PART III.

Spherically-Symmetric Motions in Stellar Atmospheres.

C. - Non-Catastrophic Mass-Loss from Stars.

Summary Introduction.

A. DEUTSCH

Mt. Wilson and Palomar Observatories

1. - Introduction.

Gas can be observed to flow from a wide variety of stars into the interstellar medium, and to do so in a fascinating variety of ways. In some flows, the temperature exceeds a million degrees; in others, it cannot be distinguished from zero. Flow velocities range from 5 000 km/s down to one or two km/s at the limit of detection. The particle densities in some flows are at least 10^9 times larger than in others. The rates of mass-loss span the range from zero to probably more than 10^{26} g/s.

The gas dynamics of most of these flows are still but poorly understood. That the flows must occur widely throughout the galaxy, however, is clear not only from the observation, but as well from our understanding of the laws of stellar structure. These insure the impossibility of any hydrostatic equilibrium in a star more massive than about 1.2 solar masses, once it has exhausted its thermonuclear energy resources. And the time scale for this exhaustion is, for all massive stars, much less than the age of the galaxy. With a recent model of the galaxy, SCHMIDT (1959) found that for every two grams now in the stars, nearly one gram has been ejected from past generations of stars, whose relics are now the white dwarfs. Moreover, the cosmic abundances of the various elements appear to substantiate the view that a large fraction of the matter in the galaxy has already been ejected from stars, in the interiors of which it was subjected to processes of nucleosynthesis (BURBIDGE and BURBIDGE, 1958).

In this review we shall be principally concerned with the recently-discovered ejection processes that occur in the late-type giants and supergiants. Unlike the spectacular mass-loss phenomena which occur in the planetary nebulae, the novae, and the supernovae, these flows are relatively unobtrusive. Despite their comparative mildness, they probably represent the process chiefly responsible for transferring matter from aging stars back to the interstellar medium. Moreover, conditions in these flows appear to be nearly steady, and this should facilitate their theoretical description. Flows of this kind, in which the gas is cold and the velocities of the order of ten km/s, can be spectroscopically detected in most or all of the stars which lie in the hatched region of the Hertzsprung-Russell diagram of Fig. 1.

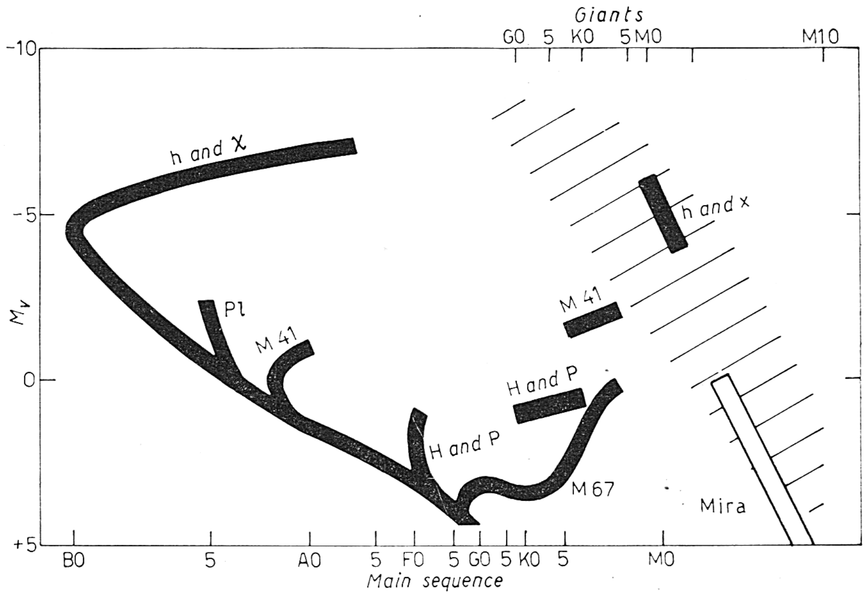

Fig. 1. – Composite HR diagram for open clusters, adapted from SANDAGE (1957). Field stars in the hatched area show circumstellar absorption lines which indicate a slow, cold outflow.

In Section **2**, we shall survey the observational evidence for these cold outflows, and in Section **3** we shall review the sparse information available on the thermodynamic variables and their gradients within the flows from the two best-studied stars. In Section **4** we shall then examine the theory of stationary, spherically-symmetric, laminar gas flows under gravity alone, in order to assess what additional theoretical work is needed for an understanding of the observations.

However, before turning to these subjects, we shall look briefly at the other varieties of mass-flows observed in stars. Table I is an attempt to summarize the observations in a form suitable for an orientation to the gas-dynamics of the phenomenon. In each of the flows that are tabulated, the mean-free-paths are likely to be short compared with the scale of the flow, and a hydrodynamical description will therefore be valid for most purposes.

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Within any group of objects, like the Be stars, for example, some of the tabulated quantities will differ by factors of three or four from one star to another. These differences override the uncertainties in the values chosen to characterize each class of objects. The second column to the sixth give values

TABLE I. – *Observed properties of outflows from stars* (*R is the radial distance at which the outflow is observed*).

Kind of star	R/R^*	Flow velocity (km/s)	Thermal Velocity (Km/s) [1]	Escape velocity (km/s)	Particle density (cm^{-3})	Mass loss ($M_\odot$/yr)	References
Sun	200	500 18	100 30	45 45	100 30	$6\cdot10^{-13}$ $4\cdot10^{-15}$	PARKER, 1960 *a*, *b* CHAMBERLAIN, 1961
Close binaries	2	100	25	400	10^{11}	10^{-5}	SAHADE, 1960 STRUVE, 1958
Be stars	3	50	25	250	10^{11}	10^{-7}	BOYARCHUK, 1958 SOBOLEV, 1960
WR stars	5	1 200	25	500	10^{11}	10^{-5}	UNDERHILL, 1959, 1960
P Cygni stars	1.3	100	20	800	10^{11}	10^{-5}	UNDERHILL, 1960, 1961 PAGEL, 1958
M Giants	100	10	4	15	10^{3}	10^{-9}	DEUTSCH, 1960
Planetary nebulae	10^{6}	20	20	0.3	10^{4}	(10^{-1})	ALLER, 1956 SEATON, 1960
Novae	1 to 10^{5}	2000 to 200	20	100 to 5	10^{10} to 10^{4}	(10^{-4})	GAPOSCHKIN, 1957
Supernovae	?	5000	?	?	?	(1)	GAPOSCHKIN, 1957

[1] *Ed. Note*: The temperatures on which these thermal velocities are based have not been determined by one single method, common to all entries. For details on any particular case, inquiry should be addressed to Deutsch.

that refer to the level that is observed in the outflow; typically, the level where the optical depth is of order unity in the spectrum lines produced in the flow. This probably represents a severe idealization; in some cases, it is probable that the physical variables change by several orders of magnitude over the range of distances R which contribute to the observed spectra.

The last three rows of the table refer to objects in which the mass-loss is catastrophic, or, at least, very far from steady. That is to say, the ejection process changes appreciably before the ejected matter may be considered to have mixed with the interstellar gas. For these catastrophic processes, the number in the last column gives the total amount of mass loss, in solar units. Considering all the flows together, and recalling that in some Be stars the expansion velocity may vanish or temporarily become negative, we find that the Mach number may clearly be either large, small, or comparable with unity. Similarly, the flow velocity may evidently be either large, small, or comparable with the local velocity of escape.

