there exists a configuration of 19 unit circles satisfying A. We consider the following three cases :

(1) M has 12 members and N has 7 ;
(2) M has 11 members and N has 8 ;
(3) M has 10 members and N has 9.

By the corollaries to lemmas 2, 3, these three cases exhaust the possibilities.

Case 1. If C_i, C_j are two consecutive members of M, then by lemma 2 $\angle O_iOO_j > 28\frac{3}{4}^\circ$, and so $\angle O_iOO_j < 360^\circ - 11\ \ 28\frac{3}{4}^\circ < 45^\circ$. Thus by lemma 6 there is no N-circle C_k, $OO_k > 1{\cdot}2$. Hence the angle between two N-circles is greater than $49\frac{1}{7}^\circ$ (lemma 3). Then the sum S of the angular separations of consecutive N-circles is such that (lemmas 2, 3, 5)

$$S > 7\ \ 49\tfrac{1}{7}^\circ + \alpha \,.\, O + \beta(53^\circ - 49\tfrac{1}{7}^\circ) + \gamma(57\tfrac{1}{2}^\circ - 49\tfrac{1}{7}^\circ) + \delta(86\tfrac{1}{4}^\circ - 49\tfrac{1}{7}^\circ),$$
$$\alpha + 2\beta + 3\gamma + 4\delta = 12, \quad \alpha + \beta + \gamma + \delta \leqslant 7$$

Thus $S > 360^\circ - 16^\circ + 3\frac{6}{7}\beta + 8\frac{1}{4}\gamma + 36\delta > 360^\circ$, which is impossible.

Case 2. For the sum S of the angular separations of consecutive N-circles we have

$$S > 310^\circ + \alpha(41\tfrac{1}{3}^\circ - 38\tfrac{3}{4}^\circ) + \beta(52\tfrac{1}{7}^\circ - 38\tfrac{3}{4}^\circ) + \gamma(57\tfrac{1}{2}^\circ - 38\tfrac{3}{4}^\circ),$$
$$\alpha + 2\beta + 3\gamma = 11, \quad \alpha + \beta + \gamma \leqslant 8.$$

Thus $S > 360^\circ$ unless $\alpha = 6$, $\beta = 1$, $\gamma = 1$, when $S = 357\frac{1}{2}^\circ$, and so the angle between two N-circles is less than $2\frac{1}{2}^\circ$ plus the relevant minimum used above. In the case of the six pairs of N-circles enclosing one M-circle this is less than $44^\circ < 49\frac{1}{7}^\circ$. If C_1C_2 is one such pair, then by lemma 3 max $(OO_1, OO_2) > 1{\cdot}2$, say $OO_2 > 1{\cdot}2$, and so if C_3, C_4 are the M-circles enclosing C_2, $\angle O_3OO_4 > 45^\circ$ by lemma 6. There are at least three such pairs of M-circles, and so if S' is the sum of the angular separations of the M-circles, $S' > 3 \,.\, 45^\circ + 8 \,.\, 28\frac{3}{4}^\circ = 365^\circ$, which is impossible.

Case 3. The total angular separation S of the N-circles is such that, (by lemmas 2, 3, 4, 5),

$$S > 348\tfrac{3}{4}^\circ + \alpha \,.\, 2\tfrac{7}{12}^\circ + \beta \,.\, 13\tfrac{1}{4}^\circ + \gamma \,.\, 18\tfrac{3}{4}^\circ,$$
$$\alpha + 2\beta + 3\gamma = 10, \quad \alpha + \beta + \gamma \leqslant 9.$$

Hence $S > 360^\circ$, which is impossible.

Also by lemma 2 we can draw 12 unit circles touching C and 6 unit circles with their centres on C. Thus the theorem is proved. E. R. R.

JOHN WALTER BROOKS.

John Walter Brooks, B.Sc., Honorary Secretary of the North-East Branch of the Mathematical Association, died very suddenly on 2nd November, 1948. He had been associated with the Branch since its inception in 1928, and for all but about two years since then had served it in the capacity of Secretary. His life was marked by qualities of sincerity and integrity, which were evident in all his activities.

John Brooks was born in 1894, and was educated at Rutherford College School, Newcastle, and at Liverpool University, where he graduated with second class honours in mathematics. He served during the first world war as an infantry officer, and in later years in the Army Cadet Forces and the Air Training Corps.

After teaching for a few years in Newcastle elementary schools, he was appointed mathematics master at Westoe Secondary School, and later at South Shields High School. He was a fine teacher, and was greatly loved and respected by boys and colleagues. J. H.

https://doi.org/10.1017/S0025557200019185 Published online by Cambridge University Press