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ON THE ORDER OF ARC-STABILIZERS IN ARC-TRANSITIVE GRAPHS

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Abstract

Let p be a prime. We say that a transitive action of a group L on a set Ω is *p-sub-regular* if there exist $x, y \in \Omega$ such that $\langle L_x, L_y\rangle = L$ and $L_x^Y \cong \mathbb{Z}_p$, where $Y = y^{L_x}$ is the orbit of y under L_x. Our main result is that if Γ is a G-arc-transitive graph and the permutation group induced by the action of G_v on $\Gamma(v)$ is p-sub-regular, then the order of a G-arc-stabilizer is equal to p^{s-1} where $s \leq 7$, $s \neq 6$, and moreover, if $p = 2$, then $s \leq 5$. This generalizes a classical result of Tutte on cubic arc-transitive graphs as well as some more recent results. We also give a characterization of p-sub-regular actions in terms of arc-regular actions on digraphs and discuss some interesting examples of small degree.

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1. Introduction

Throughout this paper, all graphs considered will be finite, simple and connected. Let Γ be such a graph and let $G \leq \mathrm{Aut}(\Gamma)$. Then Γ is said to be *G-arc-transitive (G-arc-regular)* if G acts transitively (regularly) on arcs of Γ. The study of arc-transitive graphs was initiated by Tutte [16]. His classical result is that if Γ is a cubic G-arc-transitive graph, then G acts regularly on s-arcs for some $s \leq 5$ and the order of the arc-stabilizer G_{uv} is 2^{s-1}.

Since then, arc-transitive graphs have been widely studied. It is well known that the order of an arc-stabilizer is unbounded for arc-transitive graphs in general. Nevertheless, some results can be obtained by considering the *local action* of an arc-transitive group, which we now define.

If G is a group acting on a set and Θ is a union of orbits under this action, then G^{Θ} will denote the permutation group induced by the action of G on Θ. Let Γ be a G-arc-transitive graph, let v be a vertex of Γ and let $\Gamma(v)$ denote the neighborhood of v in Γ. We call $G_v^{\Gamma(v)}$ the *local action of* (Γ, G). Moreover, if P is a permutation group property and $G_v^{\Gamma(v)}$ has property P, then we say that (Γ, G) is a *locally P pair*.

Let k denote the valency of Γ. Note that, since Γ is G-arc-transitive, $G_v^{\Gamma(v)}$ is a transitive subgroup of the symmetric group S_k and, up to equivalence, does not depend on the choice of v. Sami [10] studied locally D_k pairs (Γ, G) where D_k denotes the dihedral group of order $2k$ acting naturally on a set of cardinality k. In particular, he showed that, if k is odd, the order of a G-arc-stabilizer divides 2^4, generalizing Tutte's result. In this paper, we further extend this result by obtaining similar bounds for a wider class of local actions, which we now define.

DEFINITION 1.1. Let p be a prime. We say that a transitive action of a group L on a set Ω is *p-sub-regular* if there exist $x, y \in \Omega$ such that $\langle L_x, L_y\rangle = L$ and $L_x^Y \cong \mathbb{Z}_p$, where $Y = y^{L_x}$ is the orbit of y under L_x.

One can easily see that the natural action of D_k is 2-sub-regular for every odd integer k. A characterization of p-sub-regular actions in terms of half-arc-transitive actions on graphs will be given in Section 3, and several families of p-sub-regular actions will be described. With this definition, we can now state our main result.

THEOREM 1.2. *Let (Γ, G) be a locally p-sub-regular pair and let uv be an arc of Γ. Then the order of the arc-stabilizer G_{uv} is equal to p^{s-1} where $s \le 7$, $s \ne 6$. Moreover, if $p = 2$, then $s \le 5$.*

Since dihedral groups of odd degree are 2-sub-regular, this is a generalization of Sami's (and hence Tutte's) result. These results can be viewed as part of a more general problem. Call a transitive permutation groups L *restrictive* if there exists a constant $c(L)$ such that, for any locally L pair (Γ, G), the inequality $|G_{uv}| \le c(L)$ holds.