It is outside the scope of this paper to detail the analytical methods that have led to the results summarized in Table I. The references of column 8 will serve to start the interested reader on a study of the literature bearing on these questions.

2. – Circumstellar envelopes of M giants.

In this section we shall briefly review the evidence for cool, quasi-steady outflows of gas from the giants and supergiants in the hatched area of Fig. 1. A more detailed discussion has recently been given by DEUTSCH (1960). The phenomenon was first observed by ADAMS and MCCORMACK (1935) in several M-type supergiants. In the spectrum of *α Orionis* (M2 Ib), for instance, all strong zero-volt absorption lines were found to be double. Examples may be seen in the enlarged spectrogram of this star in Fig. 2.

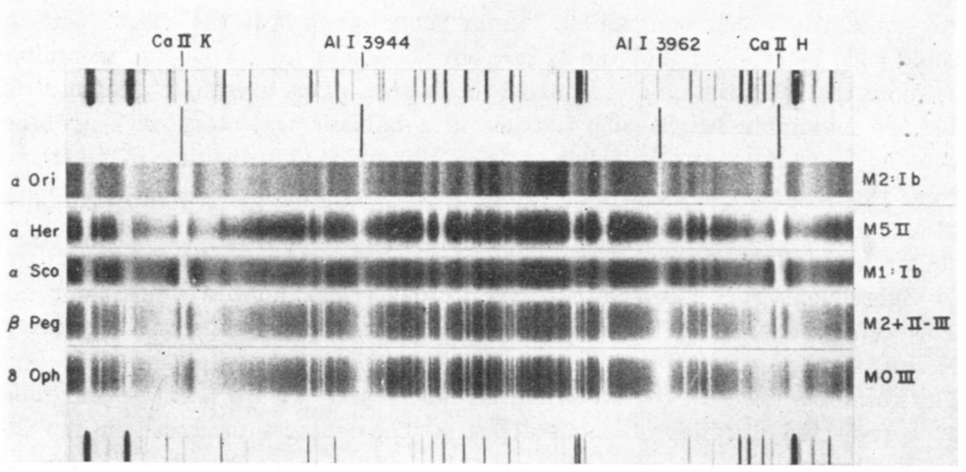

Fig. 2. – Enlargements from spectrograms of M stars. Original dispersion, 4.5 Å/mm. The resonance lines of Ca II show circumstellar components in all M giants and supergiants. When these lines are very strong, circumstellar resonance lines of Al I (and other elements) also appear.

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One component of these double lines is evidently formed in an expanding shell or envelope, where very low values prevail for the excitation temperature and pressure. This expanding envelope is in a quasi-steady state, for it has given no indication of changes in velocity or other characteristics over the 25 years it has been observed. The other component of the double lines must be attributed to the reversing layer, which also produces all the other relatively wide and shallow lines that characterize the spectrum. These « normal » lines indicate an excitation temperature which lies much closer to the effective temperature of the star. Their radial velocity shows a slow, irregular variation, with a range of about 8 km/s. Relative to the mean velocity of the reversing layer, the circumstellar (CS) lines yield an expansion velocity $|\Delta A|$ of 8 km/s.

These spectroscopic observation at once suggest that we have to do with a star which suffers small and irregular pulsations, and which is surrounded by an expanding low-temperature envelope. But the escape velocity at the surface of α *Orionis* is probably of the order of 100 km/s, and in any case is far greater than the 8 km/s expansion that is observed. It was therefore supposed that the gas must fall back into the star sometime after it is observed in slow ascent. Moreover, since no cases were known where the envelope is in contraction, one also had to suppose that on its return to the star the gas is in an unobservable state of ionization.

However, it was then found by DEUTSCH (1956) that among the CS lines in the spectrum of α^1 *Herculis* (M5 II), with an expansion velocity of 10 km/s, there are many which can also be seen in the spectrum of its visual companion, α^2 *Herculis* (G0 II-III). Since no comparable zero-volt cores are known in other G stars, it was concluded that the circumstellar shell of this M star actually envelopes the visual companion. From the geometry of the visual pair, it could then be inferred that the M star envelope is at least 1000 a.u. in radius, or more than 350 times the radius of the M star. This exceeds by a factor of 10^4 the maximum height of a particle in a ballistic trajectory with a radial velocity of 10 km/s at the stellar surface. Moreover, at a distance of 1000 a.u. from the M star, the observed expansion velocity exceeds the local velocity of escape. It was therefore concluded that α^1 *Herculis* loses mass to the interstellar medium in the quasi-steady outflow that produces the CS spectrum.

Subsequent work has isolated several other visual binaries which show the α *Herculis* phenomenon: the lines produced in the CS envelope of the primary M star, superposed onto the spectrum of the earlier-type companion. Fig. 3(a) illustrates the effect in η *Geminorum*, an M3 II star with a G8 III companion at a projected distance of about 100 a.u. The features produced in the CS envelope are the sharp, deep absorption lines on the shortward (left) edge of the emission lines at Ca II H and K. Along the line of sight to the M star, these absorption lines indicate an expansion velocity of about 28 km/s.

Only a small number of visual binaries are suitable for observations of this

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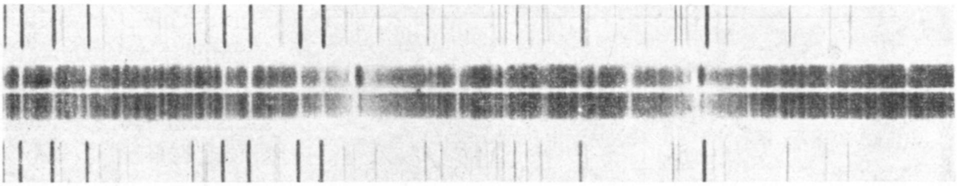
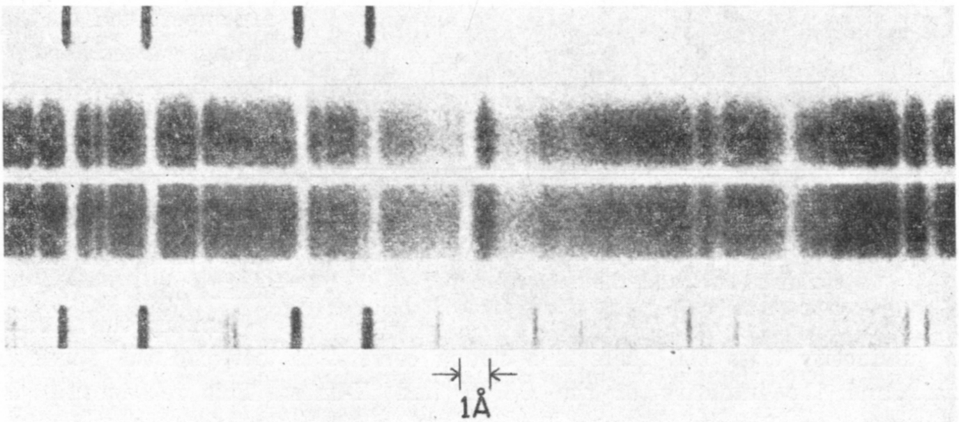
kind. But there are also a few spectroscopic binaries which can provide evidence on the dimensions of CS envelopes. Fig. 3(*b*) shows the motion of the CS *K*-line relative to the normal reversing-layer lines in the spectrum of one



a)



b)

Fig. 3. – *a*) The region of Ca II *H* and *K* in the spectra of η *Geminorum* (M3 II) and its visual companion (G8 III). *b*) The *K*-line in the spectrum of ***RR Ursae Minoris*** (M5 III); the observations were made at two different phases in the 750 day cycle of this spectroscopic binary. Original dispersion of both plates, 10 Å/mm.

such star. Actually, of course, it is the normal lines which exhibit a variable Doppler shift due to orbital motion; the stationarity of the CS *K*-line indicates that it must arise in an envelope which is large compared with the spectroscopic orbit.

