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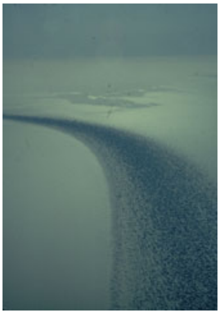
The motion of ice stream margins



A. C. Fowler†

MACSI, Department of Mathematics and Statistics, University of Limerick, Limerick, Ireland
OCIAM, Mathematical Institute, University of Oxford, Oxford OX1 3LB, UK

The recent article by Schoof (*J. Fluid Mech.*, vol. 712, 2012, pp. 552–578) provides a technically demanding solution to the problem of determining ice-stream margin evolution. It is important in opening the way to the future theoretical description of how the ice sheets will melt and sea level will rise as the climate warms. But the sophistication of the mathematics should not operate as a mask to an examination of the credibility of the model.

Key words: ice sheets, lubrication theory, shear layers

1. Introduction

Ice streams are those fast moving rivers of ice which are the primary means by which ice sheets such as that of Antarctica discharge into the ocean. They have typical widths of 50 km, and extend for many hundreds of kilometres upstream into the inland ice. Figure 1 shows the ice streams of Antarctica, indicated in blue. The figure by the title shows the margin of the Bindschadler ice stream (D) (81°S, 142°W) in the Siple Coast of West Antarctica. Flow is away from the viewer, and the width of the crevassed margin is approximately 2 km. (Photograph reproduced courtesy of Charlie Raymond, and kindly provided by Bob Bindschadler.) There is much interest in the formation and evolution of ice streams, because they appear to be the major agent whereby ice sheet collapse and resultant sea level rise will occur in a warming climate. Pine Island Glacier in West Antarctica and Jakobhavn Isbrae in Greenland are just two examples of outlet glaciers which have rapidly accelerated in the last few decades.

The mechanism by which ice streams form is not properly understood, but it seems likely that some form of instability mechanism is involved (Fowler & Johnson 1996; Hindmarsh 2009). It is also known that they are time-evolving systems, since the

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† Email address for correspondence: fowler@maths.ox.ac.uk

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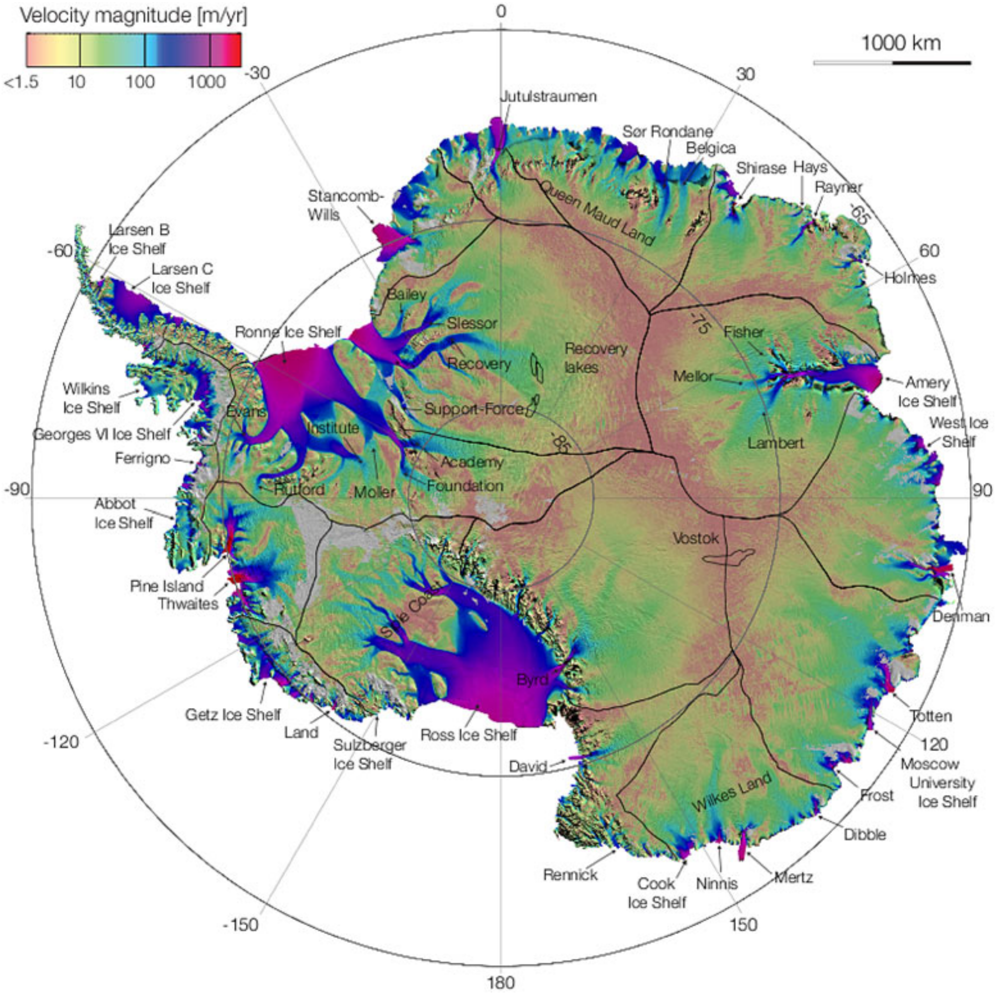

FIGURE 1. The ice streams of Antarctica, indicated in the blue parts of this map coloured with respect to magnitude of velocity (Rignot, Mouginot & Scheuchl 2011). The two large blue and purple areas are the (floating) Ronne-Filchner (upper) and Ross (lower) ice shelves. The image is available at http://earthobservatory.nasa.gov/IOTD/view.php?id=51781, and is reproduced courtesy of Eric Rignot, NASA Jet Propulsion Laboratory and University of California, Irvine.

presence of buried crevasses in the Kamb ice stream (C) in Antarctica indicates that the presently slowly moving ice was flowing rapidly some 200 years ago.

