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A Note on 3-choosability of Planar Graphs Related to Montanssier's Conjecture

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Abstract. For a given list assignment $L = \{L(v) : v \in V(G)\}$, a graph $G = (V, E)$ is L-colorable if there exists a proper coloring c of G such that $c(v) \in L(v)$ for all $v \in V$. If G is L-colorable for every list assignment L having $|L(v)| \geq k$ for all $v \in V$, then G is said to be *k-choosable*. Montassier (Inform. Process. Lett. 99 (2006) 68-71) conjectured that every planar graph without cycles of length 4, 5, 6, is 3-choosable. In this paper, we prove that every planar graph without 5-, 6- and 10-cycles, and without two triangles at distance less than 3 is 3-choosable.

1 Introduction

All graphs considered in this paper are finite, simple planar graphs. A graph G is planar if G can be drawn on the plane so that its edges meet only at the vertices of the graph. A plane graph is such a particular drawing of a planar graph.

For a given list assignment $L = \{L(v) : v \in V(G)\}$, a graph $G = (V, E)$ is L-colorable if there exists a proper coloring c of G such that $c(v) \in L(v)$ for all $v \in V$. This coloring is also called an *L-coloring* of G. If G is L-colorable for every list assignment L having $|L(v)| \geq k$ for all $v \in V$, then G is said to be *k-choosable* or *k-list colorable*. The *choice number* of G, denoted by $\chi_l(G)$ or $ch(G)$, is the minimum k such that G is k-choosable.

All 2-choosable graphs were characterized completely in [5]. Thomassen proved that every planar graph is 5-choosable [16]. Examples of planar graphs that are not 4-choosable were given by Voigt [18] and by Mirzakhani [13], independently. Voigt and Writh [19], and Gutner [8] independently, presented some planar graphs of girth 4 that are not 3-choosable. Thus, it remains to determine whether a given planar graph is 3- or 4-choosable. In [8], Gutner proved that these problems are NP-complete. Therefore, many authors tried to find sufficient conditions for a planar graph to be 3- or 4-choosable. Alon and Tarsi [1] proved that every planar bipartite graph is 3-choosable. Thomassen [17] proved that every planar graph of girth 5 is 3-choosable. Lam *et al.* [9, 10] proved that plane graphs without i-cycles are 4-choosable, for $i = 3, 4, 5$, or 6. Independently, Wang and Lih [21] proved that planar graphs without 5-cycles are 4-choosable. Fijavž *et al.* [7] proved that planar graphs without 6-cycles are 4-choosable. Farzad [6] showed that a planar graph without 7-cycles is 4-choosable.

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Xu [24], and Wang and Lih [22] independently, proved that plane graphs without two triangles sharing a common vertex are 4-choosable.

For more new sufficient conditions for a planar graph to be 3-choosable, see [2–4, 11, 20, 23, 25]. The distance between two vertices x and y, denoted by $\operatorname{dist}(x, y)$, is the length of a shortest path connecting them in G. The distance between two triangles T and T' is defined to be the value $\min\{\operatorname{dist}(x, y) \mid x \in V(T) \text{ and } y \in V(T')\}$. In [14], Montassier et al. proved that every planar graph either without 4- and 5-cycles and without triangles at distance less than 4, or without 4-, 5-, and 6-cycles and without triangles at distance less than 3 is 3-choosable. In [15] Montassier proposed a conjecture that every planar graph without cycles of length 4, 5, 6, is 3-choosable.

Let $\mathcal{G}$ denote the set of planar graphs without triangles at distance less than 3 and without 5-, 6-, and 10-cycles. In this article, we focus on the 3-choosability of graphs in $\mathcal{G}$. More precisely, we prove the following result.

Theorem 1.1 *Every planar graph without 5-, 6-, and 10-cycles, and without two triangles at distance less than 3 is 3-choosable*

A result of Lovász [12] will be used in the proof of Lemma 3.2.

Theorem 1.2 (L. Lovász [12]) *Suppose that a list $L(x)$ of colours is associated with each vertex x of a graph G with $|L(x)| = d(x)$ for each x and $L(v) \neq L(w)$ for some v, w. Further assume G to be 2-connected. Then G admits a proper colouring that uses an element of $L(x)$ to colour x for each vertex x.*

Theorem 1.2 also shows that the only 2-connected graphs that are not L-list colorable with $|L(v)| = \Delta(G)$ for all $v \in V(G)$ are the complete graphs and the odd cycle with identical lists for all vertices.