Besides these observations of M giants and supergiants in binary systems, high dispersion spectrograms have been obtained for the detection of CS lines in more than a hundred single giant stars of late spectral type. This material indicates that CS lines due to cool expanding envelopes can be recognized in the spectra of all giants later than type M0. Normal (Class III) giants earlier than M0 never show CS features; but comparable lines have been found in a variety of supergiants with earlier types.

The *H* and *K* lines of Ca II are always the strongest of the CS lines, and often can be seen at a dispersion which is too low to reveal any of the others.

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Fig. 2 shows the region of H and K in five representative stars. It is important to note that only the sharp, deep absorption components at H and K can be attributed to the outflowing CS envelope. These features are normally superposed on emission lines, which arise in a chromosphere that itself appears to expand at about 40% of the speed of the envelope (WILSON, 1960). The emission lines, in turn, are superposed on the strong, winged absorption lines that originate in the reversing layer of the star. The spectra of M giants in binary systems confirm the validity of these attributions.

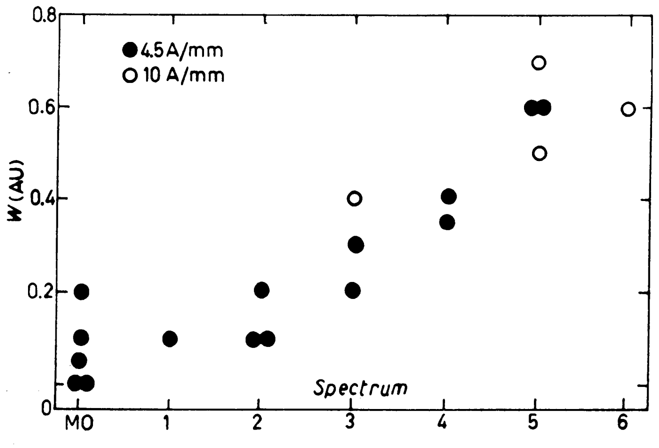

Fig. 4. – The correlation for giants with M_v fainter than -2.5 of the estimated strength W of the circumstellar K-line, and the spectral type.

Among the giants of luminosity class III, there is a strong correlation between the spectral type and the intensity of the CS H and K-lines. This relationship is illustrated in Fig. 4.

The expansion velocities also correlate with spectral type, as is shown in Fig. 5. On the assumption that most of the Ca in the envelope resides in the singly-ionized state, DEUTSCH (1960) has derived the rates of mass-loss which these observations imply. The surface density of Ca II was first obtained from the curve of growth. Since the profiles of the CS lines give no evidence for a velocity gradient in the envelope, the gas density was supposed to vary as r^{-2}. With the additional assumption

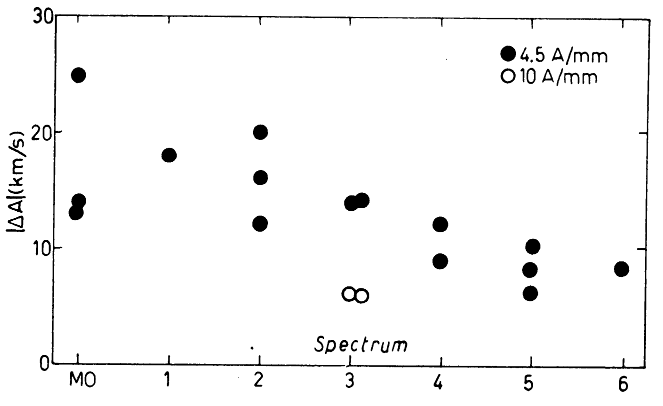

Fig. 5. – The correlation for M-type giants with M_v fainter than -2.5 of the expansion velocity $|\Delta A|$ from the circumstellar K-line, and the spectral type.

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of normal (solar) abundances for other atoms relative to Ca, the rate of mass-loss, $-\dot{M}$, could then be derived. At M1, the rate is $2\cdot10^{14}$ g/s; at M3, $2\cdot10^{16}$ g/s; at M5, $6\cdot10^{18}$ g/s. Near type M3, these results are impaired by an extreme sensitivity to the value that is chosen for the Doppler width, b, of the absorption coefficient. At its worst, this sensitivity, could cause the result for type M3 to be wrong by a factor of ten in either direction.

The correlation in Fig. 4 may indeed indicate that the cooler stars support more massive outflows, in accord with these estimates by DEUTSCH. But it may equally well be simply an effect of excitation. The latter explanation requires that a large proportion of the CS Ca be doubly ionized in the hotter M stars, and it invalidates Deutsch's calculations of $-\dot{M}$. The absence of CS lines in giants earlier than M0 would then not preclude the occurrence of mass-loss from these objects. In fact, the ultraviolet continuum of a K giant is probably sufficient for double ionization of virtually all the Ca that may be in the envelope.

If we admit this possibility, we must also admit that invisible mass-loss may occur from the G and K giants at rates comparable with the rate of visible mass-loss in the late M giants. Considerations of stellar evolution suggest that this must be true. A star, which evolves from A0 on the main sequence to M5, cannot remain a late M star long enough to lose the requisite amount of mass for transformation to a white dwarf. But its total lifetime on the red side of the Hertzsprung gap is amply long for the requisite mass-loss, if this proceeds uniformly at approximately the rate we observe in the late M giants.

3. – The spectra of two M-type supergiants.

Except for the observations we have just discussed of Ca II *H* and *K* and a few other zero-volt lines in the spectra of several score M giants, we have little information relating to the structure of the expanding envelopes around M giants. For two supergiants in which the CS spectrum is very well developed, more detailed studies have been made. These stars are *α Herculis* and *α Orionis*. Table II gives the values that have been adopted for the dimensions and other physical parameters of these stars. The distances and masses are not well determined, and could conceivably be wrong by factors of about two.

We shall first discuss the spectrum of *α Herculis*, and the inferences that have been drawn from it regarding the structure of the atmosphere and the CS envelope. The reversing layer produces wide, shallow absorption lines, in which the central intensity increases systematically with excitation potential for lines of the same width. SPITZER (1939) finds that the line profiles indicate

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microturbulence with a velocity spectrum that approximates a dispersion law. The fictitious damping constant γ, which gives the half-width of the line at half-intensity, is about 4 km/s.

TABLE II.

Spectral type	α *Herculis* M5 II	α *Orionis* M2 I*b*
Distance (pc)	150	210
Mean absolute magnitude, M_v	-2.4	-5.7
Mean bolometric magnitude, M_B	-5.9	-9
Effective temperature, T_E (°K)	2 700	3 150
Radius ($\odot = 1$)	580	1 010
Mass ($\odot = 1$)	15	20
Escape velocity, V_{es} (km/s)	99	87
Thermal velocity at T_E, V_{th} (km/s)	7.0	7.5
Flow velocity, $\bar{V}$ (km/s)	10	8

A number of emission lines occur, which are attributed to a chromosphere, or region of inverted temperature gradient. The strongest of these lines are at Ca II *H* and *K* (see Fig. 2); others lie in the wings of *K* and in the far ultraviolet (HERZBERG, 1948).

