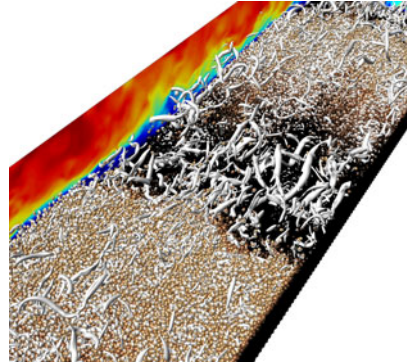


## A decade's investigation of the stability of erodible stream beds

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This contribution is a remake of the one published under the same title by Reynolds (*Nord. Hydrol.*, vol. 7, 1976, pp. 161–183). As in that paper, attention is given to the latest developments in the field of river morphodynamics and, in particular, on the formation of bed patterns. Stimulated by the work by Kidanemariam and Uhlmann (*J. Fluid Mech.*, vol. 750, 2014, R2) the opportunity is taken to examine the most recent developments in terms of sediment transport models.

**Key words:** multiphase and particle-laden flows, sediment transport

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### 1. Introduction

Legend has it that Albert Einstein's son once asked his father about the opportunity to carry out research on sediment transport. Allegedly, Einstein advised his son not to pursue this study, advice that, indeed, was promptly ignored, as Hans Albert Einstein soon became a prominent scientist in this field. Certainly, that was not the first time (nor the last) a father's advice had gone unheeded, but what made the conversation remarkable was the justification apparently given by Einstein: sediment transport was a far too complicated topic to be studied!

Not surprisingly, this anecdote has been proudly handed down from one generation to the next in the sediment transport community, even though the chances are the whole story was nothing more than 'a construction of frustrated researchers' (Gyr & Hoyer 2006), since Einstein wrote in his own hand to Meyer-Peter, coauthor of one of the most famous sediment transport formulae, to recommend his son for a doctoral position. Nonetheless, as there is a grain of truth in every story, let us assume for a moment that Einstein's concern had foundation and ask what makes flow above an erodible bottom such an intricate problem to study. Sediment transport by geophysical flows is governed by the mutual interaction between fluid and sediment particles. The flow moves the sediment through the forces acting on each grain, the motion of the

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sediment alters the shape of the flow domain through erosion and deposition, and the flow, in turn, is altered both by the presence of the sediment and by the spatial and temporal changes of the domain shape.

This is the world of ‘morphodynamics’, the science that describes the processes whereby natural fluids shape the Earth’s surface (Seminara 2010). It is a fairly young discipline, its origin dating back to the work of Felix Maria Exner (1876–1930), an Austrian meteorologist and geophysicist who can be regarded as the father of morphodynamics, even though the term itself was introduced many decades afterward. Exner’s main contribution was the formulation of a sediment continuity equation (Exner 1925), named after him, which relates the time evolution of the bed to the spatial variations of the sediment load carried by the flow. Coupled to a suitable flow model, this equation has been successfully used in the past (and it is still widely used nowadays) to study morphodynamic problems. In particular, we focus here on the most common bed forms encountered in rivers: ripples, dunes, antidunes and bars.

In a period of approximately thirty years, from the 1960s to the 1980s, several seminal studies on the stability of bed forms were produced (see the reviews by Reynolds 1976; Seminara 2010). These analyses, based upon the Exner equation and on relatively simple flow models, shed some light on the main mechanisms which drive the instability process and allowed for the identification of the unstable regions in the parameter space where different bed forms were expected to form. Recently, research implementing more refined Reynolds-averaged Navier–Stokes (RANS) flow models have been mostly devoted to understanding transition and competition between bed forms within the same stability framework, thus laying the foundations for the formulation of a unified theoretical regime diagram for river bed forms in terms of the main flow and sediment parameters (Colombini & Stocchino 2011, 2012).

Interestingly, about a century after its formulation, most of the sediment transport models adopted in morphodynamics still rely on the Exner equation and on a suitable empirical relationship for the sediment discharge, usually set equal to the ‘transport capacity’ of the flow under the assumption that sediment responds instantaneously to the action of shear. To date, the most popular alternative to this approach is related to the concept of saturation length: the distance needed by sediment discharge to adapt to a change in shear. Non-equilibrium models based upon a sediment momentum equation have been formulated to quantify the effect of sediment inertia (Parker 1975), although the actual role of saturation length in subaqueous environments is still debated. Finally, some recent attempts to replace the classic Exner equation should be mentioned. In Furbish *et al.* (2012) and Ancy & Heyman (2014), by means of a probabilistic interpretation of the bed load transport, alternative stochastic forms of the Exner equation are formulated.

