ON THE ION BROADENING OF THE 12 μm LINES OF ATOMIC MAGNESIUM

D. HOANG-BINH and H. VAN REGEMORTER
LAM, Observatoire de Paris, F-92195 Meudon Cédex, France

Abstract. We have investigated the impact broadening of the 12 μm lines. Ion broadening is found to follow the adiabatic theory, whereas electron broadening needs a non-adiabatic treatment. This relaxes the fixed width/shift ratio, as found by Chang and Schoenfeld (1991), using a pure adiabatic analysis.

Key words: atomic processes – infrared: stars – Mg I

1. Introduction

Electron broadening of the 12 μm lines has been shown to be important by Chang and Schoenfeld (1991). Ion broadening of the far-infrared lines of hydrogen has been shown to be more important than electron broadening by Hoang-Binh (1982), and Hoang-Binh *et al.* (1987). Hoang-Binh (1982) has estimated the half-width at half-maximum (HWHM), γ, of the far-infrared lines of hydrogen, broadened by elastic collisions with protons, and found that it is much larger than the HWHM due to inelastic collisions with electrons. This is due to the complete degeneracy of the (n, l) and $(n, l \pm 1)$ levels, which gives transitions rates scaling as the square root of the perturber mass, transition rates for $n \rightarrow n' \neq n$ being negligible. Now, in the case of the 12 μm ($6h$–$7i$) and ($6g$–$7h$) lines, the broadening is due to collisional transitions between nearly degenerate levels, and it is of interest to find out whether collisions with ions are still predominant in the broadening process.

2. Theory

The number, P, of perturbers in the sphere of Weisskopf radius, for a dipole interaction between degenerate H levels, is

$$P \approx N_p \left[Z_p\, n(n-1)\, e^2(2\pi a_0)/hv\right]^3 \qquad (1)$$

where n is the principal quantum number, v is the relative velocity of the perturber and emitter, and $Z_p e$ is the charge of the perturber.

In the following, we will set $v =< v >= (2kT/\mu)^{1/2}$, where μ is the reduced mass if the interacting particles and the kinetic temperature is taken to be 5,000 K. Let N_1 be the value of N_p corresponding to $P = 1$. The usual condition for validity of the impact approximation, applicable to H, is $P < 1$, or $N_p < N_1$. Thus, using this condition for the 12 μm lines, the impact approximation is applicable for $N_p(\mathrm{Mg}^+) < 1.57 \times 10^{11}$ cm^{-3}, $N_p(\mathrm{H}^+) < 6.93 \times 10^{12}$ cm^{-3}, and electron density $N_e < 5.13 \times 10^{17}$ cm^{-3} (Table 1). But the use of this criterion for the non-hydrogenic case has been much criticized (see *e.g.*, Sahal-Brechot 1969). Although often reliable for electrons, it breaks down for protons and ions. It can be shown that, for the correct effective adiabatic polarization interaction examined below, the impact approximation is valid for $N_p(\mathrm{Mg}^+) < 5 \times 10^{14}$ cm^{-3} and $T = 5,000$ K.

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TABLE I

Electron and ion (H^+ and Mg^+) densities (cm^{-3}), below which the impact approximation is applicable ($P < 1$), for the case of degenerate levels: $T = 5,000$ K.

n	$N_1(e)$	$N_1(H^+)$	$N_1(Mg^+)$
5	4.75E+18	6.42E+13	1.45E+12
6	1.41E+18	1.90E+13	4.30E+11
7	5.13E+17	6.93E+12	1.57E+11
8	2.17E+17	2.92E+12	6.62E+10
9	1.02E+17	1.38E+12	3.11E+10
10	5.22E+16	7.04E+11	1.59E+10

For a line $a \rightarrow b$, the collision width, in angular frequency units, is given by the Baranger formula

$$\gamma = N_p < [\sum_i Q(a \rightarrow i) + \sum_j Q(b \rightarrow j)] > + \gamma \text{ (elastic)}, \tag{2}$$

where the Q are collisional cross-sections, and, for our case, $a = 6g(6h), b = 7h\ (7i)$, and the summations are over levels $i \neq a, j \neq b$.