The CS envelope produces violet-shifted absorption cores in all strong lines arising from the ground level. In addition, CS cores occur in the strongest lines that arise from excited sub-levels within the ground term; two examples are Al I 3962, with an excitation potential of 0.01 eV (see Fig. 2), and Fe I 3856, with an excitation potential of 0.05 eV. The central intensities of the CS lines are indistinguishable from zero when they are fully resolved on the spectrograms.

A similar CS spectrum is seen in the visual companion; however, all the CS lines are weaker there than in the M star, and the lines from excited sub-levels are completely absent. The excitation temperature is then very low, but not zero, near the M star; and it is indistinguishable from zero in the outer parts of the envelope which are traversed by the line of sight to the G star. Neither the CS line profiles nor their radial velocities give any evidence for a velocity gradient in the envelope, except that *H* and *K*, which are the strongest CS lines, yield an expansion velocity significantly smaller (by 4 km/s) than that of 10 km/s which characterizes the other CS lines.

In his 1956 discussion of the CS spectrum, DEUTSCH found the surface density of Ca II to be 28 times that of Ca I along the line of sight to the G star. This is a surprisingly low degree of ionization for so rarefied a gas as that in the outer envelope. Indeed, on the basis of certain reasonable assumptions regarding the geometry of the system and the source of ionizing radiation, DEUTSCH found it impossible to account for so low an ionization,

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unless the gas were concentrated in clouds which fill the volume of the envelope with a packing fraction of only 10^{-7}.

On the basis of new information, there is now reason to believe that this awkward requirement can be significantly relaxed. However, it is still not clear that one can understand the ionization without postulating some degree of lumpiness in the outer envelope. The phenomenon may be related to the one which maintains condensations in the high chromospheres of the supergiant stars which have been studied at chromospheric eclipses (WILSON, 1960*b*).

In the spectrum of *α Orionis* one can see in a more exaggerated form all the same peculiarities that occur in *α Herculis*. SPITZER (1939) finds that the reversing-layer lines have the profiles of dispersion curves, with the parameter $\gamma = 8$ km/s. He also obtains from the spectrum a variety of conventionally defined « temperatures », the differences among which are taken to indicate very strong deviations from local thermodynamic equilibrium. These determinations range from an excitation temperature of 2100° for Fe I lines, to a kinetic temperature of 200000° for Fe atoms with $(\overline{v}^2)^{\frac{1}{2}} = 10$ km/s, corresponding to the observed dispersion profiles.

The CS spectrum of this star has recently been examined in detail by WEYMANN (1961). He adopts a model for the envelope in which the ionization temperature $T_i = 2560°$, which is just the boundary temperature for a gray atmosphere with $T_{\mathrm{eff}} = 3150°$. With a total surface density of $N = 1.2 \cdot 10^{22}$ atoms per cm² in the envelope, he finds that double ionization is everywhere negligible. Moreover, without any condensations to inhibit ionization, the model yields CS line strengths which are in reasonable accord with the observations for the lines of neutral Al, Na, Ti, and Cr, and for the lines of singly-ionized Ca, Sc, Ti, Sr, and Ba. However, the model does predicts CS lines which are too strong for Mn I and Fe I—a discrepancy which WEYMANN attributes to its neglect of an important contribution from the chromosphere to the ionization continuum of these atoms.

WEYMANN treats as a disposable parameter the radius r_0 of the inner boundary of the observed envelope. Its value depends sensitively on the value of T_{eff}. He finds that r_0 is about 11 times the stellar radius. At this height the flow velocity is still only 30% of the local escape velocity. The gas density is $8 \cdot 10^{-16}$ g/cm³; the electron density 300 cm^{-3}; and the kinetic temperature about 1000°. Collisional excitation in these conditions can then populate the fine-structure levels in agreement with the observations. The rate of mass-loss is $2.6 \cdot 10^{20}$ g/s, or about ten times the rate of thermonuclear conversion of H to He in this star. For $r > r_0$, the density falls as r^{-2}; the flow velocity V and the ionization remain constant. At large distances V also falls, as indicated by Ca II H and K. As in *α Herculis*, the expansion velocity from these very strong lines is significantly smaller than from all the other CS lines.

On this model, we do not observe any CS gas below r_0. The gas must therefore be too hot there to contribute to the resonance lines seen in the CS spectrum. It is not yet clear what limits must be placed on the temperature, however, to insure that this region of the model will not yield emission lines which have no counterpart in the real star.

4. – Spherically-symmetric flows in a gravitational field.

In order to assess the theoretical implications of the massive outflows observed in many stars, we shall now survey some of the properties of spherically-symmetric stationary flows in an inverse-square gravitational field. Several authors have discussed this problem from the point of view of coronal evaporation (Rubbra and Cowling, 1960; Chamberlain, 1960; de Jager, 1960). The first of these papers has also given brief consideration to flows in which electromagnetic forces are important. Wilson (1960*a*) has explored the possibility that the envelope is expelled by forces arising from radiation pressure in Lyman α. The theory is difficult because of the effects of non-coherent scattering; however, there can be no doubt that this mechanism requires an exceedingly high mean intensity of Lyman α within the envelope. Weymann (1961) has concluded that in α *Orionis* the requisite intensity of Lyman α would doubly ionize Ba to an extent inconsistent with the observed strength of the line Ba II 4554.

In the present context, it is appropriate to assume that the mean-free-path of an atom or ion is sufficiently short so that a hydrodynamical description is a valid one. We shall also assume that the only forces acting on the gas are those due to the pressure gradient and to gravity. The Eulerian equation of motion may then be written as

$$V\frac{\mathrm{d}V}{\mathrm{d}r} = -\frac{1}{\varrho}\frac{\mathrm{d}P}{\mathrm{d}r} - \frac{GM}{r^2}\,. \tag{1}$$

Here V, ϱ, and P are, respectively, the flow velocity, the density, and the gas pressure at distance r from the center of a star of mass M. We also have the continuity equation

$$4\pi r^2 \varrho V = -\dot{M} = \text{const}\,, \tag{2}$$

and the gas law (in the usual notation)

$$P = \frac{k}{\mu m_{\mathrm{H}}}\varrho T\,. \tag{3}$$

We shall suppose that the gas is monatomic, with a ratio of specific heats $c_p/c_v = \frac{5}{3}$. We shall approximate the actual flows with the polytropic law

$$P\varrho^{-\gamma} = a = \text{const}\,, \tag{4}$$

where γ may have any constant value in the range $0 \leqslant \gamma \leqslant \frac{5}{3}$. We are therefore not limited to the discussion of isentropic flows.

The integral of eq. (1) follows, using eq. (4), and is the equivalent of the Bernoulli equation. For $\gamma \neq 1$, it is

$$\frac{1}{2}V^2 - \frac{GM}{r} + \frac{P/\varrho}{1-1/\gamma} = K = \text{const}, \tag{5}$$

and for $\gamma = 1$,

$$\frac{1}{2}V^2 - \frac{GM}{r} + \frac{P}{\varrho}\ln P = K = \text{const}\,. \tag{6}$$

In recent years a number of authors have discussed these equations, and the associated equation for the transport of energy. BONDI (1952) has considered the problem in connection with the accretion of interstellar gas by continuous inflow to a star. MCCREA (1956) has generalized Bondi's results for the case where the inflow possesses a standing shock-wave. PARKER (1958, 1960*a*, 1960*b*) and CHAMBERLAIN (1960, 1961) have tried to represent the solar corona by different models of outflow based on the same equations. ROGERS (1956) and WEYMANN (1960) have attempted to apply them to the phenomena observed in the M-type supergiants. STANYUKOVICH (1959) has also studied these and similar equations in the context of various astrophysical processes.

