SOME DIFFICULTIES IN THE THEORY OF NUTATION

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Fedorov (1958) made a splendid discussion of data on the nutations; Vicente and I made a theory, taking account of a fluid core and of an elastic shell. The agreement was far better than had been found before, but some inconsistencies remained.

A doubtful point in our theory concerns the structure of the core. In my first paper on the subject (1949, especially pp. 672-3) I gave equations of motion for a homogeneous incompressible fluid core of small viscosity ν; the solution depends on a sort of stream function satisfying

$$(i\nu\nabla^2 + \gamma^2)\nabla^2\Omega = 4\omega^2\partial^2\Omega/\partial z^2$$

reducing for $\nu = 0$ to

$$\gamma^2\left(\frac{\partial^2\Omega}{\partial x^2} + \frac{\partial^2\Omega}{\partial y^2}\right) + (\gamma^2 - 4\omega^2)\frac{\partial^2\Omega}{\partial z^2} = 0 \quad .$$

If Ω is of the form $(\ell x+my)z$ both equations are satisfied exactly, and this can be used as an approximation even if the fluid is neither homogeneous nor incompressible. This was used by Vicente and me (1957) for the core, putting in higher powers of x, y, z, when required.

The inner core introduces difficulties. In a potential problem we should have to introduce terms in $(\ell x+my)z/r^5$ to satisfy the boundary conditions. In this case, if we take out a factor in $\tan^{-1}(y/x)$ and if $|\gamma| < 2\omega$, the differential equation is hyperbolic in $\tilde{\omega}$ and z, with characteristic cones going right through the outer core. This applies to all diurnal and long-period tides. I had a similar problem in 1924 in a paper on oscillations of an elliptic lake. There are cases where the motion is strong around the minor axis with a maximum some way off it, but dies down rapidly towards the ends. Some analogue must occur here.

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Vicente and I avoided this difficulty by considering two models, one with an incompressible core of uniform density and a massive particle at the centre; the other, called the Roche model, attributed the whole variation of density to compression. Both were adjusted to give the right mass and moment of inertia according to a 1942 model of Bullen. The Roche model gives too high a compressibility, and the central particle one is obviously impossible, but it would be expected that the truth would lie between them.

Jackson (1930) had pointed out that the observed value of the principal nutation was less than that calculated for a rigid Earth. Federov was the first to have determined the nutations in obliquity and longitude separately. The observed coefficients and our calculated ones were as follows, in seconds of arc:

	Observed	Central Particle Model	Roche Model
Obliquity	9.198±0.004	9.2015	9.2187
Longitude	6.853±0.004	6.820	6.849

The central particle model agrees with observation for the obliquity, but for longitude it differs by about 7 times the apparent standard error; the Roche model agrees for longitude but differs by 5 times the standard error for obliquity. Means between them would be inconsistent for both. For the fortnightly terms both models agreed with observation. The comparison for the semiannual terms was as follows (observed values quoted by Fedorov from Popov)

	Observed	Central Particle Model	Roche Model
Obliquity	0.578±0.004	0.5734	0.5403
Longitude	0.533±0.004	0.5232	0.4883

The differences for the central particle model are about 1.1 and 2.5 times the standard errors; for the Roche model about 10 and 5 times the standard errors. Clearly the solution for these terms depends greatly on the structure assumed for the core.

A substantial difference between the two models was found also from the periods of the free nutations. Both give the usual Eulerian nutation. The central particle model has another free period, of 447 days, with the displacements of the core about -9 times those of the shell. The Roche model gives a period of about 250 days, with similar displacements, and a third with a period of about 140 days, entirely due to the variation of density within the core. Since half a year is

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183 days it appears that approach to resonance will be important for the semiannual period.

The radius of the inner core, according to my solution of 1939 (Jeffreys, 1976, p. 156), is about 0.36 of that of the main core. Later revisions, especially by Bullen (1975) have not altered this greatly. There is substantial evidence now that the inner core is solid. It would now be possible to make much more detailed calculations of its effects.

Vicente and I used Takeuchi's (1950) model and his solutions for the shell, which we transformed to a more convenient form. It may be better to revise the whole of the calculation with a more recent model, such as Bullen and Haddon's A (Bullen, 1975; Jeffreys, 1976, p. 212).

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