2 Terminology and Notation

Let $G = (V, E, F)$ denote a planar graph, with V, E, and F being the set of vertices, edges, and faces of G, respectively. We use $b(f)$ to denote the boundary walk of a face f and write $b(f) = [v_1 v_2 v_3 \cdots v_n]$ if $v_1, v_2, v_3, \ldots, v_n$ are the vertices of $b(f)$ in a cyclic order. A face f is *incident* with all vertices and edges on $b(f)$. The *degree* of a face f of G, denoted by $d_G(f)$, is the number of edges incident with it, where cut edges are counted twice. A vertex (face) of degree k is called a *k-vertex* (*k-face*). If $r \leq k$ or $1 \leq k \leq r$, then a k-vertex (k-face) is called an r^+- or r^--vertex (r^+- or r^--face), respectively. If $S \subset V(G)$, then $G - S$ is the subgraph obtained from G by deleting the vertices in S and all the edges incident with some vertices in S. As usual, $G[S]$ is the subgraph of G induced by S. A k-cycle is a cycle with k edges. A chord of a k-cycle ($k \geq 4$) is an edge joining two nonconsecutive vertices on C.

Two adjacent faces are *normally adjacent* if they have only two vertices in common (clearly, the two common vertices are adjacent), or are *abnormally adjacent* (that is, they have at least three vertices in common). A triangle is synonymous with a 3-cycle.

For $x \in V(G) \cup F(G)$, we use $F_k(x)$ to denote the set of all k-faces that are incident with or adjacent to x, and $V_k(x)$ to denote the set of all k-vertices that are incident with or adjacent to x. Let v be a k-vertex. If $|F_3(v)| = 1$ and each other face incident with v is of degree at least 7, then v is *light*. If $|F_4(v)| = 1$ and each other face incident with v is of degree at least 7, then v is *sublight*. If v is only incident with 7^+-faces, then v is *good*. For convenience, we use $N(f)$ and $V(f)$ to denote the set of faces adjacent to the face f and vertices incident with f, respectively.

A face f of G is called a simple face if $b(f)$ forms a cycle. Obviously, when $\delta(G) \geq 2$ for $k \leq 5$, or G is 2-connected, each k-face is a simple face. A face f is *frugal* if some vertex t is incident to it at least twice. Obviously, any 5-face, 6-face, or 10-face in G must be a *frugal* face for its properties. Moreover, G contains no *frugal* 6-face because any *frugal* 6-face must be constructed by two intersecting 3-cycles, it is a contradiction to the triangles-distance condition. A list of faces of a vertex v is *consecutive* if it is a sublist of the list of faces incident to v in cyclic order.

3 Structures of a Minimum Counterexample

In this section, we always assume that G is a counterexample of Theorem 1.1 with $|V(G)|$ minimum. Then G is connected, having neither a 5-, 6-, or 10-cycle nor two triangles at distance less than 3. Clearly $|V(G)| \geq 3$, and $|F_3(f)| \leq \lfloor \frac{d(f)}{4} \rfloor$ for G contains no triangles at distance less than 3.

Lemma 3.1 *G does not contain any vertices of degree less than* 3*.*

Proof Suppose that G contains a vertex u of degree less than 3. We choose a vertex v from $V(G) \backslash \{u\}$ as the precolored one. By the minimality of G, $G - u$ admits an L-coloring c. In G, we can color u with a color in $L(u)$ different from the colors of its neighbors to extend c to an L-coloring of G. This is a contradiction. ■

Lemma 3.2 *Any even circuit C in G contains at least one 4^+-vertex.*

Proof Let $V(C) = \{v_1, v_2, \dots, v_{2n}\}$. Suppose $d_G(v) = 3$ for all $v \in C$ by Lemma 3.1, and L is a color-list of G with $|L(v)| = 3$ for all $v \in V(G)$. By assumption, there exists an L_0-coloring ϕ_0 of $G_0 = G - C$, where L_0 is the restriction of L to $V(G_0)$. Let $L' = \{L'(v_i) : 1 \leq i \leq 2n\}$, where $L'(v_i) = L(v_i) \setminus \{\phi_0(u) : u \in N_G(v_i) \setminus C\}$.