In this paper we shall briefly examine some simple solutions for the outflow from a star of prescribed radius r_0 and mass M. If we arbitrarily assign the polytropic exponent γ, and the values at the base of the flow of the velocity, density, and temperature, we may then derive from the equations given above the formal solutions for $V(r)$, $\varrho(r)$, $T(r)$, and $P(r)$ But included in the manifold of formal solutions obtained in this way, there will be many which are inadmissible in the present context. We shall limit ourselves here to the discussion of solutions which are single-valued and positive real between the base of the flow and infinity, and in which the acceleration of the gas remains finite. The first of these restrictions is a necessary consequence of the assumption in eq. (2) that the flow describes a single polytrope from r_0 to ∞. The arbitrariness of this assumption should be kept in mind when we consider the consequences below.

It will be convenient for us to introduce the following notation. We let χ be the distance in units of the stellar radius; c the local velocity of sound for

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a monatomic gas; β the local Mach number of the flow; and α proportional to the ratio of gravitational potential energy to thermal energy at the base of the flow. We also let the local escape velocity and thermal velocity be V_{es} and V_{th}, respectively. The subscript zero will always denote the base of the flow. We then have the following relations:

$$
(7) \qquad \begin{cases} \chi = r/r_0\,, \qquad c^2 = \dfrac{5kT}{3\mu m_{\mathrm{H}}}\,, \qquad \beta = V/c\,, \\[2ex] \alpha = \dfrac{GM}{2r_0c_0^2} = \dfrac{9}{20}\,(V_{es}/V_{th})_0^2\,. \end{cases}
$$

For a star of given escape velocity $(V_{es})_0$, the specification of T_0 and V_0 is equivalent to the specification of α and β_0.

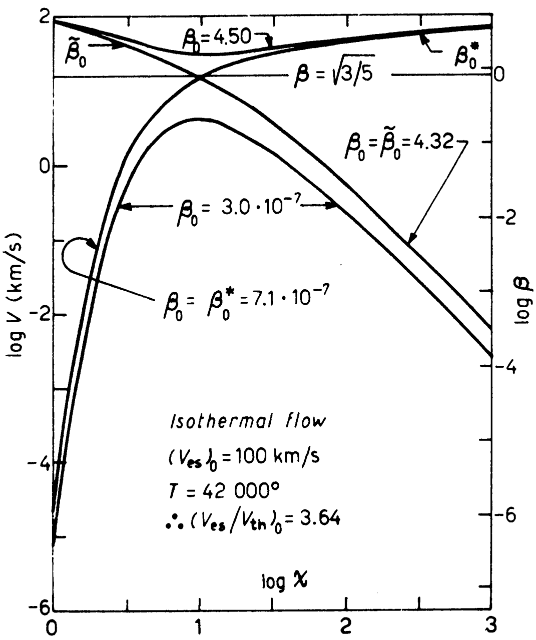

Fig. 6. – The flow velocity V as a function of $\chi = r/r_0$, in four representative isothermal flows. The subsonic flow and the decelerating transsonic flow go to zero velocity as $\chi \to \infty$.

A) *The isothermal case.* We shall first discuss the case of isothermal flow, $\gamma = 1$. When we solve for the local Mach number, we obtain the equation

$$
(8) \qquad \frac{1}{2}(\beta^2 - \beta_0^2) - \frac{3}{5}\ln(\beta/\beta_0) = \frac{6}{5}\ln\chi - 2\alpha(1 - 1/\chi)\,.
$$

The character of this solution depends on the values of α and β_0. When $\alpha \geqslant \frac{3}{5}$, the velocity profiles are like one of the examples in Fig. 6. For β_0 sufficiently small, β remains less than $\sqrt{\frac{3}{5}}$; it rises to a maximum at $\chi = \frac{5}{3}\alpha$, and at large distance vanishes as χ^{-2}. For β_0 sufficiently large, β remains greater than $\sqrt{\frac{3}{5}}$; it falls to a minimum at $\chi = \frac{5}{3}\alpha$, and at large distances increases as $(\ln\chi)^{\frac{1}{2}}$. The critical values of β_0 which delimit these two regimes (see Fig. 7) are the zeros of eq. (8) when $\chi = \frac{5}{3}\alpha$ and $\beta = \sqrt{\frac{3}{5}}$. Let us call these zeros β_0^* and $\tilde{\beta}_0 \geqslant \beta_0^*$. For $\beta_0^* < \beta_0 < \tilde{\beta}_0$, there is a discontinuity in the acceleration of the gas at the point where $\beta = \sqrt{\frac{3}{5}}$, and these solutions are inadmissible.

For $\beta_0 = \beta_0^*$, a singular solution exists in which β increases monotonically with χ; the flow is subsonic for small χ and supersonic for large χ, with β diverging logarithmically as $\chi \to \infty$. For $\beta_0 = \tilde{\beta}_0$, a second singular solution exists, in which β decreases monotonically; the flow is supersonic for small χ and subsonic for large χ, with β going to zero with χ^{-2}.

When $\alpha < \frac{3}{5}$, β_0 may take any positive value except $\sqrt{\frac{3}{5}}$. In these solutions, β decreases monotonically if $\beta_0 < \sqrt{\frac{3}{5}}$, and it increases monotonically if $\beta_0 > \sqrt{\frac{3}{5}}$. The asymptotic forms are the same as for the case where $\alpha \geqslant \frac{3}{5}$.

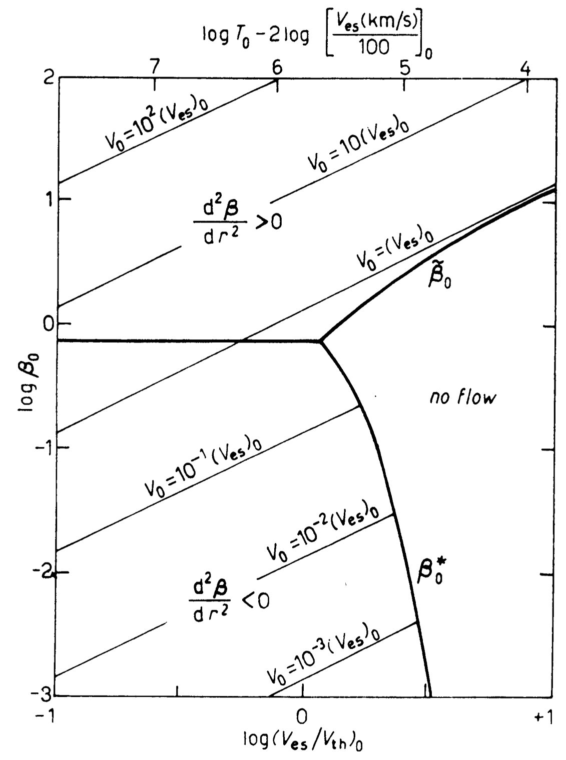

Fig. 7. – The regimes of isothermal flow. The scale at the top is drawn for $\mu = 1.38$.

If we seek to approximate the observed outflows from M giants by these isothermal solutions, it seems appropriate to impose additional boundary conditions upon them which will insure a fit with the interstellar medium as $\chi \to \infty$.