PROBLEM 1.3. Characterize restrictive permutation groups.

A lot of work has been done on this problem. A trivial observation is that regular groups are restrictive. The previously mentioned results of Tutte and Sami show that odd dihedral groups are restrictive. Gardiner [4, 5] showed that doubly primitive groups are restrictive. In the same vein, Weiss [18] conjectured that primitive groups are restrictive. The conjecture was recently verified in the 2-transitive case [12–15] but the general conjecture is still quite open. See [2, 7] for some recent results.

Besides purely theoretical benefits, obtaining such upper bounds on the order of arc-stabilizers for graphs of a certain family also yields an efficient way to enumerate the graphs of this family up to a certain order. See [1, 3, 9] for examples.

Apart from D_4, all transitive actions of degree at most 5 are either regular, 2-transitive or p-sub-regular, for some p. It is well known that the order of arc-stabilizers of locally D_4 pairs is unbounded. Hence, with respect to Problem 1.3, the smallest open degree is 6. As a starting point to solving Problem 1.3, it would therefore be interesting to classify transitive groups of degree 6 according to the existence of such a bounding constant.

Section 2 will be devoted to the proof of Theorem 1.2. In Section 3, we give a characterization of p-sub-regular actions. We also give a list of p-sub-regular actions that is exhaustive up to degree 26.

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2. Proof of Theorem 1.2

Our group theory notation is rather standard and follows Wielandt [19]. If x, g are elements of a group G, we will denote by $x^g = g^{-1}xg$ the conjugate of x by g. Similarly, if $X \subseteq G$, then X^g will denote $\{x^g \mid x \in X\}$. Finally, if G is a group acting on a set Ω and $x \in \Omega$, then x^G will denote the orbit of x under G.

Our notation and terminology with respect to graphs is also fairly standard. Even though our main interest lies in simple graphs, it will be convenient for us to view graphs in a slightly more general context of digraphs. A *digraph* $\Gamma = (V, A)$ consists of a finite set of vertices V and a set of arcs $A \subseteq (V \times V)\backslash\{(v, v) \mid v \in V\}$. For an arc (u, v) of a digraph Γ, we say that u and v are the vertices of (u, v). Also, we say that v is an *out-neighbor* of u and that u is an *in-neighbor* of v. The symbols $\Gamma^+(v)$, $\Gamma^-(v)$ and $\Gamma(v)$ denote the set of out-neighbors of v, the set of in-neighbors of v and the union $\Gamma^+(v) \cup \Gamma^-(v)$, respectively. We call $|\Gamma^+(v)|$ and $|\Gamma^-(v)|$ the *out-degree* and the *in-degree* of v, respectively. If the out-degree (in-degree) is constant for all $v \in V$, we will call it the *out-valency* (*in-valency*) of Γ. If the in-valency and the out-valency are equal, we call it simply the *valency*.

If for all $u, v \in V$, we find that $(u, v) \in A$ whenever $(v, u) \in A$, we say that Γ is a *graph*. An *edge* of a graph is an unordered pair $\{u, v\}$ (also denoted by uv) such that (u, v) is an arc of the graph.

An *n-arc* $U = (u_0, \ldots, u_n)$ is a sequence of $n + 1$ vertices of Γ such that $(u_i, u_{i+1}) \in A$ for $0 \leq i \leq n - 1$ and $u_i \neq u_{i+2}$ for $0 \leq i \leq n - 2$. We will call u_0 the *start-vertex* of U and u_n the *end-vertex* of U. Similarly, if $n \geq 1$, then (u_0, u_1) will be called the *starting arc* of U and (u_{n-1}, u_n) will be called the *ending arc* of U. A subsequence of U of the form $(u_i, u_{i+1}, \ldots, u_{j-1}, u_j)$ for some $0 \leq i \leq j \leq n$ is called a *sub-$(j - i)$-arc* of U. An n-arc of the form $(u_1, \ldots, u_n, u)$ will be called a *successor* of U. Note that the graph Γ is connected if and only if, for any pair of vertices $x, y \in V$, there is an n-arc (for some n) with start-vertex x and end-vertex y.

