MODE DISCRIMINATION IN NONRADIALLY OSCILLATING STARS FROM LIGHT, COLOUR AND VELOCITY OBSERVATIONS

L.A. Balona
South African Astronomical Observatory

R.S. Stobie
Royal Observatory Edinburgh

ABSTRACT

Expressions for the amplitudes and phases of the light, colour and radial velocity variations are derived for a star in nonradial oscillation. For stars in the cepheid instability strip the spherical harmonic mode of the oscillation can be obtained from the phase difference between the light and colour variations. For β Cep stars the mode can be estimated from the amplitude ratio of the light and colour variations.

1. THEORY

Consider oscillations corresponding to an arbitrary spherical harmonic distortion of a non-rotating star whose equilibrium structure may be treated as spherically symmetric. We assume that amplitudes are sufficiently small that the linear approximation is adequate.

The variation of the stellar radius about the equilibrium value R_o is, in spherical co-ordinates (R, θ, ϕ):

$$R = R_o\{1 + \varepsilon P_{\ell m}(\cos\theta)\cos(\omega t + m\phi)\}$$

where $\varepsilon \ll 1$ is the relative semi-amplitude and $P_{\ell m}(\cos\theta)$ is the Legendre function. The variation of the local emergent flux is:

$$F = F_o\{1 + \varepsilon f P_{\ell m}(\cos\theta)\cos(\omega t + m\phi + \psi)\}$$

where f is a scaling factor for the amplitude and ψ is the phase lag of the flux variation relative to the radius variation. We assume a limb-darkening law of the form:

$$h = (2 - \beta) + 1.5\beta(\underline{n}.\underline{u})$$

where $\underline{n}$ is a unit vector normal to the surface and $\underline{u}$ is a unit vector in the direction of the observer. Finally, we assume a surface

Space Science Reviews 27 (1980) 371–376. 0038–6308/80/0273–371 $00.90.

https://doi.org/10.1017/S0252921100081781 Published online by Cambridge University Press

velocity field with components:

$$v_R = \frac{\partial R}{\partial t}; \qquad v_\theta = \alpha_H \frac{\partial v_R}{\partial \theta}; \qquad v_\phi = \frac{\alpha_H}{\sin\theta}\frac{\partial v_R}{\partial \phi}$$

where $\alpha_H = 74.6Q^2$ and Q is the pulsation constant in days. These relationships were first described by Dziembowski (1977) and further discussed by Balona and Stobie (1979a, b).

By transforming to the observer's co-ordinate system and integrating over the visible hemisphere, we obtain the light, colour and radial velocity variations (correct to first order):

$$\Delta V = 1.086\ \hat{\varepsilon} F_V \cos(\omega t + \phi_V)$$

$$\Delta C = (1.086\ \hat{\varepsilon} F_C/A) \cos(\omega t + \phi_C)$$

$$\Delta V_r = \hat{\varepsilon} \omega R_o F_{RV} \cos(\omega t - \pi/2)$$

where

$$F_V \sin\phi_V = -f b_\ell \sin\psi$$

$$F_V \cos\phi_V = -(f b_\ell \cos\psi + g_\ell)$$

$$F_C \sin\phi_C = -f b_\ell \sin\psi$$

$$F_C \cos\phi_C = -(f b_\ell \cos\psi + h_\ell)$$

$$F_{RV} = u_\ell + \alpha_H v_\ell$$

The quantities b_ℓ, g_ℓ, h_ℓ, u_ℓ and v_ℓ are definite integrals which depend on the spherical harmonic order ℓ and the limb-darkening coefficient, β. They can be written as:

$$b_\ell = (2 - \beta)I_1 + 1.5\beta I_2$$

$$g_\ell = \{2 - \ell(\ell + 1)\}b_\ell$$

$$h_\ell = \{2 - \ell(\ell + 1)\}(b_\ell - 2I_1)$$

$$u_\ell = (2 - \beta)I_2 + 1.5\beta I_3$$

$$v_\ell = \tfrac{1}{2}\ell(\ell + 1)\{(2 - \beta)I_2 + \beta I_3\}$$

where

$$I_n = \int_0^1 \mu^n P_\ell(\mu)\, d\mu$$

and $P_\ell(\mu)$ is the Legendre polynomial with $\mu = \cos\theta$. The quantity $\hat{\varepsilon} = \varepsilon P_{\ell m}(\cos i)$ is the projected semi-amplitude where i is the angle of inclination of the pole of oscillation to the line of sight.

