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Influence of turbulence on the shape of a spectral line: the analytical approach

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Abstract. We consider the propagation of spectral-line radiation in a correlated turbulent atmosphere. The ensembles of turbulent velocities $\mathbf{u}(\mathbf{r},t)$ and optical depths, τ_ν, are assumed to be Gaussian. We investigate the explicit analytical solution of the stochastic radiative transfer equation for the intensity I_ν of radiation. The scattering term is not taken into account. It is shown that, in addition to the usual Doppler broadening of the spectral line, correlated turbulent motions of atoms and molecules give rise to considerable changes in the shape of a spectral line. We find that the mean intensity $I_\nu^{(0)}$ ($I_\nu = I_\nu^{(0)} + I'_\nu$, $\langle I'_\nu \rangle = 0$) obeys the usual radiative transfer equation with renormalized extinction factor $\alpha_\nu^{\rm eff}$ if the correlation length R_0 of the turbulence is small as compared to a photon free path. A simple analytical expression for $\alpha_\nu^{\rm eff}$ is given. This expression integrally depends on the two-point correlation function of the turbulent velocity field.

Keywords. turbulence, molecular processes, radio lines: ISM, radio lines: stars, masers: circumstellar matter

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In a turbulent layer the effective absorption coefficient $\alpha_\nu^{\rm eff}$ does not coincide with the mean absorption coefficient $\alpha_\nu^{(0)} \equiv \langle \alpha_\nu \rangle$. The transition $\alpha_\nu^{\rm eff} \to \alpha_\nu^{(0)}$ takes place if the optical depth at the line center τ_1 of correlation length R_0 tends to zero (the case of short-correlated turbulence), or if the characteristic thermal velocity $u_{\rm Th}$ is equal to or greater than characteristic turbulent velocity u_k (parameter $\xi = u_{\rm Th}/u_k \geqslant 1$).

It is found that $\alpha_\nu^{\rm eff} \leqslant \alpha_\nu^{(0)}$. Statistically, a turbulent layer with a finite correlation length is more transparent than a layer of short-correlated turbulence for all the frequencies of a spectral line. It can be estimated even from the simple example of the averaging of intensities corresponding to two realizations with optical lengths $\tau_\nu^{(0)} + \tau'_\nu$ and $\tau_\nu^{(0)} - \tau'_\nu$.

The averaged spectral line intensity $I_\nu^{(0)}$ is narrower than that described by the radiative transfer equation with an averaged extinction factor $\alpha_\nu^{(0)}$ (see Fig. 1). This means that turbulence with a finite correlation length diminishes the equivalent width of an absorption line as compared with short-correlated turbulence.

The distortion of the shape increases with increasing optical depth τ_0 of the turbulent layer at the line center. An increase in the parameter τ_1 and decrease in ξ also give rise to an increase in the distortion.

If the turbulent layer contains many ($\tau_0/\tau_1 \geqslant 1$) turbulent cells (rotating curls), then the effects of finite correlation length turbulence disappear. If the number of turbulent

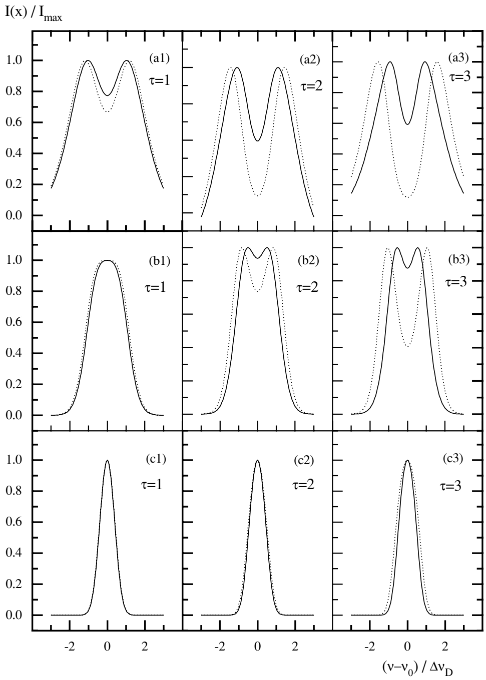

Figure 1. Normalized shape $I_\nu(x)/I_{\nu\,\mathrm{max}}$ of the maser line with a Gaussian initial profile after the passage of a turbulent cloud (solid lines). The dotted lines correspond to the usual model of short-correlated turbulence. The cases (an), (bn) and (cn) represent $\Delta\nu_D/\Delta\nu_S = 0.5$, 1 and 2, respectively; $\Delta\nu_D$ and $\Delta\nu_S$ are the Doppler widths in the cloud and maser source. The optical depth of the cloud τ at the line center takes the values $1, 2$ and 3. Turbulence correlation length R_0 is taken to be one-fifth of the layer's depth, and parameter $\xi = u_{\mathrm{Th}}/u_k = 0.1$; u_{Th} and u_k-values are characteristic thermal and turbulent velocities in the cloud.

cells is not large, but $u_{Th} \geqslant u_k$, the finite correlation length effects also tends to disappear. In the first case the situation looks like a cloud of many chaotically moving absorbing atoms. The second case corresponds to a very small shift of the center of the α_ν − profile for either correlated or uncorrelated turbulence.

These effects were mentioned earlier in various papers as the results of purely numerical calculations (see, for example, Gail *et al.* 1974, Levshakov *et al.* 1997, Magnan *et al.* 1976), without a simple physical explanation. The simple analytical formulae derived in our paper (Silant'ev *et al.* 2006) allow us to understand the expected effects of turbulence without complicated numerical calculations.

In our paper (Silant'ev *et al.* 2006) we discuss the problem of how to obtain the main turbulent parameters from the analysis of the shape of an absorption line or maser line propagating through a turbulent interstellar cloud. It should be emphasized that in the case of finite correlation length turbulence one can estimate separately the characteristic thermal velocity u_{Th} and turbulent velocity u_k. This is impossible if one uses the short-correlated model of turbulence.

As an illustrative example, we have estimated the possible parameters of the H_2O maser source in S252A (Berulis *et al.* 1996). We hope that the obtained analytical formulae will be useful for a detailed explanation of various observational data.

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