Thin Layer Approximation in 3-D

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Abstract. Equations are derived which describe the propagation of strong shocks in the interstellar matter, without any restrictions for symmetry, in a thin layer approximation (2.5 dimensions). Using these equations permits to calculate the propagation of shock waves from nonsymmetric supernovae explosions in a medium with arbitrary density distribution and the formation of superbubbles in galaxies.

1 Thin Layer Approximation

A thin shell approximation for discription of strong shocks is based on two simplifications. First, it is assumed that all swept-up intercloud gas accumulates into the thin shell just behind the shock front and moves with the velocity u. Second, the pressure distribution inside the cavity $P_{\rm in}(r,t)$ is taken to be uniform. The equations of mass and momentum conservation in spherically symmetric case may be expressed as follows (Chernyi 1957)

$$M = M_0 + 4\pi \int_0^R \rho(r) r^2 dr, \quad \frac{d(Mu)}{dt} = 4\pi R^2 (P_{\rm in} - P) + Mg, \tag{1}$$

where M is the mass of the shell, M_0 is the ejected mass, R is the shock radius and u is the gas velocity behind the shock; $\rho(r)$ and P are the density and the pressure of the ambient gas, g is the external gravitational field. For the adiabatic blastwave without gravity

$$1)\, \frac{dR}{dt} = \frac{\gamma+1}{2} u \quad \text{or} \quad 2)\, \frac{dR}{dt} = u; \qquad E_0 = E_{\rm th} + \frac{1}{2} M u^2, \tag{2}$$

where $E_0 = \text{const}$ is the energy of the explosion, $E_{\rm th} = \frac{4\pi}{3(\gamma-1)} P_{\rm in} R^3$ is the thermal energy of the blastwave, and γ is the adiabatic index. In (2) the radius R is related to the shock front in the case 1) and to a sphere inside a thin layer in the case 2). Equations (1)–(2) have a simple solution for the homogeneous case if the swept-up mass is much greater than the ejected one: $R = \xi_0 \left[\frac{E_0}{\rho_0}\right]^{1/5} t^{2/5}$, where (Chernyi 1957, Bisnovatyi-Kogan and Blinnikov 1982)

$$1)\, \xi_0 = \left[\frac{75(\gamma-1)(\gamma+1)^2}{16\pi(3\gamma-1)}\right]^{1/5}, \quad 2)\, \xi_0 = \left[\frac{75(\gamma-1)}{8\pi}\right]^{1/5}. \tag{3}$$

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Comparison with an exact self-similar (SS) solution (Sedov 1946) shows, that the case 2) gives much better precision, $\xi_0 = 1.033,\ 1.014\ 1.036$ for (SS), (1), (2) cases respectively at $\gamma = 1.4$ (molecular cloud); and similarly $\xi_0 = 1.15,\ 1.12,\ 1.15$ for $\gamma = 5/3$.

2 Three-Dimensional Shocks

Introduce, following Bisnovatyi-Kogan and Silich (1991), Silich (1992) (see also Palouš, 1990; Bisnovatyi-Kogan and Silich, 1995) a Cartesian coordinate system (x, y, z). Let m be the mass, $\mathbf{r}$ the radius-vector, $\mathbf{u}$ the velocity of a particular Lagrangian element of the shock, $\rho(x,y,z) = \rho_0 f(x,y,z)$ the ambient gas density, $\mathbf{n}$ the unity vector normal to the shock front, $\mathbf{g}$ the acceleration of the external gravitational field, $\mathbf{V}$ the velocity field of the undisturbed gas flow, Σ the surface area of the Lagrangian element, $m = \sigma\Sigma$, σ the surface density, and $\Delta P = P_{\rm in} - P$ the pressure difference between the hot interior and warm (cold) external gas. The pressure $P_{\rm in} = (\gamma - 1)E_{\rm th}/\Omega$ of the hot tenuous gas within the cavity is a function of the thermal energy, $E_{\rm th}$, of the bubble and the volume Ω. To describe the expansion of the shock we must introduce the surface area element and define the volume of any closed three-dimensional region. It is well known from differential geometry that any surface may be specified parametrically, with Cartesian coordinates at any point on the surface being a function of two parameters, λ_1 and λ_2: $x = x(\lambda_1, \lambda_2)$, $y = y(\lambda_1, \lambda_2)$, $z = z(\lambda_1, \lambda_2)$. Then the element of the surface area may be defined by the expression (Budak and Fomin, 1965):

$$d\Sigma = S(\lambda_1, \lambda_2) d\lambda_1 d\lambda_2\ , \tag{4}$$

$$S(\lambda_1, \lambda_2) = \left\{ \left[\frac{\partial(y,z)}{\partial(\lambda_1,\lambda_2)}\right]^2 + \left[\frac{\partial(z,x)}{\partial(\lambda_1,\lambda_2)}\right]^2 + \left[\frac{\partial(x,y)}{\partial(\lambda_1,\lambda_2)}\right]^2 \right\}^{1/2} . \tag{5}$$