In deriving the colour variation we have made the assumption that the surface brightness variation is proportional to the colour variation so that $\Delta S = A\ \Delta C$. Balona (1981) has shown that the effect of gravity variation does not seriously affect the problem of mode estimation.

The radial velocity variation calculated above is defined as the variation of the centroid of the line profile.

2. MODE IDENTIFICATION

The expressions for the light, colour and radial velocity variations contain six unknowns: ℓ, f, ψ, $\hat{\varepsilon}$, A and R_o. The observations provide five quantities - three amplitudes and two phase differences. Since ℓ is an integer, certain constraints are placed in the set of possible solutions. This is best illustrated by considering the phase differences between the light and colour variations and the light and radial velocity variations.

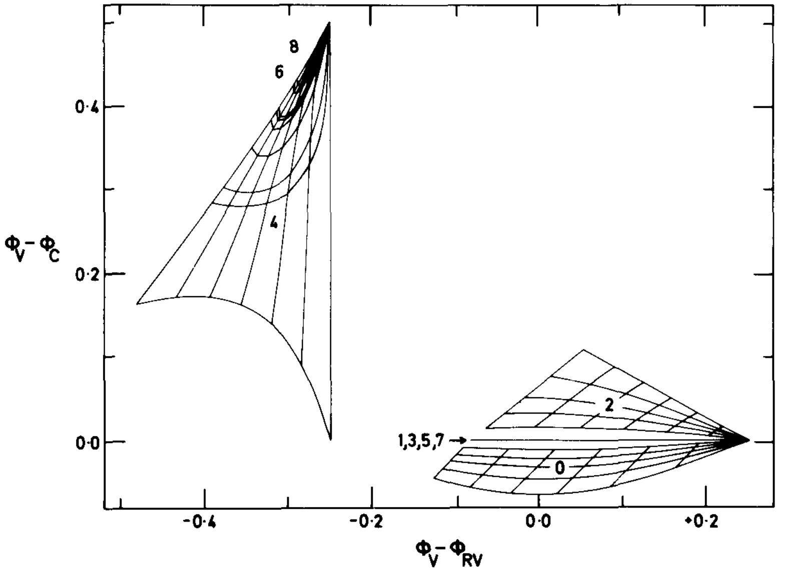

Fig. 1: Mode discrimination is obtained from the phase difference between the light and colour variations and from the phase difference between the light and radial velocity variations. The numbers indicate the spherical harmonic mode. For each mode lines of constant f and ψ are shown.

https://doi.org/10.1017/S0252921100081781 Published online by Cambridge University Press

Fig. 1 shows such a plot for a limb-darkening constant $\beta = 1$. For each mode lines of constant f for f = 5, 7, 10, 15, 30 and of constant ψ for ψ = 60, 80, 100, 120, 140, 160, 180° are shown. In general the different modes are well separated except for ψ near 180°. In particular the radial ($\ell = 0$), dipole ($\ell = 1$) and quadrupole ($\ell = 2$) modes are distinguished by negative, zero and positive phase shifts between the light and colour variations.

For stars where ψ is close to 180°, i.e. the β Cep variables, another method of mode identification must be used. Fig. 2 shows the quantities F_V/F_C and F_V/F_{RV} for f between 5 and 30 and for ψ = 160, 170, 180°. Good mode discrimination is also obtained; in particular ℓ = 0, 1 and 2 can be distinguished by the ratio F_V/F_C being less than one, equal to one or greater than one respectively. In order to apply this method, which is essentially the same as that proposed by Stamford and Watson (1978), the values of A and R_o must be known since

$$F_V/F_C = |\Delta V|/A|\Delta C| \quad \text{and} \quad F_V/F_{RV} = \omega R_o|\Delta V|/1.086|\Delta V_r|.$$

