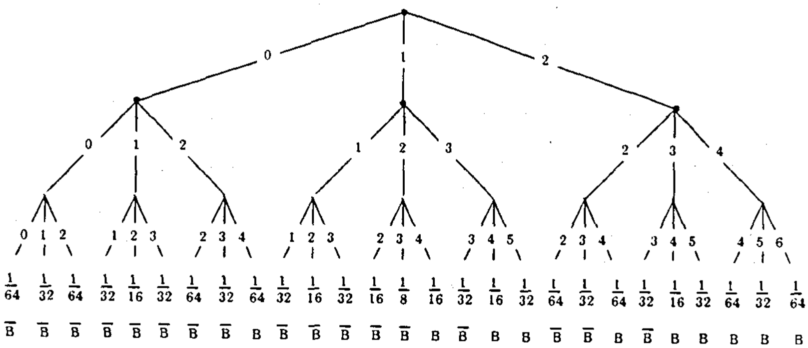

Figure 1

The middle path leading from each node has probability 1/2 while each of the outside paths has probability 1/4. The fraction at the bottom of each complete path gives the objective probability that the system will develop along that path. The numbers on the paths represent the state of system S at those points in its development. Finally, B occurs at the bottom of each path which would lead to the system's "blowing up", while $\bar{B}$ occurs at the bottom of those paths which would not lead to the system's "blowing up". Now suppose that a particular system develops along the starred path in figure 1 (i.e., there are two decay events the first moment, none the second moment, and two again the third moment of the system's development). Let t_0 be the moment at which the system "blows up", and for each n let t_{-n} be the nth moment preceding the "blow up" and let $P_{t-n}(B)$ be the objective probability at moment t_{-n} that the system will "blow up". Then, it is easily seen that for a particular system which follows the starred path the objective probabilities are: $P_{t-3}(B) = 5/16$, $P_{t-2}(B) = 11/16$, and $P_{t-1}(B) = 4/16$. Thus the system evolves from a state in which the explosion is improbable to one in which it is more probable, back to a less probable state, and finally to a state in which the event has occurred.

The relevant objective probabilities evolve discontinuously in the simple system described in our example. However, if the system were to develop for, say, 1200 moments (rather than just 3) and were so constructed that it would "blow up" if and only if at least 1600 decays had occurred in that interval, then the objective probabilities would evolve in a more nearly continuous fashion. Yet they still might, in a particular case, follow a tortuous path (up and down) before ultimately converging to one.

https://doi.org/10.1086/psaprocbienmeetp.1986.1.193142 Published online by Cambridge University Press

If we ask, returning to our specific example, whether or not the reference class A (of systems which develop over an interval of three moments) is objectively homogeneous relative to attribute B, the answer is obviously that it is not. Specifically, let $A=\{x_1,x_2,...\}$ denote the sequence of final states of the system S (so that for each i, x_i represents either the system's having "blown up" or not having "blown up") and let $D = \{y_1,y_2,...\}$ denote the sequence of states of the system at t_{-2}, two moments prior to the final state of the system (when attribute B either occurs or does not occur). Then D is an associated sequence for A. Moreover, the class C={yi| exactly 2 decay events have occurred up to moment t_{-2}} is an admissible selective class for A. Since P(B|A.C)=11/16 is different from P(B|A)=5/16, it follows in accordance with Definition 4 that A is not an objectively homogeneous reference class relative to B. It can easily be shown by considering associated sequences determined by the state of the system at moment t_{-1} that the subclass C of the reference class A also fails to be objectively homogeneous.

These considerations motivate the following generalization of Salmon's concept of objective homogeneity.

Definition 1'. Let A be a reference class consisting of a sequence of events $x_1,x_2,...$ For each i, consider a temporal coordinate system whose zero point t_0 is the moment at which x_i reaches its final state. For each n, let t_{-n} represent the nth moment prior to t_0. Any infinite sequence D consisting of events $y_1,y_2,...$ will be called a (t-n)-*associated sequence* if each event y_i occurs in the absolute past (past light cone) of the state of system x_i at the moment t_{-n}.

Definition 2'. Let $D=\{y_1,y_2,...\}$ be a (t-n)-*associated sequence* of a reference class $A= \{x_1,x_2,...\}$. The class C qualifies as a (t-n)-*admissible selective class* of the associated sequence D if and only if for any y_i in D, the membership of y_i in C or $\bar{C}$ could, in principle, be ascertained by a computer that receives information from a physical detector responding to y_i, but that receives no information gathered by the detector (or from any other source) in response to any events z_i in the absolute future of y_i.