If C has no chord, it is clear that $|L'(v_i)| \geq 2$. Since every even cycle is 2-choosable, there exists an L'-coloring ϕ' on C. An L-coloring of G immediately follows by combining ϕ_0 and ϕ'. If C contains chords. It is easy to see that in this case $|L'(v)|$ is equal to the degree of v in the subgraph of G induced by $V(C)$, and this subgraph is 2-connected and distinct from K_4. If there exists a vertex with degree 2 in $G[C]$, then we get the proof by Theorem 1.2. Otherwise, all the vertices in $V(G[C])$ are of degree 3. We also get an L'-coloring of C, because C is an even circuit. ■

Lemma 3.3 *Neither a 3-face nor a 4-face is adjacent to a 4-face.*

Proof Because $\delta(G) \geq 3$, any 3-faces and 4-faces are simple faces. If the 3-face or 4-face is normally adjacent to a 4-face, a 5-cycle or 6-cycle will appear, contradicting

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the fact G contains no 5-cycles and 6-cycles. But if a 3-face or a 4-face is abnormally adjacent to a 4-face, we will get two triangles with distance less than 3. ■

Lemma 3.4 *Neither a 3-face nor a 4-face is* abnormally adjacent *to a 7-face.*

Proof Let the 7-face $b(f_1) = [v_1v_2u_1u_2u_3u_4u_5]$. First, we consider f_1 is a *frugal* 7-face, as depicted in Figure 1. In this case, we see that f_1 cannot be adjacent to any 3-face, because of the triangles-distance condition, and f_1 cannot be adjacent to any 4-face by Lemma 3.3. So we assume that f_1 is a simple face in the following discussion.

Figure 1: Two kinds of frugal 7-faces

Let f be a 3-face with $b(f) = [v_1v_2v_3]$, and let f_1 be a 7-face with $b(f_1) = [v_1v_2u_1u_2u_3u_4u_5]$. We now prove that $v_3 \cap \{u_1, u_2, u_3, \ldots, u_5\} = \phi$. Clearly we have $v_3 \neq u_1$, or we will get that $d(v_2) = 2$, a contradiction to $\delta(G) \geq 3$. Next we can get $v_3 \neq u_2$; otherwise, we have two adjacent triangles $v_1v_2u_2$ and $v_2u_1u_2$. Then $v_3 \neq u_3$, or G contains a 5-cycle $v_1v_2u_1u_2u_3v_1$. By similar argument, we can also get that $v_3 \notin \{u_4, u_5\}$.

Let f_1 be an arbitrary 4-face with $b(f_1) = [v_1v_2v_3v_4]$ that is abnormally adjacent to a 7-face f_2 with $b(f_2) = [v_1v_2u_1u_2u_3u_4u_5]$. Then $\{v_3, v_4\} \cap \{u_1, u_2, u_3, u_4, u_5\} \neq \phi$. But this is impossible. First we can show that $v_3 \neq u_1$, or there will be a vertex v_2 of degree 2. By similar analysis, we have $v_3 \neq u_2$, $v_3 \neq u_3$, $v_3 \neq u_4$, and $v_3 \neq u_5$, since G has no 5- and 6-cycles. So $v_3 \notin \{u_1, u_2, u_3, u_4, u_5\}$. Similarly, $v_4 \notin \{u_1, u_2, u_3, u_4, u_5\}$. ■

Lemma 3.5 *Let f_1 be a 7-face in $F(G)$. Then $|F_4(f_1)| = 0$ if $|F_3(f_1)| = 1$.*

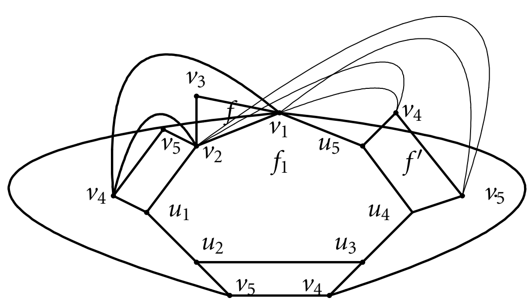

Figure 2. Case in Lemma 3.5

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Proof A *frugal* 7-face cannot be adjacent to a 3-face by the proof of Lemma 3.2. So we can consider f_1 to be a simple 7-face. Let f be a 3-face with $b(f) = [v_1v_2v_3]$, and let f_1 be a 7-face with $b(f_1) = [v_1v_2u_1u_2u_3u_4u_5]$. By Lemma 3.4, any 3-face cannot be abnormally adjacent to f_1. Then the 3-face f is normally adjacent to the 7-face f_1.