## 2. Overview

How and to what extent is the model proposed by Kidanemariam & Uhlmann (2014) innovative? The first novelty is the use of a direct numerical simulation (DNS) for the flow, whereby the Navier–Stokes equations are numerically solved using a grid (and a time-step) small enough to resolve adequately all the relevant spatial (and temporal) scales of the turbulent flow. Hence, there is no need to introduce any ‘artificial’ turbulence model (i.e. to parameterize turbulence through an eddy viscosity) as in RANS models. Considering that the latter are still the most popular choice in the study of bed form instability, this is, in itself, a giant leap towards an accurate representation of the dynamics of the small eddies that are significant in the near-wall

region, where sediment transport takes place. Unfortunately, the smallest grid size in a DNS is required to be of the order of the Kolmogorov scale, which decreases as the flow Reynolds number increases.

The second novelty is the way the presence of sediment inside the flow is accounted for. A clever use of the immersed boundary technique, in fact, allows the imposition of no-slip conditions at the surface of each grain by means of a localized force term which is added to the flow equations. Considering that the effect of the sediment on the flow is usually completely disregarded (only the effect of a change of shape is accounted for, via the Exner equation) this is another important step towards a realistic representation of fluid-sediment interactions. Unfortunately, this imposes another limit on the smallest grid size to be adopted in the computation since there should be a reasonable number of computational points (approximately 500 in the present case) within the volume of a single grain.

The third novelty is the way sediment transport is modelled: particle motion is obtained via the integration, for each moving grain, of the Newton equation driven by the forces acting on the sediment, thus including hydrodynamic forces, gravity and grain-to-grain contacts. In particular, a linear mass–spring–damper system is employed to model the collision forces, which play an important role in the case of bedload transport (i.e. when the moving sediment is concentrated in a thin layer close to the bed). Moreover, the fluid–bed interface is defined dynamically at each time step and it changes shape as a consequence of the sediment motion. Considering that the Exner equation has been the cornerstone upon which most of the morphodynamic theories of the last century have been built, this is another important achievement: no need for an Exner equation means, in turn, no need for any empirical estimate of the solid discharge in terms of the flow and sediment parameters.

Unfortunately, since the timescales of the particle collision process are much shorter than those characteristic of the smallest eddies, the Newton equations have to be solved with a timestep approximately 100 times smaller than the one (already small) used for the flow. Moreover, the number of moving particles (and so of the equations to be solved) dramatically increases for turbulent simulations.

### 3. Future

When looking at the turbulent simulation presented in the supplementary movie available with (Kidanemariam & Uhlmann 2014), the first thought that comes to mind is that Einstein was definitely right: this is indeed a far too complex problem to study. However, simulations are so realistic that one seems to be watching an actual laboratory experiment. A fairly common question has then to be asked once again: will numerical simulations become the laboratory of the future? Let us give some figures: in six years of hard work, Guy, Simons & Richardson (1966) compiled the most famous experimental bed-form dataset, composed of exactly 339 runs. Counting eight hours a day and six days a week (humans are not computers), this gives an average of 50 hours per run. The single turbulent simulation presented in Kidanemariam & Uhlmann (2014) took approximately three months to be completed. Counting 24 hours a day and seven days a week (computers are not humans), this gives more than 2000 wall-clock hours per run. And, contrary to their experimental counterparts, ‘numerical’ bed forms did not even reach a final equilibrium amplitude.

Furthermore, is this a river we are looking at? A creek? A rivulet? Is the bed form we are observing a ripple or a dune? Assuming a 1 mm sand, the control volume is approximately 30 cm long, 4 cm high and 8 cm wide, which would be a small

laboratory channel. The value of the roughness Reynolds number, approximately 30, and the bed-form wavelength, between 10 and 15 cm, seem to qualify it as a ripple. Note that the last question is not irrelevant, since it is still debated whether ripples and dunes are different objects, their instability being controlled by different parameters, or they are just the shorter and longer version of the same bed form.

To conclude, this is indeed an excellent piece of work, whereby the complexity of the model has been pushed to a level never before reached in the field of sediment transport modelling. Having said so, will this be the future in morphodynamics? This is hard to say: though, on one hand, CPU power is constantly increasing, on the other hand the simulations described in the highlighted paper consumed a total of approximately 5 million CPU hours on the currently tenth fastest computer system worldwide (SuperMUC at the Leibniz Supercomputing Centre in München). And we are still very far from simulating a simple dune, which would require a much longer computational box, a free surface and a larger roughness Reynolds number. But this, I have to admit, sounds as the comment of someone who has spent the last 20 years working on RANS models and the Exner equation. The interested reader, is now ready to provide their own answer to this burning question.

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