The elastic part is given in terms of the phase shifts ϕ_a and ϕ_b, by the equation

$$\begin{aligned} \gamma\,(\text{elastic}) &= N_p < v [\int |f_a(\Omega) - f_b(\Omega)|^2 \, d\Omega > \\ &= N_p < v \int 8\pi\rho \, d\rho \sin^2[\phi_a - \phi_b] > \end{aligned} \tag{3}$$

where $f_a(\Omega)$, $f_b(\Omega)$ are scattering amplitudes, and

$$\phi_a = (I_H/kT)\,(\mu/m) \sum_i f_{a,i}\,(I_H/\Delta E_{a,i})\,(1/\rho^2)\,B(\beta_{a,i}\,a_0^2), \tag{4}$$

and ϕ_b is given by the same expression with b, j replacing a, i. The quantities $f_{a,i}$ and $f_{b,j}$ are oscillator strengths, while $\beta_{a,i} = 2\pi\rho\,\Delta E_{a,i}/hv$, and a corresponding expression holds for $\beta_{b,j}$.

Tables of the well-known functions $A(\beta)$ and $\zeta(\beta)$ (see below) and $B(\beta)$ may be found in Griem's (1974) book. The corresponding shift is

$$d = N_p < v \int 2\pi\rho \, d\rho \sin[2(\phi_a - \phi_b)] > \tag{5}$$

Let us consider a collisional transition $(n, l) \rightarrow (n, l+1)$ corresponding to a line of angular frequency $\omega(l, l+1)$. In the process of line broadening by collisions, adiabatic theory will be applicable if

$$\omega(l, l+1)\,\rho_\epsilon/v \geq 1, \tag{6}$$

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TABLE II
Energy of triplet levels of Mg I: T is 5,000 K.

n	l	$E(\text{cm}^{-1})$	$\Delta E(\text{cm}^{-1})$	$\Delta E/kT$
6	3	58575.46	3.534E+01	1.017E-02
6	4	58610.80	8.140E+00	2.343E-03
6	5	58618.94		
7	3	59400.77	2.277E+01	6.552E-03
7	4	59423.54	5.316E+00	1.530E-03
7	5	59428.85	1.664E+00	4.789E-04
7	6	59430.52		

where ρ_ϵ is the effective impact parameter. Let E_l and E_{l+1} be the energies of levels (n, l) and $(n, l+1)$, respectively, and writing $\Delta E_{l,l+1} = (E_{l+1} - E_l)$, expression (6) is equivalent to

$$\Delta E_{l,l+1}/kT \geq 2/l_p \tag{7}$$

where $l_p = \mu v \rho_\epsilon/(h/2\pi)$ is the angular momentum of the perturber.

3. Electron impact widths

For electron impacts, μ is the electron mass, the important values of l_p $(= l_e)$ are 1, 2, 3, 4; thus adiabaticity obtains for $\Delta E_{l,l+1}/kT \geq 0.5$ to 2. Now, Table 2 shows that for the transitions of interest, $\Delta E_{l,l+1}/kT << 1$; thus, electronic collisions are strongly non-adiabatic. As equation (2) shows, the calculation of $\gamma\,(= \gamma_e)$ requires knowledge of cross-sections of collisional transitions between levels a, b and other atomic levels. We will consider only the most important $(\Delta n = 0)$ transitions $(l \rightarrow l \pm 1))$. Using the impact parameter theory in the dipole approximation (Seaton 1962, Sahal-Brechot 1969), the cross-section for $a \rightarrow i$ is

$$Q(a \rightarrow i) = 8\,(I_{\text{H}}/kT)\,(I_{\text{H}} f_{a,i}/\Delta E_{a,i})[\zeta(\beta_1)/2 + \phi(\beta_1)]\,\pi a_0^2, \tag{8}$$

where I_{H} is the Rybderg energy and β_1 and $\phi(\beta_1)$ are defined and tabulated by Seaton (1962).

In the case of electrons, when $kT >> \Delta E$, the relevant values of $\beta_{a,i}$ and $\beta_{b,j}$ are very small, and $B\,(\beta) = -\pi\,\beta^2[1/2 + \ln(1.78\,\beta/2)]$. It is easy to show that the phase shifts ϕ_a and ϕ_b are also very small, and γ_e(elastic) and d_e(elastic) are negligible. Using equation (2) and values listed in Table 3, we have calculated the electron widths γ_e listed in Table 4.