In particular, we shall require that $V \to 0$ and $P \to P_I$, whe the subscript I denotes the interstellar medium. These far boundary conditions may be satisfied only by the subsonic solutions, with $\beta_0 < \beta^*$; and by the decelerated critical solution, with $\beta_0 = \tilde{\beta}_0$. In either of these cases, it may be shown that to produce a mass-loss $(-\dot{M})$ we must have

$$(9) \qquad \alpha \geqslant \left| \frac{10kT_I(-\dot{M})}{\pi(5e/3)^{\frac{3}{2}} \cdot 3\mu m_{\rm H} r_0^2 (V_{es})_0^3 \varrho_I} \right| .$$

As representative of an M giant star in an H I region of the interstellar medium, we take $r_0 = 100\, R_\odot$, $(V_{es})_0 = 100$ km/s, $T_I = 200°$, $\varrho_I = 2 \cdot 10^{-25}$ g/cm³, $-\dot{M} > 10^{14}$ g/s. We then find that $\alpha > 175$; and, from eq. (7), $T < 1\,400$. Furthermore, we may show that for subsonic flows with $\alpha \gg 1$, we have

$$(10) \qquad \varrho_0 \geqslant \frac{3\mu m_{\rm H}(V_{es})_0^2 \varrho_I}{20kT_I} (5e/3)^{\frac{3}{2}} \alpha \exp\left| \frac{10}{3} \alpha \right| .$$

With $\alpha > 175$, this becomes $\varrho_0 > 10^{233}\,\mathrm{g/cm^3}$! This result reflects the circumstance, that, in a massive subsonic isothermal flow, the velocity must be extremely small near the base of the flow. It is clear that we cannot hope to represent the observed flows with such a model, regardless of any reasonable changes in the parameters of eq. (9) and (10) such as T_I, r_0, etc.

Let us now consider the decelerating critical flow. In this case, we have

$$\varrho_0 \geqslant \frac{3\mu m_{\mathrm{H}}(V_{\mathrm{es}})_0^2 \varrho_I}{10kT_I}(5e/3)^{\frac{3}{2}}\alpha^{\frac{1}{2}} \geqslant 6\cdot 10^{-20}\ \mathrm{g/cm^3}\,. \tag{11}$$

Thus, this flow cannot be ruled out on the grounds of too high a density near the star. However, it may be shown that, when $\alpha \gg 1$ in this flow,

$$V_0 = \sqrt{2}\,(V_{\mathrm{es}})_0\,. \tag{12}$$

In our example, therefore, the initial flow velocity would be 140 km/s, and this cannot be reconciled with the observations.

B) The adiabatic case. Putting $\gamma = \frac{5}{3}$ for the adiabatic flow of a monoatomic gas, we find that the solution may be written in the form

$$\chi = S^{-1}[(\beta/\beta_0)^{\frac{3}{2}} - 4\alpha/\beta_0^2 + (3/\beta_0^2)(\beta_0/\beta)^{\frac{1}{2}}]\,, \tag{13}$$

where

$$S = \frac{3 - 4\alpha}{\beta_0^2} + 1\,. \tag{14}$$

We again limit consideration to solutions which are single valued and positive real between $\chi = 1$ and ∞, and in which the acceleration of the gas remains finite. These restrictions require that we have $S \geqslant 0$. Fig. 8 shows the various flow regimes. No transsonic flows exist. When $S = 0$, the Mach

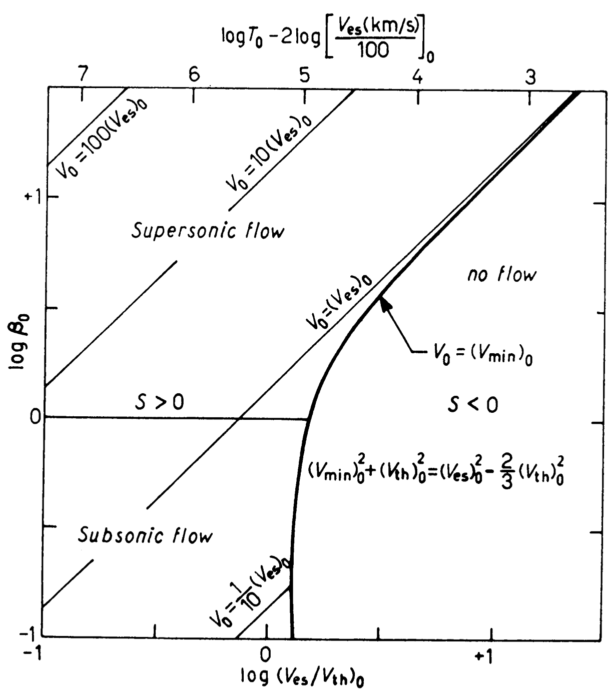

Fig. 8 – The regimes of adiabatic flow. The scale at the top is drawn for $\mu = 0.5$.

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number remains constant throughout the flow. If $S > 0$ and the flow is initially subsonic ($\beta_0 < 1$), then β decreases monotonically and the flow remains subsonic; if the flow is initially supersonic ($\beta_0 > 1$), then β increases monotonically, and the flow remains supersonic. If initially sonic ($\beta_0 = 1$), the flow remains sonic ($\beta \equiv 1$); this case can occur only when $S = 0$.

In flows with $S = 0$, T and V both vanish as $\chi \to \infty$. Along the curve $S = 0$, then, the gas at the base of the flow has just enough energy to reach infinity. For given values of the escape velocity and the thermal velocity at the base of the flow, therefore, this limiting curve gives the corresponding minimum value of the initial flow velocity. We find that

$$\frac{1}{2}(V_{\min})_0^2 + \frac{1}{2}(V_{\rm th})_0^2 = \frac{1}{2}(V_{\rm es})_0^2 - \frac{1}{3}(V_{\rm th})_0^2 \,. \tag{15}$$

In this form, the equation points up the fact that, although the flow is adiabatic, a unit mass of gas may move to infinity down the existing pressure gradient, even though its energy content would be insufficient for ballistic escape.

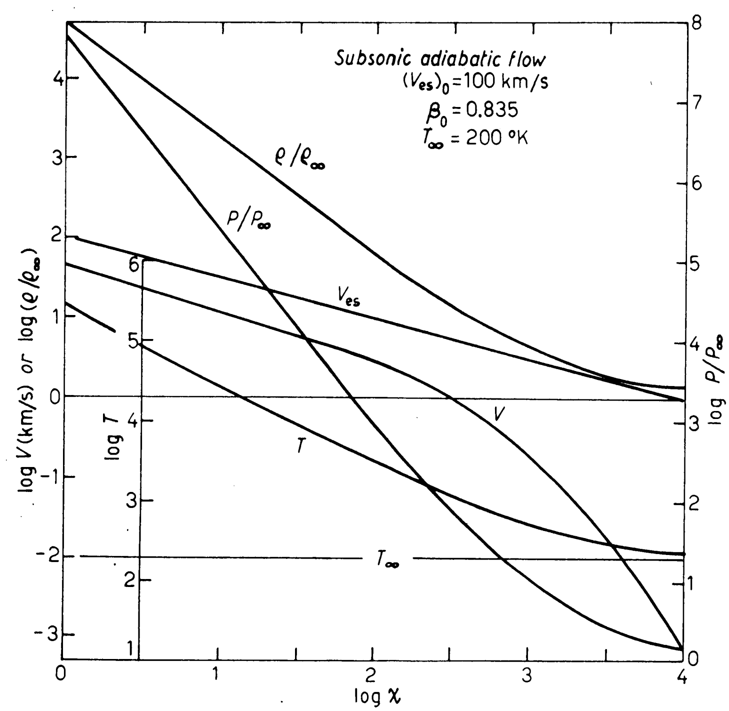

Fig. 9. – A representative subsonic adiabatic flow, with $\mu = 1.38$. As $\chi \to \infty$, $V \to 0$, and the temperature goes to the interstellar value, $T_I = 200°$.