We now introduce the notion of *nice* arcs.

DEFINITION 2.1. Let Γ be a graph, let $G \leq \mathrm{Aut}(\Gamma)$, let (x, y, z) be a 2-arc of Γ and let $n \geq 2$ be an integer. An n-arc of Γ will be called *nice (with respect to* (x, y, z) *and* G*)* if each of its sub-2-arcs is in the orbit of (x, z, y) under G. A 1-arc of Γ is called nice if it belongs in the G-orbit of the arc (x, y).

We will simply call an n-arc nice and omit the mention of (x, y, z) and G when ambiguity is unlikely. Note that every 1-arc is nice if and only if G is arc-transitive. Similarly, every n-arc of Γ with $n \geq 2$ is nice if and only if G acts transitively on 2-arcs of Γ. The 2-arc-transitive graphs have been extensively studied and, as we will see, the notion of nice n-arcs allows us to apply some of the techniques to the 1-arc-transitive case.

The following basic result will be used a few times. The proof is a simple exercise.

LEMMA 2.2. *Let G be a finite group acting transitively on a set Ω and let R be a transitive nonempty relation on Ω. If R is preserved by G, then it is an equivalence relation.*

Our next aim is to give sufficient conditions for G to act regularly on nice s-arcs for some $s \geq 2$.

LEMMA 2.3. *Let p be a prime, let (Γ, G) be a locally p-sub-regular pair, let z be a vertex of Γ and let $L = G_z^{\Gamma(z)}$. Let $x, y \in \Gamma(z)$ such that $L = \langle L_x, L_y \rangle$ and $L_x^Y \cong \mathbb{Z}_p$, where $Y = y^{L_x}$. Let s be the largest integer such that G acts transitively on s-arcs of Γ that are nice with respect to (x, z, y) and G. Let $(v_0, \ldots, v_s)$ be a nice s-arc of Γ and let G_i be the pointwise stabilizer of $(v_0, \ldots, v_{s-i})$. Then, for $0 \leq i \leq s-1$, it follows that $|G_i| = p^i$. In particular, $|G_0| = 1$ and G acts regularly on nice s-arcs.*

PROOF. Note that $|y^{L_x}| = p$ implies that a nice t-arc has exactly p nice successors. For $1 \leq i \leq s-1$, it follows that $|(v_{s-i+1})^{G_i}| = |G_i : G_{i-1}| = p$ and, by induction, that $|G_i| = p^i |G_0|$. It remains only to prove that $|G_0| = 1$.

For two arcs a and b, we write $a \rightsquigarrow b$ if there is a nice t-arc with starting arc a and ending arc b. This relation is transitive and is also preserved by G; hence, by Lemma 2.2, it is an equivalence relation. Let $a = (x, z)$ and let $E = [a]_{\rightsquigarrow}$ be the equivalence class of a under $\rightsquigarrow$. We will show that every arc of Γ is in E.

Let $(u, v) \in E$. By arc-transitivity, there exists a nice 2-arc (u, v, w). Clearly, $(v, w) \in E$ and then, because $\rightsquigarrow$ is an equivalence relation preserved by G, the class E is preserved setwise by both G_{uv} and G_{vw} and hence by G_v. In particular, all arcs incident to v are in E. Repeating this argument using transitivity of G and connectedness of Γ allows us to conclude that every arc of Γ is in E.

Now, let $g \in G_0$ and let A be a nice s-arc stabilized by g. If g acts transitively on the p nice successors of A then G acts transitively on nice $(s+1)$-arcs, which is a contradiction. Since $L_x^Y \cong \mathbb{Z}_p$, it follows that g stabilizes each nice successor of A. As we showed above, every arc of Γ is in $[(v_{s-1}, v_s)]_{\rightsquigarrow}$ and therefore $g = 1$. □

Our next goal is to bound s. First, we need the following result.

