NOTATION

A few preliminary words are necessary in respect to the following scheme of notation.

In the first place the notation should be uniform, that is to say, when a definite point connected with the triangle is under consideration it should always be denoted by the same letter, and not by one letter at one time and by another letter at another. But the converse practice of never using a letter which has denoted one special point to denote any other point need not be carried out, and indeed cannot be, unless the number of properties investigated be very small, or unless recourse be had to letters ornamented with an intolerable number of suffixes or accents. To illustrate by an example. When the circumcentre of a triangle ABC has to be mentioned, it is invariably called O, but in cases where the circumcentre does not come into consideration at all, there is no adequate reason why O should not be used to denote some other point such as the point of concurrency of three angular transversals.

In the second place capital letters, loaded or not as the case may be with an accent or a suffix or even with both, should denote points, and small letters should denote lines. I am aware that such a rule is not adhered to in some of the best geometrical treatises, for instance those of Chasles who constantly uses both capitals and small letters to denote points. The extremely prevalent, though not universal, practice of denoting the radius of the circumcircle by R shows that one deviation at least must be made from the rule if accordance with general usage is to be secured. For the distances between the incentre, the excentres, and the vertices of a triangle Greek letters have been employed. This was found to be unavoidable, if conciseness was to be aimed at. These Greek letters are the notation adopted by the discoverers of not a few of the properties* connected with the incentre, etc., and I could not invent a better.

In the third place the notation should conform as far as may be to that actually in use among geometers. I have had this consideration continually before me, and it has caused me a world of trouble. For the notation has been changed, rechanged, and changed again; and though I dare not hope that the compromise which has been

* The properties referred to are not contained in the section which is here printed.

come to will commend itself to everybody as the best in the circumstances, yet if it were needful I could support with a considerable weight of authority drawn from one country or another, the designation of every important point to which objection might be taken.

POINTS.

A, B, C,	= vertices of the fundamental triangle. When the sides of triangle ABC are spoken of, they are understood to be taken in the order BC, CA, AB.
A', B', C'	= mid points of the sides BC, CA, AB.
A_∞, B_∞, C_∞	= harmonic conjugates of A', B', C', for* BC, CA, AB.
A_1, B_1, C_1 A_2, B_2, C_2	= vertices of some triangle related to ABC, for example, anticomplementary, inscribed or circumscribed.
D, E, F	= points of contact of sides with incircle. = points of intersection of sides with transversal. = points of intersection of sides with angular transversals. = projections of a point on the sides.
D_1, E_1, F_1 D_2, E_2, F_2 D_3, E_3, F_3	= points of contact of sides with 1st excircle. 2nd excircle. 3rd excircle.
D', E', F'	= harmonic conjugates of D, E, F for BC, CA, AB. = those points in $A'B'C'$ which correspond to D, E, F in ABC. Similarly for D_1', E_1', F_1', etc.
G, G_a, G_b, G_c	= centroids of ABC, HCB, CHA, BAH.
G_0, G_1, G_2, G_3	= centroids of $I_1I_2I_3$, $I\,I_3I_2$, $I_3\,I\,I_1$, $I_2I_1\,I$.

* Following the example of Mr W. J. Russell, I have used the word "for" instead of the phrase "with respect to."

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Γ	= point of concurrency of AD, BE, CF.
Γ_1, Γ_2, Γ_3	= points of concurrency of AD_1, BE_1, CF_1, and so on.
	These four points are frequently called the Gergonne points of ABC.
H	= orthocentre of ABC.
I	= incentre of ABC.
I_1, I_2, I_3	= 1st, 2nd, 3rd excentres of ABC.
J	= incentre of $A_1B_1C_1$, the triangle anticomplementary to ABC.
J_1, J_2, J_3	= 1st, 2nd, 3rd excentres of $A_1B_1C_1$.
	These four points are frequently called the Nagel points of ABC.
K	= insymmedian point of ABC (Lemoine's point).
K_1, K_2, K_3	= 1st, 2nd, 3rd exsymmedian points of ABC.
L, M, N	= feet of internal angular bisectors of ABC.
	= projections of the symmedian point on the sides of ABC.
L′, M′, N′	= feet of external angular bisectors of ABC.
L_1, M_1, N_1	= projections of the first exsymmedian point on the sides of ABC.
	Similarly for L_2, M_2, N_2, etc.
O	= circumcentre of ABC.
	= point of intersection of three concurrent lines.
O′	= circumcentre of A′B′C′, or nine-point centre of ABC.
O_a, O_b, O_c	= circumcentres of HCB, CHA, BAH.
O_0, O_1, O_2, O_3	= circumcentres of $I_1I_2I_3$, II_3I_2, I_3II_1, I_2I_1I.
O_1, O_2, O_3	= circumcentres of ΩCA, ΩAB, ΩBC.
O_1', O_2', O_3'	= circumcentres of Ω'CA, Ω'AB, Ω'BC.
O, O′	= pairs of isogonal points with respect to ABC.

