Cvetković, D. M., Doob, M. and Sachs, H. *Spectra of graphs* (3rd edition) (Johann Ambrosius Barth Verlag, Heidelberg-Leipzig 1995), 447 pp., 3 335 00407 8, DM 168.

The first edition of this book, which appeared in 1980, was the first monograph on graph spectra; it was encyclopaedic in nature and its bibliography included virtually all relevant material published before 1979—over 550 references in all. Publication of a third edition is a testament to its continuing usefulness, not only as a source of information, but also as a readable introduction to the subject. Unlike the second edition the third is a considerably expanded version of the original. For better or worse the additional material appears in two supplements (and a further bibliography) after a reprint of the second edition. Naturally, a disadvantage of this arrangement is the lack of an integrated treatment of some topics; moreover the index has not been extended to cover the new portions of text. The decision to separate the old from the new is however justified in as far as it leaves a clear dividing line between the comprehensive treatment of pre-1979 material and the subsequent results selected for presentation without proof. Selectivity is inevitable in view of the rapid growth of the subject over the last fifteen years; indeed the volume of results [2] compliled by Cvetković *et al.* in 1988 lists over 700 additional references, while distance-regular graphs (known as metrically regular graphs in the first edition) are the subject of a recent monograph [1] with 800 references.

We review the 75 pages of additional material after giving a brief description of what comes before. The first part of the book (essentially the original edition) is self-contained except for a few results from matrix theory such as the Perron–Frobenius and interlacing theorems, which are stated without proof in Chapter 0. The book deals primarily with the spectrum of a (0,1)-adjacency matrix of a finite undirected graph. Such a matrix is regarded as a matrix with real entries and Chapters 1–5 are concerned with the relation between its spectrum and the structure of the underlying graph. Chapter 6 is devoted to the characterization of graphs by spectral properties and Chapter 7 to the use of spectral techniques in proving graph-theoretical results which are formulated without reference to graph spectra. Such results include, for example, necessary conditions on the parameters of strongly regular graphs. Chapter 8 explores roots of the subject in the physical sciences, in particular by reference to the equations of motion of a vibrating membrane and to Hückel's theory of molecular orbitals. The final chapter contains a few miscellaneous items and the appendix includes lists of spectra of (amongst others) all connected graphs with at most 5 vertices, all trees with at most 10 vertices and all cubic graphs with at most 12 vertices.

The additional text in the third edition starts with Appendix A, which consists of three pages of specific comments on the previous editions and a page of errata. Appendix B begins with a review of relevant texts and survey papers which have appeared in the last 15 years, including several books concerned with the broader topic of algebraic combinatorics. The subsequent sections survey the more important developments in the decade 1984–1993: these include the classification of graphs with least eigenvalue -2, the ordering of graphs by eigenvalues (with particular reference to the largest and second largest eigenvalues) and the use of graph angles as algebraic invariants. The angles in question are those between coordinate axes and eigenspaces and one problem mentioned is the construction of trees with prescribed eigenvalues and angles. A further section is devoted to star partitions, another important notion related to eigenspaces: such a partition of the vertices of a graph provides (i) a natural way of associating eigenvalues with vertices, (ii) a means of explaining the role of an individual eigenvalue in the structure of a graph and (iii) an approach to the graph isomorphism problem. There follows a short section on the Laplacian spectrum, a survey of published tables of eigenvalues and angles, some notes on the computer packages GRAPH and GRAFFITI, some more miscellaneous results and finally an additional bibliography of 300 items.

There is little doubt that the text will remain 'the bible' for researchers in the field. Newcomers will forgive the slightly cumbersome arrangement of material in the new edition and recognise that it offers not only the detailed groundwork of the original but also a valuable overview of recent developments.

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REFERENCES

1. A. E. Brower, A. M. Cohen and A. Neumaier, *Distance-regular graphs* (Springer-Verlag, Berlin, 1989).

2. D. M. Cvetković, M. Doob, I. Gutman and A. Torgašev, *Recent results in the theory of graph spectra* (North-Holland, Amsterdam, 1988).

Ziegler, G. M. *Lectures on polytopes* (Graduate Texts in Mathematics, Vol. 152, Springer-Verlag, Berlin-Heidelberg-New York-London-Paris-Tokyo-Hong Kong 1995), ix+370 pp., softcover: 3 540 94365 X, £21, hardcover: 3 540 94329 3, £47.

Since Branko Grünbaum's monumental book *Convex polytopes* (Wiley-Interscience) appeared in 1967, several texts have been written on or around the same subject. However, few of them deserve more than a tepid recommendation (which is why I shall not mention their authors). Further, the exceptions to this stricture have usually been volumes of collected articles, which were not devoted solely to polytopes. Here, though, we do have a book which can be welcomed with considerable enthusiasm. While it does not attempt to emulate the (then) comprehensiveness of Grünbaum's work, it provides more than adequate compensation with its coverage of theories which have developed over the last nearly thirty years.

Let us recall that a *convex polytope* is simultaneously definable as the convex hull of some finite point-set in a euclidean space or as a bounded intersection of finitely many closed half-spaces. A polytope P (we shall usually drop the qualification 'convex' in what follows) has *faces* of each dimension up to $\dim P$, which are its intersections with supporting hyperplanes. The family $\mathscr{F}(P)$ of these faces (including $\emptyset$ and P itself) forms a lattice, partially ordered by inclusion. Two polytopes P and Q are *isomorphic* (or *combinatorially equivalent*) if their face-lattices $\mathscr{F}(P)$ and $\mathscr{F}(Q)$ are isomorphic (as lattices).

Like Grünbaum's book Ziegler's is largely concerned with the combinatorics of polytopes. Central to this area is the *Steinitz problem*, which asks which lattices are isomorphic to face-lattices of popytopes. Up till now complete answers are only available for d-polytopes (that is d-dimensional polytopes) with d at most 3 or with at most $d+3$ vertices (or facets). These polytopes are also 'nice' in that there always exist isomorphic polytopes with rational vertices and the spaces of realizations of isomorphic polytopes (factored out by affinities) are contractible. It has recently become clear that for all other polytopes the situation is completely different: all subfields of the algebraic numbers may be needed for realizations and the realization spaces may be arbitrarily complicated. However, one aspect of this new work is only mentioned in the book and the other is too new even for that. It should be emphasized, though, that polytopes occur in many different contexts; the connexions with algebraic geometry appear to be particularly deep and extensive.

Ziegler's book provides a fine introduction to these newly developing areas. The treatment of the equivalence of the two definitions of polytopes (given above) in Lecture 1 is non-standard, using the computational technique of Fourier–Motzkin elimination. This sets a tone, that of doing things in often slightly unusual ways. The basic results about face-lattices are established in Lecture 2. Graphs of polytopes (formed by their vertices and edges) are dealt with in Lecture 3; here we see recent important theorems such as Kalai's pseudopolynomial bound on the edge-diameter of a d-polytope with n facets and his proof of the fact that the graph of a simple d-polytope (one with just d facets through each vertex) determines its combinatorial type.

The next two lectures concern Steinitz-type problems. Lecture 4 treats 3-polytopes, giving the positive answer mentioned above. Lecture 5 looks at 4-polytopes, giving examples which show that objects which look as if they ought to be projections (in some sense) of 4-polytopes in fact need not be.

A Gale diagram of a d-polytope with n vertices is a set of n points in $(n-d-1)$-dimensional space and encodes its combinatorial properties. Gale diagrams formed a very new topic in