LEMMA 2.4 [6, Lemma 1]. *Let x and g be elements of G. Put $x_i = x^{g^i}$ for $i \in \mathbb{Z}$ and define $H_i = \langle x_1, \ldots, x_i \rangle$ for each $i \geq 1$. Let $H_0 = 1$. Suppose that x has prime order p and that there exist positive integers t and n such that:*

(1) *$\langle H_t, g \rangle = G$;*
(2) *$|H_i : H_{i-1}| = p$, for $1 \leq i \leq t$; and*
(3) *H_t contains no nonidentity normal subgroup of G and no nonidentity subgroup of the center of H_{t+n}.*

Then $t \leq 3n$ and $t \neq 3n-1$. Moreover, if $n = 2$, $p = 2$, and $t = 6$, then H_t contains a nonidentity normal subgroup of H_8.

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The lemma as stated above is due to Glauberman but the proof is essentially due to Sims [11, Proposition 2.6]. It draws heavily on ideas contained in two papers of Tutte [16, 17]. We are now ready to prove our main result.

PROOF OF THEOREM 1.2. Let (Γ, G) be a locally p-sub-regular pair. We need to show that the order of G_{uv} is equal to p^{s-1} where $s \leq 7$, $s \neq 6$, and that, if $p = 2$, then $s \leq 5$. Let z be a vertex of Γ and let $L = G_z^{\Gamma(z)}$. Since (Γ, G) is a locally p-sub-regular pair, there exist neighbors $x, y \in \Gamma(z)$ such that $L_x^Y \cong \mathbb{Z}_p$ and $L = L^\circ$, where $Y = y^{L_x}$ and $L^\circ = \langle L_x, L_y\rangle$. Since $L = L^\circ$ and L is transitive, $x^{L^\circ} = \Gamma(z)$. This shows that the hypothesis of Lemma 2.3 is satisfied.

Let s be the largest integer such that G acts transitively on s-arcs of Γ that are nice with respect to (x, z, y) and G. Let $(v_0, \ldots, v_s)$ be a nice s-arc of Γ and let G_i be the pointwise stabilizer of $(v_0, \ldots, v_{s-i})$. By Lemma 2.3, we know that $|G_i| = p^i$ for $0 \leq i \leq s-1$. In particular, $|G_1| = p$. Let x_0 be a generator of G_1, let $g \in G$ be such that $(v_1, \ldots, v_s)^g = (v_0, \ldots, v_{s-1})$ and let $x_i = x_0^{g^i}$. Finally, let $H_i = \langle x_0, \ldots, x_{i-1}\rangle$ and let $H_0 = 1$. The rest of the proof is split into three steps.

STEP 1. We will show that $G_i = H_i$ for $0 \leq i \leq s$ by induction on i. It is clearly true for $i = 0$. Let $1 \leq i \leq s$ and suppose that the statement is true for $i-1$. This implies that $H_i = \langle H_{i-1}, x_{i-1}\rangle = \langle G_{i-1}, x_{i-1}\rangle$. It is easy to check that $\langle G_{i-1}, x_{i-1}\rangle \subseteq G_i$. Note that x_0 does not fix v_s, hence $x_{i-1} = x_0^{g^{i-1}}$ does not fix $v_s^{g^{i-1}} = v_{s-i+1}$, therefore $x_{i-1} \notin G_{i-1}$. If $i < s$, we have $|G_i : G_{i-1}| = p$ and hence $G_i = \langle G_{i-1}, x_{i-1}\rangle = H_i$. If $i = s$, let $v_{-1} = v_0^g$. Clearly, (v_{-1}, v_0, v_1) is a nice 2-arc. Note that, by the Frattini argument, the fact that $L^\circ = L$ implies that $G_{v_0} = \langle G_{v_0v_1}, G_{v_{-1}v_0}\rangle$. It follows that

$$G_s = G_{v_0} = \langle G_{v_0v_1}, G_{v_{-1}v_0}\rangle = \langle G_{s-1}, (G_{s-1})^g\rangle = \langle H_{s-1}, (H_{s-1})^g\rangle = H_s.$$

This completes Step 1.