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P, Q P, P′ Q, Q′	= pairs of special points.
R, S, T	= points where the perpendiculars of a triangle meet the circumcircle. = feet of the insymmedians. = projections of Ω on BC, CA, AB.
R′, S′, T′	= feet of the exsymmedians. = projections of Ω' on BC, CA, AB.
T, T_1, T_2, T_3	= points of contact of the nine-point circle with the incircle and the excircles. = centres of the four Taylor circles.
U, U′	= ends of that diameter of the circumcircle which is perpendicular to BC. U is on the opposite side of BC from A. Similarly for V, V′ and W, W′.
U, V, W	= points in which concurrent lines from A, B, C meet the circumcircle ABC. = mid points of AH, BH, CH.
X, Y, Z	= feet of the perpendiculars from A, B, C.
X′, Y′, Z′	= harmonic conjugates of X, Y, Z for BC, CA, AB. = mid points of YZ, ZX, XY.
Ω, Ω'	= Brocard points of ABC.

Regarding the notation which may be adopted for many other important points connected with the triangle, such as the isogonic and isodynamic centres, the centres of Brocard's circle and Lemoine's first circle (triplicate ratio), Steiner's point, Tarry's point, and so on, no suggestion is offered for the present. Those who wish to see the notations which have been employed may be referred to Mr De Longchamps' *Journal de Mathématiques Élémentaires* and to *Mathesis*.

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LINES.

$a, \; b, \; c$	= the sides BC, CA, AB of ABC.
$\alpha, \; \beta, \; \gamma$	= AI, BI, CI.
$\alpha_1, \beta_1, \gamma_1$	= AI_1, BI_1, CI_1.
$\alpha_2, \beta_2, \gamma_2$	= AI_2, BI_2, CI_2.
$\alpha_3, \beta_3, \gamma_3$	= AI_3, BI_3, CI_3.
$\alpha_1 - \alpha, \; \beta_2 - \beta, \; \gamma_3 - \gamma$	= $I\,I_1$, $I\,I_2$, $I\,I_3$.
$\alpha_2 + \alpha_3, \; \beta_3 + \beta_1, \; \gamma_1 + \gamma_2$	= I_2I_3, I_3I_1, I_1I_2.
$d, \; e, \; f$	= the sides EF, FD, DE of DEF. Similarly for the sides of $D_1E_1F_1$, etc.
$h_1, \; h_2, \; h_3$	= the perpendiculars AX, BY, CZ.
$h_1', \; h_2', \; h_3'$	= the segments AH, BH, CH of the perpendiculars.
h_1'', h_2'', h_3''	= the segments HX, HY, HZ of the perpendiculars.
$k_1, \; k_2, \; k_3$	= the perpendiculars OA′, OB′, OC′ from the circumcentre.
$l_1, \; l_2, \; l_3$	= the internal angular bisectors of A, B, C.
$\lambda_1, \; \lambda_2, \; \lambda_3$	= the external angular bisectors of A, B, C.
m_1, m_2, m_3	= the medians from A, B, C.
$n_1, \; n_2, \; n_3$	= the insymmedians from A, B, C.
$\nu_1, \; \nu_2, \; \nu_3$	= the exsymmedians from A, B, C.
$p_1, \; p_2, \; p_3$	= perpendiculars from any point on BC, CA, AB.
r	= radius of the incircle.
$r_1, \; r_2, \; r_3$	= radii of the 1st, 2nd, 3rd excircles.

R	= radius of the circumcircle.
R_1, R_2, R_3	= radii of the circumcircles of ΩCA, ΩAB, ΩBC.
R_1', R_2', R_3'	= radii of the circumcircles of Ω'CA, Ω'AB, Ω'BC.
ρ	= radius of the incircle of XYZ,
ρ_1, ρ_2, ρ_3	= radii of the 1st, 2nd, 3rd excircles of XYZ.
s	= semiperimeter of ABC.
s_1, s_2, s_3	$= s - a$, $s - b$, $s - c$.
x, y, z	= the sides YZ, ZX, XY of XYZ.

AREAS.

Δ, Δ_a, Δ_b, Δ_c	= ABC, HCB, CHA, BAH.
Δ_0, Δ_1, Δ_2, Δ_3	$= I_1I_2I_3$, $I\,I_3I_2$, $I_3\,I\,I_1$, $I_2\,I_1\,I$.

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