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Some typical adiabatic flows are illustrated in Fig. 9 and 10. In the subsonic solutions V vanishes as $\chi \to \infty$; in the supersonic solutions, $V \to V_0 S^{\frac{1}{2}}$. Conversely, the pressure goes to a finite limit in the subsonic solutions,

$$P \to P_0 \beta_0^5 (S/3)^{\frac{5}{2}} \quad \text{as} \quad \chi \to \infty, \tag{16}$$

and it vanishes in the supersonic solutions. If we require a fit with the interstellar medium, we must therefore exclude the supersonic solutions. With

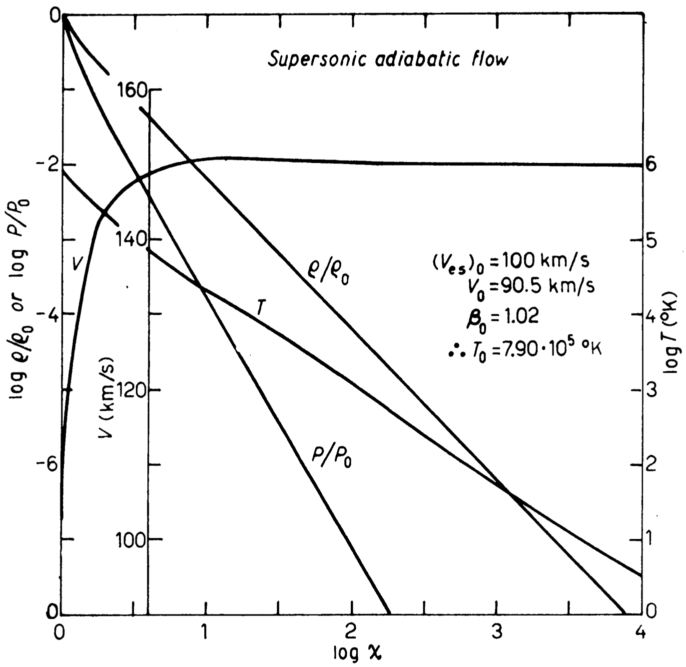

Fig. 10. — A representative supersonic adiabatic flow, with $\mu = 1.38$. This does not fit to the interstellar medium.

the condition that $P_\infty = P_I$, the subsonic solutions yield a mass-loss given by the equation

$$-\dot{M} = 2\pi \left(\frac{3\mu m_{\mathrm{H}}}{20k}\right)^{\frac{3}{2}} \alpha^{-} \beta_0 r_0^2 (V_{\mathrm{es}})_0^4 \varrho_\infty^{\frac{5}{2}} (T_I \varrho_I)^{-\frac{3}{2}} . \tag{17}$$

It is possible to show that, over the relevant range of $(V_{\mathrm{es}})_0$ and T_∞, we always have $4\alpha \gg \beta_0^2 S$. This leads to an appreciable simplification of the equations for subsonic adiabatic flow. In particular, if $P_\infty = P_I$, we find that the

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initial conditions must satisfy the following equations, with $\beta_0 < 1$:

$$\left\{\begin{aligned} V_0 &= \frac{\beta_0}{(\beta_0^2+3)^{\frac{1}{2}}}(V_{es})_0\,,\\ T_0 &= \frac{(3\mu m_H/5k)}{\beta_0^2+3}(V_{es})_0^2\,,\\ \varrho_0 &= \frac{(3\mu m_H/5k)^{\frac{3}{2}}\varrho_\infty^{\frac{5}{2}}}{(\beta_0^2+3)^{\frac{3}{2}}(\varrho_I T_I)^{\frac{3}{2}}}(V_{es})_0^3\,. \end{aligned}\right. \tag{18}$$

For the limiting cases $\beta_0 = 0$ and 1, respectively, these relations are depicted in Fig. 11. It is notable that, for a given value of $(V_{es})_0$, T_0 and ϱ_0/ϱ_∞ are both confined within very narrow limits. We find the following values when $\mu \simeq 1.38$, $(V_{es})_0 \simeq 100$ km/s, as in the M giants; and $\varrho_\infty = \varrho_I \simeq 2\cdot 10^{-25}$ g/cm³, $T_\infty = T_I \simeq 200°$, as in a typical H I region of the instellar medium:

$$\left\{\begin{aligned} 0 < \beta_0 \quad & < 1\\ 3.3\cdot 10^5 > T_0\,(\text{deg}) \quad & > 2.5\cdot 10^5\\ 2.4\cdot 10^{-20} > \varrho_0\,(\text{g/cm}^3) & > 1.8\cdot 10^{-20}\\ 0 < V_0\,(\text{km/s}) & < 50\,. \end{aligned}\right. \tag{19}$$

If such flows occur in M giants, they therefore endow these stars with structures which resemble the solar corona but are about ten times cooler.

In the solar corona itself, the flow must be very nearly adiabatic at sufficiently large distances from the sun. PARKER (1960*a*, *b*) has taken the position that in this

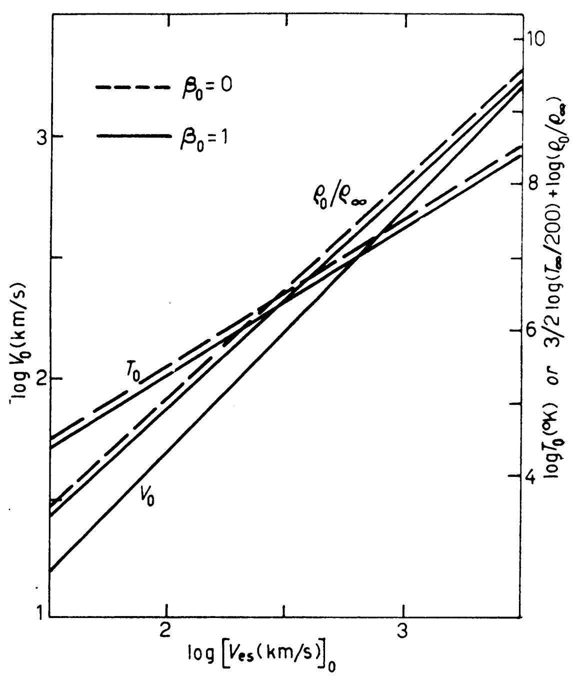

Fig. 11. – Initial conditions for adiabatic flows in which the pressure goes to the interstellar value at large distances. The initial velocity may lie anywhere below the line labeled V_0.

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region the flow must be supersonic, for the reason that in the subsonic adiabatic solutions the pressure goes to a limit which greatly exceeds the interstellar value. This conclusion, however, appears to be the result of representing the isothermal part of the flow by the accelerating critical solution, and joining an adiabatic solution to this at a point where the flow is already supersonic. CHAMBERLAIN (1961) finds it possible to represent the outer corona with a subsonic adiabatic flow going to zero pressure at ∞.