Similar expressions can be obtained for Ω and $\mathbf{n}$. If parameters λ_1 and λ_2 are considered as the Lagrangian coordinates of the shock front, then the equations for 3-D shock propagation may be written for the mass $\mu = \sigma S(\lambda_1, \lambda_2)$ per unit of Lagrangian square on the surface of parameters (λ_1, λ_2), in a compact form, convenient for numerical integration

$$\frac{d\mu}{dt} = \rho\chi, \tag{6}$$

$$\frac{du_x}{dt} = \frac{\Delta P}{\mu}\frac{\partial(y,z)}{\partial(\lambda_1\lambda_2)} - \frac{u_x - V_x}{\mu}\rho\chi + g_x, \tag{7}$$

$$\frac{du_y}{dt} = \frac{\Delta P}{\mu}\frac{\partial(z,x)}{\partial(\lambda_1\lambda_2)} - \frac{u_y - V_y}{\mu}\rho\chi + g_y, \tag{8}$$

$$\frac{du_z}{dt} = \frac{\Delta P}{\mu}\frac{\partial(x,y)}{\partial(\lambda_1\lambda_2)} - \frac{u_z - V_z}{\mu}\rho\chi + g_z, \tag{9}$$

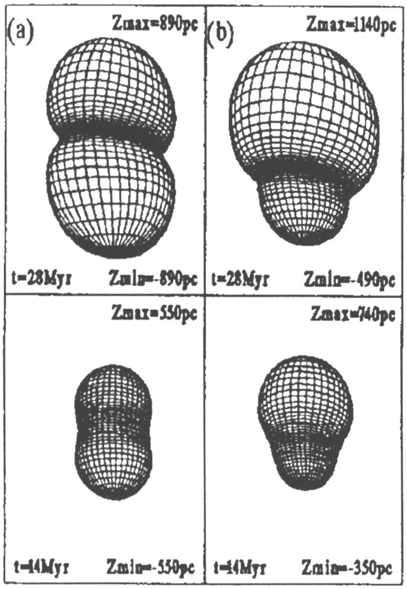

Fig. 1. Galactic superbubble morphology for different locations of the parent OB-association relative to the galactic plane. Left: the OB-association is at the midplane of the Galaxy. Right: the OB-association is 50pc above the Galactic plane.

Fig. 2. Scheme of the HI holes orientation in a spiral galaxy in the plane of view. Projection of the rotational axis to the plane of view is directed upward, determining a motion of the left side to the observer. The indicated direction of elongation of HI hole determines the orientation of the galaxy, shown in the figure.

$$\frac{\mathrm{d}x}{\mathrm{d}t} = u_x, \quad \frac{\mathrm{d}y}{\mathrm{d}t} = u_y, \quad \frac{\mathrm{d}z}{\mathrm{d}t} = u_z. \tag{10}$$

Here function χ is defined as follows:

$$\chi = (u_x - V_x)\frac{\partial(y,z)}{\partial(\lambda_1,\lambda_2)} + (u_y - V_y)\frac{\partial(z,x)}{\partial(\lambda_1,\lambda_2)} + (U_z - V_z)\frac{\partial(x,y)}{\partial(\lambda_1,\lambda_2)}. \tag{11}$$

Approximating the shock front by a number N of Lagrangian elements one gets a system of $7N$ differential equations for mass and momentum conservation. This set of equations is coupled with the equation for the gas pressure within the cavity, and the equation of total energy $E_{\mathrm{tot}} = E_{\mathrm{th}} + E_{\mathrm{k}} + E_{\mathrm{g}}$, consisting of the thermal energy of the hot bubble interior, kinetic and gravitational energies of the shell. The kinetic and gravitational energies of the

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shell are determined by corresponding surface integrals. Variations of the total energy E_{tot} of a remnant or a bubble throughout the adiabatic stage of evolution are defined by the energy input rate $L(t)$, kinetic and thermal energies of the swept-up interstellar gas with temperature $T(x, y, z)$:

$$E_{\text{tot}} = E_0 + \int_0^t \left[L(t) + \frac{1}{2} \int_{\lambda_{1,min}}^{\lambda_{1,max}} \int_{\lambda_{2,min}}^{\lambda_{2,max}} \dot{\mu}(V^2 + 3kT/\eta) \mathrm{d}\lambda_1 \mathrm{d}\lambda_2 \right] \mathrm{d}t, \quad (12)$$

where E_0 is the initially deposited energy and η is the mean mass per particle. Here E_g is neglected, and $L(t) = 0$ for SNR. During the radiative phase of expansion the gas behind the shock front cools so quickly that it does not add to the total energy of the remnant. Rarefied hot gas inside the cavity expands adiabatically and accelerates the surrounding dense shell. The time-derivative of the thermal energy of the remnant is defined then by the equation, which is used instead of (12):

$$\frac{dE_{\text{th}}}{dt} = L(t) - \int_{\lambda_{1,min}}^{\lambda_{1,max}} \int_{\lambda_{2,min}}^{\lambda_{2,max}} P_{\text{in}} u_n S(\lambda_1, \lambda_2) \mathrm{d}\lambda_1 \mathrm{d}\lambda_2, \quad (13)$$

where u_n is the velocity component normal to the shock front. The set of $(7N + 1)$ equations is solved using Adams method of 12 order. Numerical calculations of supershell formation in the plane-stratified and differentially rotating Galactic disk have been performed by Silich *et al.* (1994) and are shown in Fig. 1. An hourglass remnant with a noticeable degree of deformation by the Galactic shear has developed. Formation of elongated superbubbles due to differential galactic rotation gives a possibility to determine unambiguously the direction of galactic rotation (Mashchenko and Silich, 1995). The means of this determination is shown in the Fig. 2.

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