Schoof's (2012) article on the motion of ice streams represents a first theoretical step in the direction of describing the time evolution of ice streams, through a description of the way in which the ice-stream margins migrate. The margins of ice streams are heavily crevassed regions where ice velocity jumps from (rapid) speeds of 500 m y^{-1} to speeds of less than 10 m y^{-1} in just a few kilometres. Just as shock waves migrate, so it is reasonable to suppose that ice streams may come and go through evolution of their margins. Schoof's work is therefore important in seeking to establish the mechanism whereby this occurs.

2. Overview

Schoof (2012) considers a model for the transverse sectional structure of a downstream ice flow, consisting of an ice stream on the right ($x > 0$), and slower moving ice on the left ($x < 0$). The ice is Newtonian viscous, and thus satisfies the Stokes equations, with the stream flow being driven by a lateral shear stress far to the right. The basal conditions are taken to be those of no shear stress in $x > 0$ and no slip in $x < 0$. These represent an idealized transition from cold, frozen conditions to the left, and temperate, lubricated conditions to the right.

The ice flow solution for this geometry is easily solved using complex-variable methods, but the consistency of the solution with the assumed thermal conditions requires determination of the temperature field also. This satisfies a Poisson equation in the ice, where the source represents the viscous dissipation due to ice flow. This problem can also be solved by formulating it as a Wiener–Hopf problem, and the wizardry of complex variables allows Schoof to determine the leftwards migration speed V in terms of a single dimensionless parameter α which represents the magnitude of the driving lateral stress. Schoof here assumes $V > 0$, indicating expansion of the stream into the frozen ice; his solution for the opposite case (when α is small) is being presented separately.

Interestingly, the solution for the temperature allows the transition point speed to be determined on the basis of constraints associated with obstacle problems, in a manner very similar to that used in studying the dynamics of grounding lines (Nowicki & Wingham 2008): on the frozen side, we require $T < T_m$, where T_m is the melting temperature; on the temperate side, we require the net conductive heat flux to the bed to be non-negative.

3. Future outlook

The production of a specific relationship between ice-stream marginal movement and the driving lateral shear stress represents a significant advance in addressing the issue of satisfactory incorporation of ice-stream mechanics into ice-sheet modelling. Usual grid scales for ice-sheet models only allow one or two grid points across ice streams, and the evolution of their margins will forever lie beyond direct numerical implementation. Schoof's marginal condition thus represents a step on the road to more realistic ice-sheet modelling. It is akin to the derivation of a Rankine–Hugoniot condition for shock evolution, and it allows the possibility for the development of self-consistent models of ice-stream evolution.

Like all theoretical advances, however, the technical skill and the novelty of the achievement must be tempered by the legitimacy of the model. In his discussion, Schoof raises several difficulties with his theory, and provides several pointers for future work. There are two other points worth commenting on in this regard. Modellers always want to keep things as simple as possible: simplicity breeds insight. But the simplifications need to be the right ones.

Schoof (2012) assumes the temperature field evolves purely in the transverse section. He thus neglects the downstream advection of temperature, a term $w\partial T/\partial s$ (here s is downstream distance and w is downstream velocity). While this is consistent with his model (T is assumed independent of s), it is not really correct. The downstream residence time for a 500 km long ice stream with speed 500 m y^{-1} is 1000 years, while the transverse conduction time for ice of depth 1000 m is around 25 000 years (and Schoof assumes this is short compared to the external ice flow time scale). It is possible that his argument can be adapted, but at the least it is an issue which will form a consideration in the future development of the subject.

A more fundamental question concerns the assumption of a cold/temperate transition and an associated jump from no basal slip to no basal stress. This is a subject whose theoretical origins lie in the deep past (Hutter & Olunloyo 1980), and whose popularity has endured (Barcilon & MacAyeal 1993; Moore, Iverson & Cohen 2010). As well as its current application, it has been used to explain supposed stress concentrations in bedform generation (Kleman & Hättestrand 1999). But the assumption that the transition from cold to temperate ice allows an instant transition from no-slip to free-slip conditions is controversial (see also discussion in Fowler 2011, p. 723) and may be untenable.

In the classical theory of sliding (Weertman 1957), full slip of temperate ice is associated with the presence of a thin lubricating water film between the ice and the underlying substrate. As ice is raised towards the melting temperature, it is difficult to imagine such a film appearing instantly. Rather one might imagine, at temperatures close to the melting point, a partially lubricating film, and thus partial slip.

That being so, the question then arises: What is the distance over which the lubricating film becomes established? Or, for subglacial sediments, what is the distance over which they become water-saturated? If it is a matter of kilometres, then it becomes analogous to the near-tip plastic flow in the theory of cracks, and no harm is done with the assumption of a cold–temperate discontinuity. But if this is not the case, then Schoof's theory may turn out to be a stepping stone on the way to a more physically robust model of the ice-stream margin transition zone.

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