Suppose there also is a 4-face f' with $b(f') = [x_ix_jv_4v_5]$ that is normally adjacent to f_1 ($x_ix_j \in \{v_2u_1, u_1u_2, u_2u_3, u_3u_4, u_4u_5, u_5v_1\}$), as shown in Figure 2. Obviously, we have $v_3 \in \{v_4, v_5\}$, otherwise we will get a 10-cycle, a contradiction to the choice of G. By symmetry, we consider the following three cases. While $b(f') \cap b(f_1) = v_2u_1$, if $v_3 = v_5$, there will be a 3-face f adjacent to a 4-face f', a contradiction by Lemma 3.3 If $v_3 = v_4$, two adjacent 3-cycles $v_2v_5v_4v_2$ and $v_1v_2v_4v_1$ contradict the choice of G. While $b(f') \cap b(f_1) = u_4u_5$, if $v_3 = v_4$, $v_1v_2(v_3 = v_4)v_5u_4u_5v_1$ is a 6-cycle, while $v_3 = v_5$, $v_1v_2(v_3 = v_5)u_4u_5v_1$ is a 5-cycle. While $b(f') \cap b(f_1) = u_2u_3$, if $v_3 = v_4$, $v_1(v_3 = v_4)u_3u_4u_5v_1$ is a 5-cycle, while $v_3 = v_5$, $v_1v_2u_1u_2(v_3 = v_5)v_1$ is a 5-cycle. This contradiction completes the proof. ∎

Lemma 3.6 *No* 3*-face is abnormally adjacent to an* 8*-face. Moreover, if* $d(f) = 8$, *then* $|F_3(f) \leq 1$.

Proof If the 8-face is a frugal face, then it must be isomorphic to the case in Figure 3, so a *frugal* 8-face cannot be adjacent to any 3-face by Lemma 3.3. Next we consider a simple 8-face. Suppose to the contrary that G has a 3-face f with $b(f) = [v_1v_2v_3]$ that is abnormally adjacent to an 8-face f_1 with $b(f_1) = [v_1v_2u_1u_2u_3u_4u_5u_6]$. Then $v_3 \cap \{u_1, u_2, u_3, \cdots, u_6\} \neq \phi$. We now prove that $v_3 \cap \{u_1, u_2, u_3, \cdots, u_6\} = \phi$. Clearly, we can get $v_3 \neq u_1$, or we will get that $d(v_2) = 2$, a contradiction to $\delta(G) \geq 3$. Next we show that $v_3 \neq u_2$, otherwise we have two adjacent triangles $v_1v_2u_2$ and $v_2u_1u_2$. Then $v_3 \neq u_3$, or G contains a 5-cycle $v_1v_2u_1u_2u_3v_1$. By a similar argument, we also can get that $v_3 \notin \{u_4, u_5, u_6\}$. It is easy to see that any 8-face is *normally adjacent* to at most 1 triangles; otherwise, G will contain a 10-cycle. ∎

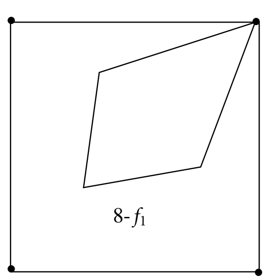

Figure 3. A frugal 8-face f_1

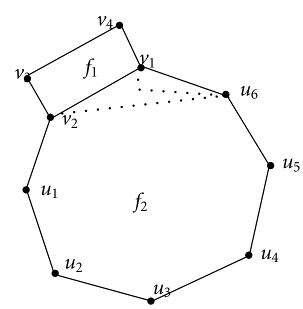

Figure 4.