Definition 3'. Let the ordered class $\{y_1,y_2,...\}$ constitute a (t_{-n})-associated sequence D with respect to the reference class A. Then a selection by a (t_{-n})-associated sequence S is any selection within A defined by the rule $x_i \epsilon S$ iff $y_i \epsilon C$, where C is a (t_{-n})-admissible selective class of the sequence D.

Definition 4'. A reference class A is *(t-n)-objectively homogeneous* with respect to an attribute B iff the probability of B within A is invariant under all selections by (t_{-n})-associated sequences.

For example, the class $A =\{x_1,x_2,...\}$ of systems described above is t_{-2} objectively homogeneous. For, given the assumption of current quantum theory that there are no hidden variables affecting the probability of a radioactive decay, it follows that no information derived from events prior to the point at which the system begins to develop (i.e., prior to t-2) is statistically relevant to attribute B. However, as demonstrated above in connection with the subclass C, A is not (t-1)-objectively homogeneous.

It follows from considerations of probability theory that if a reference class A is (t_{-m})-objectively homogeneous for a given m, then A is (t_{-n})-objectively homogeneous for all n>m (i.e., with respect to associated sequences whose information is derived from earlier moments

https://doi.org/10.1086/psaprocbienmeetp.1986.1.193142 Published online by Cambridge University Press

in the development of the system.) This fact motivates the following definition of objective homogeneity, *simpliciter*.

Definition 5. A reference class A is *objectively homogeneous* with respect to an attribute B iff the probability of B within A is invariant under all selections by (t_0)-associated sequences.

4. Conclusion

One of Salmon's purposes in Chapter 3 was to refute Hempel's thesis of the epistemic relativity of statistical explanation. Has he succeeded? In a sense, yes, because Definition 5 makes no reference to background knowledge. Nonetheless, Salmon's victory is highly qualified, since the *S-R basis* of explanations will typically (at least for macro phenomena) involve probabilities that are arbitrarily close to one. Consider for a moment the disastrous explosion of the space shuttle Challenger which occurred on January 28, 1986. Conceivably the objective probability of the explosion was close to zero in the early hours of the morning, yet seventy seconds into the flight it was evidently quite close (if not equal) to one. It follows, according to Salmon's theory, that the S-R basis for an explanation of that event would be essentially trivial (involving, perhaps, a reference class of events in which plumes of fire were issuing from the booster rocket only moments before its explosion). Of course, one could generalize the concept of the S-R basis of an explanation to include considerations of (t_{-n})-objective homogeneity for $n>0$. But there do not appear to be any objective grounds for isolating one moment rather than another in the evolution of the event as the focus of the explanation. The objective probabilities may have evolved in a complex fashion (increasing over some intervals and decreasing over others). In practice, the peaks and valleys of the objective probability wave upon which we focus our explanatory concern are likely to reflect our background knowledge and pragmatic purposes. To that extent our statistical explanations of particular events will not be objectively grounded.

Notes

[1]The present paper extends arguments developed in two of my previous papers (1981, 1983). The aim of those papers was to point out the serious implications for objectivist theories of probabilistic explanation and probabilistic causality of the fact that in an irreducibly stochastic world the objective probabilities of particular events evolve and so must be temporally relativized. Since there appears to be no objective basis for choosing the point in time relative to which the explanatorily or causally relevant probabilities should be computed (the choice will depend upon our pragmatic interests and concerns), the purported objectivity of those theories is undermined.

[2]It should also be noted that in the section of Chapter 3 entitled "Some Philosophical Reflections" Salmon reformulates his definitions in a manner that achieves some of the increased simplicity of the present account. However, his development there is based upon finite reference classes and so raises other difficult issues (for example, how to measure the degree of randomness of a finite class).

https://doi.org/10.1086/psaprocbienmeetp.1986.1.193142 Published online by Cambridge University Press

References

Hanna, J.F. (1981). "Single Case Propensities and the Explanation of Particular Events." Synthese 20: 308-344.

------- . (1983). "Probabilistic Explanation and Probabilistic Causality." In PSA 1982, Volume 2. Edited by Peter Asquith and Thomas Nickles. East Lansing, Michigan: Philosophy of Science Association. Pages 181-193.

Hempel, C.G. (1965). Aspects of Scientific Explanation and Other Essays in Philosophy of Science. New York: Free Press.

Salmon, W.C. (1971). Statistical Explanation and Statistical Relevance. Pittsburgh: University of Pittsburgh Press.

-------- . (1984). Scientific Explanation and the Causal Structure of the World. Princeton: Princeton University Press.

https://doi.org/10.1086/psaprocbienmeetp.1986.1.193142 Published online by Cambridge University Press