STEP 2. We will show that H_{s+1} acts transitively on edges of Γ and that $G = \langle H_{s+1}, g\rangle$. By Step 1, we know that $H_{s+1} = \langle H_s, (H_s)^g\rangle = \langle G_s, (G_s)^g\rangle = \langle G_{v_0}, G_{v_{-1}}\rangle$. Since Γ is connected and G-arc-transitive, this shows that H_{s+1} acts transitively on the edges of Γ. Moreover, since $v_{-1} = v_0^g$, this also shows that $G = \langle H_{s+1}, g\rangle$. This completes Step 2.

STEP 3. Note that G acts transitively on edges of Γ and, as was shown in Step 2, so does H_{s+1}. Since $H_{s-1} = G_{v_0v_1}$ is a G-arc-stabilizer, it follows that H_{s-1} contains no nonidentity normal subgroup of G and no nonidentity normal subgroup of H_{s+1}. This allows us to apply Lemma 2.4 (with $n = 2$ and $t = s-1$) to conclude that $s \leq 7$ and $s \neq 6$ and, if $p = 2$, then $s \leq 5$. This concludes the proof of Theorem 1.2. □

3. p-sub-regular actions

We will now study p-sub-regular actions in more detail. We first recall the definition. Let p be a prime. We say that a transitive action of a group L on a set Ω is *p-sub-regular* if there exist $x, y \in \Omega$ such that $L = \langle L_x, L_y \rangle$ (Condition 1) and $L_x^Y \cong \mathbb{Z}_p$, where $Y = y^{L_x}$ (Condition 2). We now need a few more definitions which will be needed to characterize p-sub-regular actions.

DEFINITION 3.1. Let $\Gamma = (V, A)$ be a digraph. For n an even natural number, an *alternating n-arc* $U = (u_0, \ldots, u_n)$ is a sequence of $n + 1$ vertices of Γ such that, for every even integer i with $0 \leq i \leq n - 2$, both (u_i, u_{i+1}) and (u_{i+2}, u_{i+1}) are in A. Γ will be called *alternating-connected* if, for any pair of vertices $x, y \in V$, there is an alternating n-arc with start-vertex x and end-vertex y.

It is worth mentioning that alternating-connected digraphs are precisely the alter-complete digraphs of alter-exponent 1, as defined in [8]. We now characterize p-sub-regular actions as groups of automorphisms of certain digraphs.

THEOREM 3.2. *Let p be a prime and let L be a group acting on a set Ω. Then this action is p-sub-regular if and only if there exists an L-arc-regular and alternating-connected digraph $\Gamma = (\Omega, A)$ of out-valency p.*

PROOF. ($\Leftarrow$) Let $\Gamma = (\Omega, A)$ be an L-arc-regular and alternating-connected digraph of out-valency p. Clearly, L is transitive on Ω. Let (x, y) be an arc of Γ. It is not hard to see that the set of end-vertices of alternating n-arcs with start-vertex x is closed under the natural action of $\langle L_x, L_y \rangle$. Since Γ is alternating-connected, this implies that $\langle L_x, L_y \rangle$ acts transitively on Ω, and Condition 1 follows. Condition 2 follows from the assumption that Γ is L-arc-regular and has out-valency p.

($\Rightarrow$) Let L act p-sub-regularly as witnessed by $x, y \in \Omega$, and let $\Gamma = (\Omega, (x, y)^L)$. By definition, Γ is L-arc-transitive. By Condition 2, Γ has out-valency p. Consider the set X of end-vertices of alternating n-arcs with start-vertex x. Clearly, $x \in X$, and X is closed under L_x. If $(x, x_1, \ldots, x_n)$ is an alternating n-arc and $g \in L_y$, then $(x, y, x^g, (x_1)^g, \ldots, (x_n)^g)$ is an alternating $(n + 2)$-arc, and hence X is closed under L_y. By Condition 1, X is closed under L. Since L is transitive on Ω, this implies that $X = \Omega$ and Γ is alternating-connected. Since Γ is L-arc-transitive, this implies that Γ is connected.