Lemma 3.7 *No* 4*-face is* normally adjacent *to an* 8*-face. And any* 8*-face is abnormally adjacent to at most one* 4*-face.*

Proof Obviously, we can assume that the 8-face here is a *simple* face. Or it cannot be adjacent to any 4-faces. Let f_1 be an arbitrary 4-face with $b(f_1) = [v_1v_2v_3v_4]$ that is adjacent to an 8-face f_2 with $b(f_2) = [v_1v_2u_1u_2u_3u_4u_5u_6]$. If f_1 is *normally adjacent*

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to f_2, then G has a 10-cycle. So two faces must be *abnormally adjacent*. It is easy to see that $v_3 \neq u_1$, or there will be a vertex v_2 of degree 2. We can also show that $v_3 \neq u_2$, $v_3 \neq u_3$, $v_3 \neq u_4$ and $v_3 \neq u_5$ since G has no 5- and 6-cycles. The only case is $v_3 = u_6$ as shown in Figure 4, at this case, f_2 is an exterior face that is adjacent to f_1 on v_1v_2 and $v_3 = u_6$, $v_4 \in \operatorname{int}(C_1)$, here $C_1 = [v_1v_2(v_3 = u_6)v_1]$. Assume that f_2 is abnormally adjacent to another 4-face f_3; then we can conclude that f_3 cannot be abnormally adjacent with f_2 on v_1v_2 or v_1u_6; otherwise, it is a contradiction to planarity of G. Note that any chord of the cycle $C_2 = v_2u_1u_2u_3u_4u_5u_6v_2$ or possible path of length 2 with its endvertices on $b(C_2)$ will induce a 5- or 6-cycle of G. ■

4 Proof of Theorem 1.1

Proof of Theorem 1.1 Proceeding by contradiction, if the conclusion is not valid, then we choose a counterexample as defined in the last section. We define a weight ω on $V \cup F$ by letting $\omega(x) = d_G(x) - 6$ if $x \in V$ and $\omega(x) = 2d_G(x) - 6$ if $x \in F$. By Euler's formula for planar graphs, we have $\sum_{x\in V\cup F} \omega(x) = -12$. If we obtain a new nonnegative weight $\omega^*(x)$ for all $x \in V \cup F$ by transferring weights from one element to another, then we have

$$-12 = \sum_{x\in V\cup F} \omega(x) = \sum_{x\in V\cup F} \omega^*(x) \geq 0.$$

This contradiction will complete the proof of Theorem 1.1.

Our transferring rules are as follows, in which, f is a l-face ($l \geq 4$) and v is a k-vertex on $b(f)$.

(R1) Charge to a 3-vertex v.

- (R1.1) Every 7^+-face f transfers 1 and $\frac{7}{6}$ to each incident good and sublight v, respectively.
- (R1.2) Every 7^+-face f transfers $\frac{3}{2}$ to each incident light v.
- (R1.3) Every 4-face f transfers $\frac{2}{3}$ to each incident sublight v.

(R2) Charge to a 4-vertex v.

Every 7^+-f transfers $\frac{1}{2}$ to each incident v in the following cases, 1 to v otherwise.

- (R2.1) v is good.
- (R2.2) v is light and f is adjacent to the 3-face in $F(v)$.
- (R2.3) v is sublight and f is adjacent to the 4-face in $F(v)$.

(R3) Every 7^+-face f transfers $\frac{1}{3}$ to a 5-vertex v.

The rules are illustrated in Figure 5. First we consider v to be a vertex with $d(v) = k$. If $d(v) \geq 6$, then $w^*(v) \geq 0$.

If $d(v) = 5$, then v is incident with at most one triangle by the distance condition, or at most two 4-faces by Lemma 3.3; that is, $|F_3(f)| \leq 1$, $|F_4(f)| \leq 2$. If $|F_4(f)| = 2$, then v is not incident with any 3-faces by Lemma 3.3, so $|F_{7^+}(v)| \geq 3$, and we have $w^*(v) \geq w(v) + \frac{1}{3} * 3 = 0$ by (R3).

If $d(v) = 4$ or $d(v) = 3$, we can easily get $w^*(v) \geq 0$ by the diagram calculations in Figure 5.

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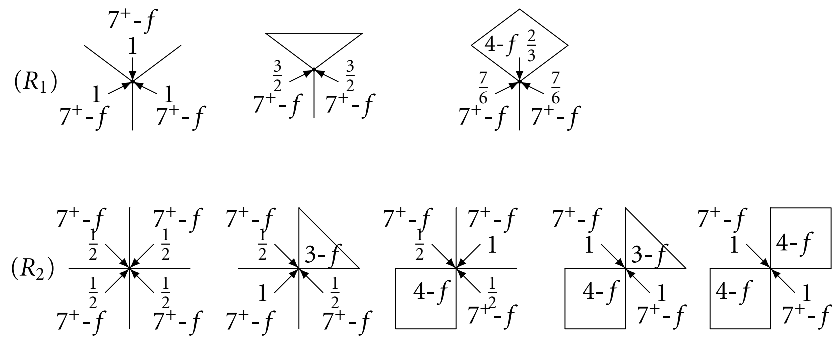

Figure 1

Now let f be a face with $d(f) = h$. The proof is divided into five cases according to the value of h.