4. Ion impact widths

For ions, $\mu >> m_e$, $l_p = l_i \approx 10^3$ to $10^4 \times l_e$, and adiabaticity obtains for $\Delta\, E_{l,l+1}/kT$ greater than about 10^{-3} to 10^{-4}. Table 2 shows that for the lines

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TABLE III

Cross Sections (Q) in cm^2 and Rates (K) in $cm^3\ s^{-1}$ for a number of transitions in a 5,000 K gas.

n	l	$Q(l \rightarrow l+1)$	$K(l \rightarrow l+1)$	$K(l+1 \rightarrow l)$
6	3	1.328E-11	5.169E-04	4.020E-04
6	4	1.380E-11	5.371E-04	4.395E-04
7	3	3.037E-11	1.182E-03	9.195E-04
7	4	3.967E-11	1.544E-03	1.264E-03
7	5	2.970E-11	1.156E-03	9.785E-04

TABLE IV

Comparison of γ_e/N_e with the adiabatic contribution γ_i/N_i given by equation (9) for Mg^+. Units are $cm^3\ s^{-1}$; T = 5,000 K.

Transition	γ_e/N_e	γ_i/N_i	d_i/N_i	C_4
$6g$–$7h$	3.36E-03	4.64E-04	3.99E-04	8.07E-11
$6h$–$7i$	1.42E-03	6.50E-04	-5.60E-04	-1.34E-10

in question, this is indeed the case. Thus, ion broadening is adiabatic. Owing to the small ion velocity and the large ρ of interest, $\beta(\text{ion}) \approx (\mu/m_e)\,\beta(\text{electron})$, for a given ΔE. When $\beta >> 1$, $B(\beta) = \pi/4\beta$ in (4), and it is a simple matter to derive the well-known Lindholm formulae (Sobelman *et al.* 1981) for the width $\gamma = \gamma_i$, and shift $d = d_i$, namely,

$$\gamma_i = 38.8\, C_4^{2/3}\, v_i^{1/3}\, N_i, \tag{9}$$

$$d_i = 33.4\, C_4^{2/3}\, v_i^{1/3}\, N_i, \tag{10}$$

and,

$$C_4 = (1/4\pi)\,[e^2/(h/2\pi)]\,(\alpha_b - \alpha_a)\,a_0^3 \tag{11}$$

where α_a, α_b are the polarizabilities in atomic units, and a_0 is the Bohr radius. For a level (n, l), we have

$$\alpha_{n,l} = 4 \sum_{n',l'} [f_{(n,l;n',l')}\,(I_H/\Delta E)^2] \tag{12}$$

where ΔE is the energy difference (in Rydberg units) between (n, l) and (n', l'). It is sufficient to consider only $n' = n$. Values of $C_4, \gamma_i/N_i$, and d_i/N_i for the lines of

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interest are listed in Table 4. It can be seen that ion impact broadening, though smaller than electron impact broadening, is not negligible.

5. Discussion

The main conclusion of our study is that, in the impact regime, adiabatic theory applies only to collision with ions, not with electrons. This is in contrast to the treatment given by Chang and Schoenfeld (1991), who use adiabatic theory to describe the broadening and shift by electron impact (ions are ignored). As expressions (9) and (10) show, adiabatic theory predicts a constant width/shift ratio, in disagreement with observations. Our results show that the line shifts are due to ions, not to electrons; hence the width/shift ratio, $(\gamma_e + \gamma_i)/d_i$, is not constant, in better qualitative agreement with observations. However, a close comparison of our results with observations is not warranted, because γ_i and d_i must refer to all ions, not just Mg^+. Further, owing to the high abundance of neutral hydrogen, broadening and shift by collisions with H atoms may be important; unfortunately, no simple reliable theory is available. The simple Fermi model overestimates l-mixing transitions rates for $n < 10$, possibly by more than one order of magnitude. The present work shows that electronic collisional l-mixing rates ($\approx 10^9$ s^{-1} for $7h \rightarrow 7i$ at $N_e = 10^{12}$ cm^{-3}) are among the largest collisional rates entering the statistical equilibrium equations (see Hoang-Binh 1991). Thus, it is not necessary to invoke collisions with H atoms to achieve equal-n-level balancing (Carlsson *et al.* 1992).

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