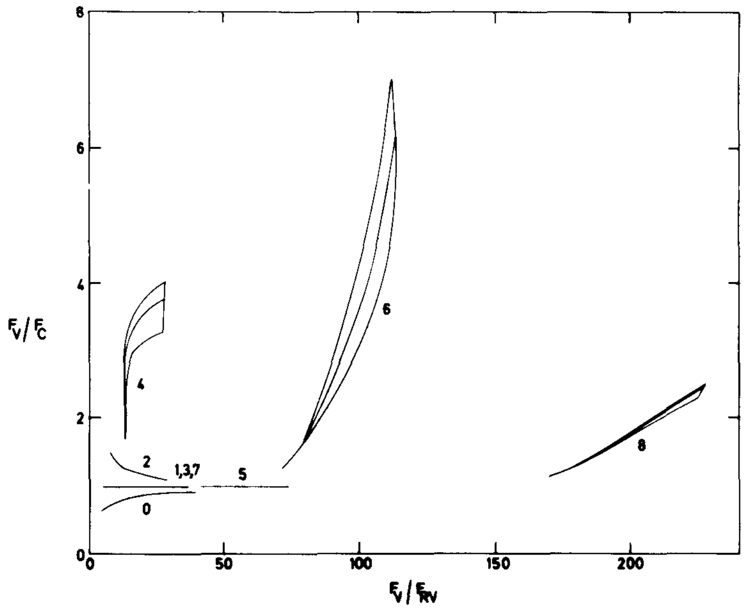

Fig. 2: For stars where ψ is close to 180° good mode discrimination is obtained from F_V/F_C and F_V/F_{RV}. These quantities involve the corresponding amplitude ratios. They are defined in the text.

https://doi.org/10.1017/S0252921100081781 Published online by Cambridge University Press

3. APPLICATION OF THE METHOD

Application of the method is very simple, even for multi-periodic stars. However, a large number of observations is required to ensure accurate phase differences or amplitudes. If σ is the standard error of one observation (in magnitudes) of the light or colour, then the standard error of the phase is:

$$\sigma_{\phi}^2 = 2\sigma^2/4\pi^2 N\Delta^2$$

where N is the number of observations and Δ is the semi-amplitude in magnitudes. Similarly the standard error in the amplitude is

$$\sigma_{\Delta}^2 = 2\sigma^2/N.$$

For a multi-periodic variable the observations must first be pre-whitened for all frequencies except the one under consideration. The phase difference between the light and colour variations or the amplitude ratio between these two variations then allows the mode to be estimated from Fig. 1 or Fig. 2. In general the variations will not be sinusoidal. However, in the linear approximation it is sufficient to take the phase difference or amplitude ratio from the fundamental component of a Fourier fit to the pre-whitened observations. Balona and Stobie (1980), Balona, Dean and Stobie (1980) and Kurtz (1980a, b) have successfully applied this method to δ Scuti variables.

REFERENCES

Balona, L.A., 1981. Mon. Not. R. astr. Soc., in press.
Balona, L.A. & Stobie, R.S., 1979a. Mon. Not. R. astr. Soc., 187, pp. 217-22.
Balona, L.A. & Stobie, R.S., 1979b. Mon. Not. R. astr. Soc., 189, pp. 649-58.
Balona, L.A. & Stobie, R.S., 1980. Mon. Not. R. astr. Soc., 190, pp 931-43.
Balona, L.A., Dean, J.F. & Stobie, R.S., 1980. Mon. Not. R. astr. Soc., in press.
Dziembowski, W., 1977. Acta Astronomica, 27, pp. 203-11.
Kurtz, D.W., 1980a. Mon. Not. R. astr. Soc., 191, pp. 115-22.
Kurtz, D.W., 1980b. Mon. Not. R. astr. Soc., in press.

DISCUSSION

SMITH: Could you comment on what the uncertainty in your technique might be due to errors in the assumed limb darkening law and due to nonadiabatic effects?

BALONA: The limb darkening law can be easily generalized and you find that it has almost no effect on the results. Our results don't really depend on whether it is adiabatic or nonadiabatic. All that it does is change your ψ and f value and we leave these as free parameters to be determined from the observations.

KEELEY: Why is it that you disregard the fact that the flux has a horizontal as well as vertical component?

BALONA: We don't. In the equations we have $\vec{n}\cdot\vec{u}$. That takes care of the line of sight.

KEELEY: Wait now. There are two components of the flux vector and you only broke down one of them, the perturbation to the radial part. What about the other part of the flux vector? You don't get it by projection.

BALONA: No, I have written down the equation giving the radiation perpendicular to the surface in, say, the V band which is the F and then applying the limb darkening effect gives the component in the direction of the observer.

HILL: You need to look at I and not at F.

BALONA: Oh, yes. I think perhaps there is confusion in the symbols that I used. Most people would use I where I used F.

BREGER: As you mentioned, the relationship between surface brightness and color should presumably have a small gravity term in it. Since you have evaluated the effect, could you tell us how important it is?

BALONA: Well, of course it depends on what color you use. Actually, I applied model atmospheres to the problem and found that there is really not much to choose between B-V and V-I. The effect just sort of distorts the phase shift diagram, but the shift is statistically of the same sort of order as you would expect from the errors of the observations.

PERCY: For β Cephei stars is the problem the lack of good quality simultaneous data, or is it the complexity involved in multimode behavior?

BALONA: I think it is more the lack of data, especially two color data.

FITCH: We also need simultaneous velocity data and there is not very much of that.