

special cases (with certain similar features) of almost Hermite spaces, which form the main topic of Chapter IX.

Many of the properties of complex and almost complex spaces can be studied in terms of two conjugate complementary distributions. In some ways, an almost complex space behaves like a kind of local product in which the components are not real. This suggests that concepts analogous to those studied in almost complex spaces might be of interest in real spaces which are locally products, or which behave in a similar way. This rather vague idea is clarified by studying what occurs in the almost complex case and making the obvious analogies. The situation is investigated in Chapters X and XI. The former deals with local product spaces and the latter with the concept of almost product space, which was initiated by Walker. Like an almost complex space, an almost product space is characterised by the existence of a mixed second order tensor field; however the square of this tensor is unity and not minus unity as in the almost complex case. One other different feature is that in the almost product case, the dimensions of the complementary distributions are not necessarily equal. In the final chapter, the author returns to almost complex spaces and studies holomorphically projective relations between connexions in such spaces.

The book is expensive, but no doubt the large numbers of formulae involving tensor components have resulted in high costs. Complex conjugates are denoted by the use of a bar; the weak-sighted may have a certain amount of difficulty with this. Apart from this, the book is very well produced and the printing is excellent. All in all it is an important addition to the literature on differential geometry and it is hoped that the author's wish that it should encourage further research is fulfilled.

E. M. PATTERSON

PONTRYAGIN, L. S., BOL'TANSKII, V. G., GAMKRELIDZE, R. S. AND MISCHENKO, E. F., *The Mathematical Theory of Optimal Processes*, translated by D. E. Brown (Pergamon Press), 338 pp., 80s.

A controlled process may be described by a system of ordinary differential equations

$$\frac{dx^i}{dt} = f^i(x^1, \dots, x^n, u^1, \dots, u^r), \quad i = 1, 2, \dots, n,$$

where the x^i are the phase coordinates of the controlled entity and the u^j are the control parameters defining the course of the process. Given the initial and final values, $x^i(t_0)$ and $x^i(t_1)$, the problem of optimal control is to find the functions $u^j(t)$ which minimise an integral functional of the form

$$J = \int_{t_0}^{t_1} f^0(x^1, \dots, x^n, u^1, \dots, u^r) dt.$$

The classical calculus of variations is not adequate for solving problems of this type when, as is the case in modern applications, the u^j are subject to restrictive conditions such as

$$|u^1| \leq 1 \text{ or } (u^1)^2 + (u^2)^2 \leq 1,$$

and the optimal control turns out to be situated at the limits of such inequalities and to involve jumps between, say, $u^1 = 1$ and $u^1 = -1$ in the above example of $|u^1| \leq 1$.

The book deals very thoroughly with the well-known and powerful Pontryagin Maximum Principle method for such problems. The presentation combines readability and rigour, and three of its seven chapters constitute an adequate account of the subject for engineers. Topics dealt with include linear time-optimal processes ($J = t_1 - t_0$), application to the approximation of functions, a pursuit problem,

processes with restrictive conditions imposed on the phase coordinates too and a treatment of a statistical control problem based on Kolmogorov's equation. The relationships to both classical variational calculus and Bellman's theory of dynamic programming are clarified. Indications are given of important still unsolved problems such as the general pursuit problem. The translation is good with few misprints.

B. MELTZER

STEIN, MARVIN L. AND MUNRO, WILLIAM D., *Computer Programming—A Mixed Language Approach* (Academic Press, 1964), 92s.

The authors' declared intention is to provide for the training both of professional programmers, and of those who, while not strictly in that class, have, nonetheless, to make frequent use of computers. Their guiding principle has been that familiarity with low-level machine codes is essential for the efficient, sympathetic use of higher level symbolic-assembly-languages and problem-oriented languages, such as FORTRAN.

The book opens with an extended account of arithmetic to a general base, *r*. Unfortunately, having once introduced the beginner to the idea of handling such numbers, on paper, outside the computer, the book stops short of describing the writing of a general decimal-to-binary conversion routine. This may be unduly critical, but the book was unfolding its material very well at this point and it was something of a disappointment to find this omission. However, the description of complement-arithmetic—the usual representation of negative quantities in a computer—and of basic machine-coding, is, in other respects through, and generously strewn with annotated examples. Later chapters discuss assembly- and interpretive-schemes, FORTRAN and the FORTRAN Symbolic Assembly Programme.

Much of the illustrative material is designed around the CDC 1604, a computer working in ones-complement arithmetic; this is fairly typical American practice, but not often found in British computers. The intending reader should bear this in mind, but on the whole it is a slight reservation. For anyone requiring an introduction to the 1604 it is an excellent book.

A particularly useful feature of the book is the great variety and number of exercises at the end of each chapter. An Appendix contains answers to certain of them.

J. A. OGDEN

CHUNG, AN-MIN, *Linear Programming* (Merrill Books, 1964), vii + 338 pp., 63s.

Linear Programming is well established as a standard method for the formulation and solution of a certain class of problems arising in business. But the mathematics involved is by no means trivial, especially for non-specialist-mathematicians working in fields such as management science. The author mentions his own experience that the subject is taught either as a mechanical rule-of-thumb for arriving at an answer, or as an involved study in advanced algebra, quite beyond the powers of non-mathematicians. The reason suggested for this unhappy state of affairs is the lack of a suitable textbook. It is at precisely this gap that the present work is directed. It is claimed that "only a knowledge of elementary algebra" has been assumed. Perhaps one would be forgiven for suggesting that a fair appetite for lengthy and heavily subscripted algebraic notation would be in order, too.

After introducing the subject-matter, via a wide-ranging set of examples, the author devotes a chapter to a concise and lucid account of the elements of matrix algebra. There follows an unhurried and careful exposition of the standard material on the properties and classification of solutions of Linear-Programming problems, and of the Simplex Method of solution. In the next two chapters two variants of this are considered—the Revised Simplex, and the Dual Simplex methods—which provide a more efficient computational procedure, and the effect upon the solution