We now show that $L_{xy} = 1$. Note that Γ is connected hence there exists a sequence $(V_0, V_1, \ldots, V_n)$ of subsets of Ω such that $V_0 = \{x\}$, $V_n = \Omega$ and $V_{i+1} = V_i \cup \Gamma^+(v_i)$ for some $v_i \in V_i$. Let L_i be the pointwise stabilizer of V_i. By Condition 2, $|L_i : L_{i+1}|$ divides p. Since $L_n = 1$, it follows that $L_0 = L_x$ is a p-group. Moreover, we know that $|L_{xy}| = |L_{xy}^{\Gamma^-(y)}||\bigcap_{v \in \Gamma^-(y)} L_{vy}|$ and, since p does not divide $|L_{xy}^{\Gamma^-(y)}|$, we conclude that $L_{xy}^{\Gamma^-(y)} = 1$. This, together with Condition 2, implies that L_{xy} fixes X pointwise. Since $X = \Omega$ we conclude that $L_{xy} = 1$. □

Hence, if L acts p-sub-regularly on Ω as witnessed by $x, y \in \Omega$, then $L_{xy} = 1$. This is the motivation for calling such actions p-sub-regular. It also suggests that p-sub-regular actions are, in some sense, close to being Frobenius. The following examples will show that many p-sub-regular actions of small degree are in fact Frobenius.

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EXAMPLES. Our first four infinite families of examples are all Frobenius groups. Hence, they can be represented as semidirect products $G = K \rtimes H$ where the action under consideration is that of G on K where K acts regularly by multiplication and H acts by conjugation.

(1) $\mathbb{Z}_n \rtimes \mathbb{Z}_2 = \langle a, b \mid a^2 = b^n = (ab)^2 = 1\rangle \cong \mathrm{D}_n$, for $n \geq 3$ odd. These are dihedral groups. Here, $p = 2$.
(2) $\mathbb{Z}_q \rtimes \mathbb{Z}_p = \langle a, b \mid a^p = b^q = a^{-1}bab^m = 1\rangle$, for odd primes p and q with $p \mid (q-1)$ and m an element of order p in $\mathbb{Z}_q^*$. These are subgroups of affine transformations of the field of order q.
(3) $(\mathbb{Z}_2)^n \rtimes \mathbb{Z}_p$, for $n \geq 2$ and $p = 2^n - 1$ a prime. This is the group of affine transformations of the field of order 2^n.
(4) $(\mathbb{Z}_n)^{p-1} \rtimes \mathbb{Z}_p = \langle b_1, \ldots, b_{p-1}\rangle \rtimes \langle a\rangle$ where $(b_i)^a = b_{i+1}$ for $1 \leq i \leq p-2$ and $(b_{p-1})^a = (b_1 \cdots b_{p-1})^{-1}$, for p an odd prime and $n \geq 2$ not divisible by 3.

The next two infinite families are closely related to each other.

(1) $SL(2, p)$ acting on the $p^2 - 1$ nonzero vectors in $(\mathbb{Z}_p)^2$, for p an odd prime. This is the usual action of matrices acting on vectors by multiplication.
(2) $PSL(2, p)$ acting on $(p^2 - 1)/2$ points, for p an odd prime. Actions in this family are induced from actions in the previous family by identifying opposite pairs of matrices and opposite pairs of vectors.

For $p > 3$, the groups in the above two families are not Frobenius. There are also some nice examples of small degree that do not fall in any of the above families, such as A_5 acting on the 20 vertices of the dodecahedron. This action is obviously not Frobenius. It is also imprimitive, as opposite vertices of the dodecahedron form a block system.

An exhaustive computer search through a library of transitive group actions of small degree reveals that, apart from A_5 acting on the vertices of the dodecahedron, all p-sub-regular actions of degree at most 26 appear in at least one of the six infinite families listed above. Some of theses actions are primitive and hence part of our work can be viewed as an effort towards the Weiss conjecture. On the other hand, the imprimitive actions also need to be considered in view of the more general Problem 1.3. From the perspective of this problem, most of the past efforts have been concentrating on the locally primitive graphs, thus working in the setting of the Weiss conjecture. We hope that our results will spur further investigation of arc-stabilizers in locally imprimitive graphs, and perhaps shed new light on the problem of bounding the order of arc-stabilizers in arc-transitive graphs in general.

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