Case 1 $h = 3$. Then $\omega^*(f) = \omega(f) = 0$, since no charge is discharged from or to f.

Case 2 $h = 4$. Then $\omega(f) = 2$.

By Lemma 3.2, f is incident with at least one 4^+-vertex. Because f transfers weight only to the incident sublight 3-vertex, we derive that $\omega^*(f) \geq \omega(f) - 3 \times \frac{2}{3} = 0$ by (R1.3).

For convenience, we denote by p the number of light 3-vertices, by q the number of sublight 3-vertices and by r the number of rest vertices on $b(f)$, respectively.

Because whatever happens, the weight $\frac{1}{3}$, which transferred from a 7^+-face to the incident 5-vertices, is less than the weight which transferred from a 7^+-face to 4^--vertices. So we only consider the 4^--vertices on $b(f)$. Moreover, because the weight transferred from a 7^+-face to light 3-vertices is $\frac{3}{2}$ by (R1.2), which is more than $\frac{7}{6}$ transferred from a 7^+-face to sublight 3-vertices by (R1.1) and is more than weight transferred from a 7^+-face to the vertices in other cases. So we consider the case that p is as large as possible, and assume f transfers 1 to every incident 4-vertex.

Case 3 $h = 7$. Then $\omega(f) = 8$.

$|F_3(f)| \leq 1$ by $|F_3(f)| \leq \lfloor \frac{d(f)}{4} \rfloor$, so $p \leq 2$. And $|F_4(f)| = 0$, while $|F_3(f)| = 1$ by Lemma 3.5. If $p = 0$, then $q \leq 6$ by Lemma 3.3, and hence $w^*(f) \geq w(f) - \frac{7}{6} \times q - 1 \times r \geq 8 - \frac{7}{6} \times 6 - 1 \times 1 = 0$. If $1 \leq p \leq 2$, then $|F_3(f)| = 1$ and $|F_4(f)| = 0$, so $q = 0$. We have $\omega^*(f) \geq \omega(f) - \frac{3}{2} \times p - \frac{7}{6} \times q - 1 \times r \geq 8 - \frac{3}{2} \times 2 - 1 \times 5 = 0$.

Case 4 $h = 8$. Then $\omega(f) = 10$.

By Lemma 3.7, $|F_4(f)| \leq 1$, so $q \leq 2$, and $p \leq 2$ for $|F_3(f)| \leq 1$ by Lemma 3.6. So $\omega^*(f) \geq \omega(f) - \frac{3}{2} \times p - \frac{7}{6} \times q - 1 \times r \geq 10 - \frac{3}{2} \times 2 - \frac{7}{6} \times 2 - 1 \times 4 = \frac{2}{3} > 0$.

Case 5 $h \geq 9$. Then $\omega(f) \geq 12$.

For $G \in \mathcal{G}$, we have $|F_3(f)| \leq \lfloor \frac{d(f)}{4} \rfloor$. Even if f is incident with $2 \times \lfloor \frac{h}{4} \rfloor$ light 3-vertices and transfers $\frac{7}{6}$ to other sublight 3-vertices on $b(f)$, then

$$\omega^*(f) \geq \omega(f) - \frac{3}{2} \times 2 \times \left\lfloor \frac{h}{4} \right\rfloor - \frac{7}{6} \times \left(h - 2 \times \left\lfloor \frac{h}{4} \right\rfloor\right) = \frac{5h}{6} - 6 - \frac{2}{3} \times \left\lfloor \frac{h}{4} \right\rfloor.$$

Consider that $\frac{h}{4} - 1 < \lfloor \frac{h}{4} \rfloor \leq \frac{h}{4}$, so $\omega^*(f) \geq \frac{5h}{6} - 6 - \frac{2}{3} \times \frac{h}{4} = \frac{4h-36}{6} \geq 0$ when $h \geq 9$. ■

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