A CRITIQUE OF THE POLARIMETRIC EVIDENCE ON THE NATURE OF CBS

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ABSTRACT. Determination of the values and confidence intervals for physical parameters and the orbital inclination of binaries from polarimetry is critically discussed. A new method based on χ^2 best fitting and incorporating constraints imposed by the canonical model is developed and applied to polarimetric data for Cygnus X-1. Whereas the χ^2 best fitting procedure yields an inclination of $|i| = 79^o$ for Cygnus X-1, the level of noise, and the model dependency of the parameter determination give rise to a very broad confidence interval around this value. Our method also indicates the accuracy of the polarimetric measurements necessary for more precise parameter determinations.

It can be shown (eg. Brown et al 1978) that polarisation of light caused by Thomson scattering in the envelope of a binary system in which (the canonical model) (i) the geometry of the system is time independent in a frame uniformly rotating with the binary period, (ii) no eclipsing or variable absorption effects take place, (iii) variations of the direct starlight intensity are small will behave according to

$$Q_t(\lambda) = p_o + q_1 \cos \lambda + p_2 \cos 2 \lambda + q_1 \sin \lambda + q_2 \sin 2 \lambda$$
$$U_t(\lambda) = u_o + u_1 \cos \lambda + u_2 \cos 2 \lambda + v_1 \sin \lambda + v_2 \sin 2 \lambda$$

(Q_t and U_t are the (theoretical) Stokes parameters, $\lambda/2\pi$ is the phase). The harmonic coefficients are functions of inclination and weighted integrals of the electron density distribution. In a natural frame in which the z-axis lies along the line of sight and the binary axis lies in the x,z plane

$$p_1/v_1 = -q_1/u_1 = \cos i, \quad p_2/v_2 = -q_2/u_2 = (1+\cos^2 i)/2 \cos i.$$

If the density distribution is symmetric about the orbital plane only second harmonic variations arise. Observations are made in a frame rotated by an angle ϕ(unknown a priori) relative to this natural frame. In principle fourier analysis of the observed $Q(\lambda)$ and $U(\lambda)$ curves would yield the p_k, q_k, u_k, v_k and hence determines, indeed overdetermines, (via the constraints above) both i and ϕ. Noise in

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the data, however, forces one to adopt a statistical procedure of 'best fitting' model and data.

Usually a simple minded formal linearized error approach is adopted when a model is used to extract parameter values from given data. Comparison of the model M {p} , where p represents the model parameters, and the data values, X_r, yields optimal values for p that depend only on X_r, i.e. $p_{opt} = P(X_r)$. If the model agrees well, and if σ_{X_r} are small then under the linear approximation

$$\sigma^2_{p_{opt}} \simeq \sum_r \left(\frac{\partial P}{\partial X_r}\right)^2_{\bar{X}_r} \sigma^2_{X_r}$$

where $\bar{X}_r$ are the expected, i.e. theoretical, values for X_r. There are a number of sources of confusion here. Firstly, even if the model conflicts with data for all p values this approach will yield formal errors in **p**, and perhaps very small ones, but say nothing of this incompatibility. Also linearity assumptions can breakdown, depending on the size of σ_{X_r} and the function form of p.

We adopt the more rigorous approach of determining the domain of parameter values for which the data and the model agree at a given significance level, and hence simultaneously test the acceptability of the canonical model which has been challenged by Kemp et al (1979) and Milgrom (1978) in the case of Cyg X-1.

Extensive polarimetric data for Cygnus X-1 have been provided by Kemp (Kemp et al 1979), and takes the form of N equally spaced phase bin averaged Stokes parameters, $Q_{ob,r}$, $U_{ob,r}$, which have been folded on a 5.600^d period. The statistic

$$\chi^2_{2N} = \sum_{r=0}^{N-1} \frac{(Q_{ob,r} - Q_{t,r})^2}{\sigma^2_r} + \frac{(U_{ob,r} - U_{t,r})^2}{\tilde{\sigma}^2_r}$$

measures the agreement between model and data, and has a chi-squared distribution with 2N degrees of freedom if $Q_{ob,r}$ and $U_{ob,r}$ are normally distributed. $Q_{t,r}$, $U_{t,r}$ are the model predicted values; and for given data χ^2_{2N} will be a function of the 8 independent model parameters, which we refer to as i,p. A 90% confidence region will be that domain of parameters for which the model must be accepted at 10% significance, i.e. $\chi^2_{2N}(i,p) < \chi^2_{2N,10\%}$. Of course, to determine this entire 8 dimensional parameter domain is computationally very difficult. Since we are interested primarily in a confidence interval for inclination, i, we can adopt a simplifying method that admits an analytical solution. Fixing i, we determine by the method of Lagrange's multipliers the optimal values of the remaining 7 independent parameters for this i and the corresponding value of χ^2_{2N}, which we call inf $\chi^2_{2N}(i)$. The 90% confidence interval for i can then be obtained by finding that range of i for which inf $\chi^2_{2N}(i) < \chi^2_{2N,10\%}$.

This analysis yields for the asymmetric and symmetric cases respectively 90% confidence intervals of $85^{\circ} \leqslant i \leqslant 150^{\circ}$ and $90^{\circ} \leqslant i \leqslant 125^{\circ}$, with optimal value $i \simeq 102^{\circ}$ ($|i| \simeq 78^{\circ}$) in both cases, the confidence interval on i greatly exceeding the formal error estimates of $\pm 7^{\circ}$ obtained as above. Optimal values for the other parameters give a lower limit on the mass of scattering material of $\sim 10^{24}$ gm, which is comparable to the mass transfer per orbital period needed to produce the X-ray luminosity. The optimal polarimetric value of $|i| \simeq 78^{\circ}$ differs greatly from $|i| = 28^{\circ}$ of the light curve analysis (Hutchings 1979). However the polarimetric confidence interval we obtain is so broad that polarimetric and photometric results are not in contradiction statistically. Furthermore the true confidence interval on i from spectrophotometric modelling may also be much larger than the formal range $28^{\circ} \pm 2^{\circ}$ usually quoted, for similar reasons (cf. Bolton, this meeting). We plan to extend our error analysis technique to the spectrophotometric case.

Though the polarimetric results for Cyg X-1 may seem disappointing, Kemp's data are very noisy, and his phase binning technique may give rise to large bin errors because of systematic changes (eg. of the system's geometry). Use of fewer orbital periods could give better parameter estimations with an adequate photon count. We are working on determining the accuracy required for useful model testing and parameter estimation.

A fuller discussion of this work can be found in Simmons et al (1979).

REFERENCES

Brown, J.C., McLean, I.S. and Emslie, A.G.: 1978, Astron. & Astrophys. 68, 415.
Hutchings, J.B.: 1974, Ap.J. 188, 341.
Kemp, J.C., Barbour, M.S., Parker, T.E. and Herman, L.C.: 1979, Ap.J. Letters 228, L.23.
Milgrom, M.: 1978, Astron. & Astrophys. 65, L.1.
Simmons, J.F.L., Aspin, C. and B rown, J.C.: 1979, preprint.

DISCUSSION FOLLOWING SIMMONS, ASPIN AND BROWN

Guinan: Have you tested your procedure of determining the orbital inclination from polarization measures against systems with i known through an eclipse analysis?

Simmons: One of our assumptions is that no eclipsing takes place. The analysis could be extended to cover this case, and we intend to do this, but the technical problems in the mathematics will be more difficult. But yes, it is a good idea to test the method in this way. Of course, it would be necessary to have good polarimetric data for an eclipsing system which could otherwise be considered to reasonably comply with the assumptions made in the canonical model.

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