If we apply eq. (18) to the sun, we also find it possible to represent the solar corona by a subsonic adiabatic flow going to any small value for the interstellar pressure. Thus, if we enter Fig. 11 with the value of $(V_{es})_0 = 308$ km/s, which is appropriate for the sun at a height of three solar radii above the photosphere, we obtain the following limits:

$$
(20) \qquad \left|\begin{array}{rll}
0 < & \beta_0 & < 1 \\
3.2\cdot 10^{6} > & T_0\,(\mathrm{deg}) & > 2.4\cdot 10^{6} \\
3.9\cdot 10^{-19} > & \varrho_0\ (\mathrm{g/cm^3}) & > 2.6\cdot 10^{-19} \\
0 < & V_0\,(\mathrm{km/s}) & < 77\,.
\end{array}\right.
$$

The limits computed for T_0 and ϱ_0 lie within a factor of two or three of the actual values for T and ϱ at this level in the solar corona. This relatively close agreement may well indicate that already within a few radii of the solar surface, the structure of the corona is largely determined by the requirement of a fit to the interstellar medium at large distances.

In contrast to the isothermal case, a massive subsonic adiabatic flow is always characterized by a very low density gradient. With the same values we have used before for an M giant in an H I region, we find from eq. (17) that $-\dot{M} = 2.8\cdot 10^{13}$ g/s when $\alpha = \beta_0 = 1$. This value can be appreciably increased only if $\alpha \ll 1$; but then we find that $(V_{th}/V_{es})_0 \gg 1$, and $\varrho_\infty/\varrho_0 \simeq 1$. Moreover, eq. (17) shows that, for a star of given mass, these results are independent of the value taken for the radius r_0 of the « base of the flow ». If the flows we observe are stationary, we may expect that at large distances they must become subsonic adiabatic. For these flows $-\dot{M}$ is so large that we must have $\alpha \ll 1$. Therefore, the subsonic adiabatic regime can set in only at distances so large that $\varrho_0 \simeq \varrho_\infty$. The assumption of a steady state may therefore be untenable for these flows. WEYMANN (1960) has concluded that small non-steady effects in a non-adiabatic flow cannot substantially increase $-\dot{M}$, without violating the condition that the mass of the star must be at least comparably large with the mass of the envelope. Possibly it will be necessary to consider the motion of the star through the interstellar gas, in order to formulate the appropriate far boundary conditions. This motion will usually be supersonic. Possibly no theory for massive outflows can succeed unless it gives up one or both of the simplifying assumptions we have adopted, *viz.*, that the flow is stationary and spherically-symmetric.

In order to estimate the strength and radial velocity of the K-line that could be expected in a typical subsonic adiabatic flow, the projected surface density of Ca II ions was computed for the flow of Fig. 9. In order to take approximate account of the ionization of calcium, the following model was assumed. The star is surrounded by a sharply defined Strömgren sphere of ionized hydrogen. Outside of this, the metals are singly ionized on the average. Ca II occurs only outside the Strömgren sphere; its ionization corresponds to an ionization temperature $T_i = 3\,000°$; an electron temperature T_e equal to the local kinetic temperature T of Fig. 9; and the appropriate factor for geometrical dilution only. The fraction of calcium in the first state of ionization then rises from .084 at $\log\chi = 0.5$ to .98 at $\log\chi = 4.0$. Taking the number density of atoms equal to 0.1 cm^{-3} in the interstellar gas, and integrating along the line of sight from $\log\chi = 0.5$ to $\log\chi = 4.0$, we then find for the surface density of Ca II $N_{\text{Ca II}} = 1\cdot 10^{10}$ cm^{-2}, and for the mean expansion velocity of these ions, $\bar{V} = 3.7$ km/s. The computed surface density is therefore a hundred times lower than is observed at type M1, and the computed expansion velocity five times too low.

C) Hydrodynamical effects of radiative cooling. In the last two sections we have briefly described the properties of isothermal and adiabatic flows. ROGERS (1956), WEYMANN (1960), and PARKER (1961) have discussed other polytropic flows in which γ assumes a constant value in the range $1 < \gamma < \frac{5}{3}$. These other polytropic flows show many of the essential features we have noted in the cases where $\gamma = 1$ and $\frac{5}{3}$, respectively. The case where $\gamma = \frac{3}{2}$ is notable in providing, for a particular value of T_0, a singular solution in which the flow velocity remains strictly constant.

In any real flow the thermodynamic behavior of the gas may depend upon a variety of processes for energy transport, and the flow may therefore depart widely from a simple polytropic law. WEYMANN has discussed some of these transport mechanisms, and particularly the effects due to radiative cooling. As has been clearly pointed out by STANYUKOVICH, radiation losses produce distinctive hydrodynamical effects, depending upon whether the flow is subsonic or supersonic. Thus, by adducing the equation for the conservation of energy, one may obtain the relation

$$\frac{\mathrm{d}V}{\mathrm{d}r} = \frac{V(2c^2/r - GM/r^2) + (\gamma - 1)(\mathrm{d}Q/\mathrm{d}t)}{c^2(\beta^2 - 1)},$$

where $\mathrm{d}Q/\mathrm{d}t$ is the net rate of heat-loss per gram of matter, and where the speed of sound c is here defined as $(\gamma p/\varrho)^{\frac{1}{2}}$. In this form, the equation shows that, other things being equal, the heat loss decelerates a subsonic flow and accelerates a supersonic flow. (The gravitational field clearly behaves in the opposite way.) The equation also indicates that the acceleration diverges at $\beta = 1$ in trans-sonic flows, except possibly in the special case where the term

$$\left[V\left(\frac{2c^2}{r} - \frac{GM}{r^2}\right) + (\gamma - 1)\left(\frac{\mathrm{d}Q}{\mathrm{d}t}\right)\right],$$

vanishes with $(\beta - 1)$.

On the basis of two extreme hypotheses about the atomic processes responsible for the radiation, WEYMANN has computed the radiative losses and their hydrodynamical effects in several cases of outflow. His calculations refer to an M-type supergiant, with $r_0 = 580\, R_\odot$ and $M = 15\, M_\odot$. For initial values, he has taken $10^5 < T_0 < 10^6$, $10^{-17} < \varrho_0 < 10^{-15}$, and $0.1 < \beta_0 < 1.5$. He finds that the radiation drastically reduces the temperature below that corresponding to adiabatic flow: to one percent of the adiabatic temperature at $\chi = 1.1$, in a typical example. Simultaneously, the subsonic flows are so sharply decelerated that the velocity falls to less than one percent of the adiabatic velocity in the same distance. At these densities, therefore, subsonic flows can be maintained only if the large radiative losses are compensated by a distributed heating mechanism. A flux of acoustic or magneto-hydrodynamic waves might provide the distributed energy source that is required. But WEYMANN finds substantial heating of this kind to be still required at very large distances from the star.

Radiative cooling has a similar effect upon the temperature distribution in the supersonic flows WEYMANN has considered; but in these flows the cooling sharply accelerates the gas. At distances larger than $\chi \simeq 2$, where the hydrogen has recombined and the radiative cooling has greatly decreased, the velocity falls below that for adiabatic flow, although the flow remains supersonic. Depending on T_0 and ϱ_0, this deceleration may be sufficient to stop the gas at a finite distance; or it may allow the gas to escape at high velocity; or it may allow the gas to escape at a velocity comparable with the low values that are observed. But in a case of the last kind, WEYMANN finds that despite the high temperatures near the star, most of the Ca I and Ca II along the line of sight lies in this region. These particles produce lines indicating an expansion velocity of about 95 km/s, instead of the 10 km/s that is observed.

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