Divide throughout by $2R \cos A \cos B \cos C$,

$$\therefore \quad \tan A + \tan B + \tan C = \tan A \tan B \tan C,$$

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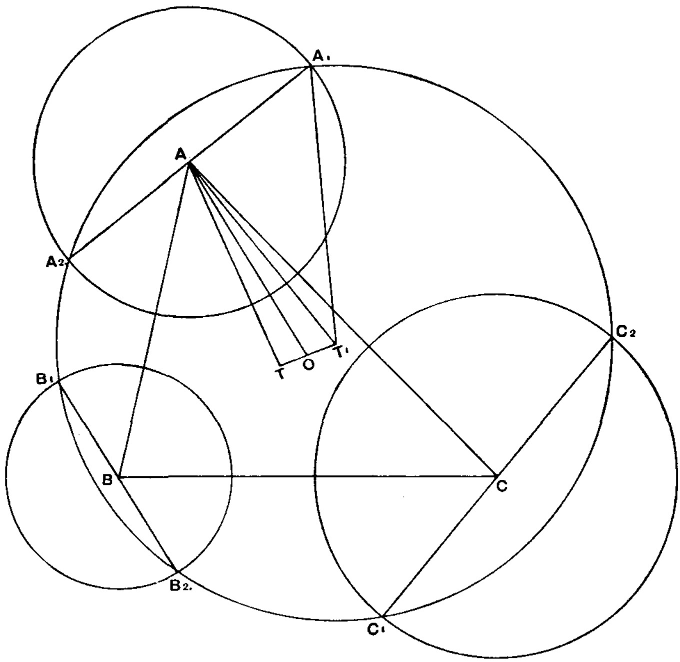
To draw a circle which cuts three circles at the ends of the diameters of these circles.—Let the three circles have centres, A, B, C, and let their radical centre be T, and the



circumcentre of $\triangle ABC$ be O. Join TO and produce it to T_1, so that $T_1O = TO$. T_1 is the centre of the required circle. Join T_1A and T_1C, and draw diameters A_1AA_2, and C_1CC_2 perpendicular to T_1A and T_1C. ⊙ with centre T_1 and radius T_1A_2 passes through A_1.

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$$A_2T_1^2 = AT_1^2 + AA_2^2 = AT_1^2 + AT^2 - (AT^2 - AA_2^2)$$
$$= 2OA^2 + 2OT^2 - (CT^2 - CC_1^2)$$
$$(T \text{ is radical centre}$$
$$\therefore\ AT^2 - AA_2^2 = CT^2 - CC_1^2)$$
$$= 2OC^2 + 2OT^2 - (CT^2 - CC_1^2)$$
$$= CT_1^2 + CT^3 - CT^2 + CC_1^2$$
$$= CT_1^2 + CC_1^2$$
$$= C_1T_1^2.$$

$\therefore$ ⊙ with centre T_1 and radius A_2T_1 passes through C_1 and hence through C_2.

Similarly, it will pass through the ends of the diameter perpendicular to T_1B.

If ρ be the radius of radical circle, and R of circumcircle of $\triangle ABC$,

$$A_2T_1^2 = AT_1^2 + AA_2^2, \text{ and } \rho^2 = AT^2 - AA_2^2$$
$$\therefore\quad A_2T_1^2 + \rho^2 = AT^2 + AT_1^2 = 2OA^2 + 2OT^2$$
$$\therefore\quad A_2T_1^2 = 2R^2 + 2OT^2 - \rho^2.$$

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Discrimination of the Roots of a Cubic Equation by Elementary Algebra.—The following note shows how the conditions for the reality or equality of the roots of a cubic can be obtained from the similar conditions for a quadratic. The method does not involve the use of the calculus or the properties of turning points, nor of the imaginary cube roots of unity.

Suppose the general cubic equation has been reduced as usual to the form

$$x^3 + px + q = 0. \quad \text{.........................} \quad (1)$$

It is certain that this has at least one real root, α, say: reduce all the roots of the equation by α and we get

$$(\xi + \alpha)^3 + p(\xi + \alpha) + q = 0,$$
$$\text{or,}\quad (\alpha^3 + p\alpha + q) + (\xi^3 + 3\alpha\xi^2 + \overline{3\alpha^2 + p}\,\xi) = 0,$$
$$\text{or,}\quad \xi(\xi^2 + 3\alpha\xi + \overline{3\alpha^2 + p}) = 0, \quad \text{............} \quad (2)$$

so that the roots of (1) are α, $\alpha + \xi_1$, $\alpha + \xi_2$, where ξ_1, ξ_2 are the roots of

$$\xi^2 + 3\alpha\xi + \overline{3\alpha^2 + p} = 0. \quad \text{............} \quad (3)$$