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EXTENDED OPTIMA AND EQUILIBRIA FOR CONTINUOUS GAMES. III. COMPARISON WITH BARGAINING EXPERIMENTS

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Abstract

The new optima and equilibria discussed in the preceding two papers are compared with the results of bargaining experiments between two and three players performed by Fouraker and Siegel. Experiments where players have complete or incomplete information are considered. There is clear evidence that the new optima are operating, and that traditional optima–Cournot, Pareto and competitive (threat)–are less satisfactory in explaining the course of the whole bargaining process.

Fouraker and Siegel [1] carried out a large number of controlled experiments which were designed to study the competitive behaviour of firms in a market. The experiments included quantity variation competition between 2 players (duopoly) and between 3 players (triopoly). Comparison of the results with traditional predictions of optimal behaviour–the *Cournot*, *Pareto* and *competitive* (or *threat*) optima–were made. In the present paper we show that the *extended optima* introduced in papers [2] and [3] give a more satisfactory agreement with the experimental results, and that, in particular, they can provide a simple and reasonable description of the bargaining sequence.

The quantity variation competition experiments in [1] take the form of a continuous *game*, in which all firms produce the same good, but choose their own levels of output, the prices paid being determined by the buyers. The profit to player i in one transaction or business period is

$$J_i(\sigma) = 0.04\sigma_i(60 - \sigma_T) \tag{1.1}$$

cents, where σ_i is the quantity produced by player i,

$$\sigma_T = \begin{cases} \sigma_1 + \sigma_2 & \text{for duopoly} \\ \sigma_1 + \sigma_2 + \sigma_3 & \text{for triopoly} \end{cases}$$

and $\boldsymbol{\sigma}$ is (σ_1, σ_2) for duopoly, $(\sigma_1, \sigma_2, \sigma_3)$ for triopoly. In each transaction, each player chooses a value σ_i for his proposed output without communication with his competitors or knowledge of their output choices. Then each player receives a profit given by (1.1). Any cash profit made by a player is his to keep, so it is reasonable to assume that all players are attempting to maximize their own J_i's.

Two versions of the game were examined, called *complete* and *incomplete information* games. In the former, each player was presented with the same profit table, which listed his own profit J_1, say, and the profit J_2 (or $J_2 + J_3$ for triopoly) of his competition, for a range of values of σ_1 and σ_2 (or $\sigma_2 + \sigma_3$ for triopoly). The values of σ_i tabulated (and hence admissible) were the integers 8, 9, 10, . . . , 32. The functions (1.1) were not revealed to the players except by implication from the table. In the game of incomplete information the profit J_2 (or $J_2 + J_3$) made by the competition was omitted from the table, but the game was otherwise the same. In all tables, a degree of rounding took place to simplify the entries; possible effects of this on the games are discussed in Sections 3, 5 and 6.

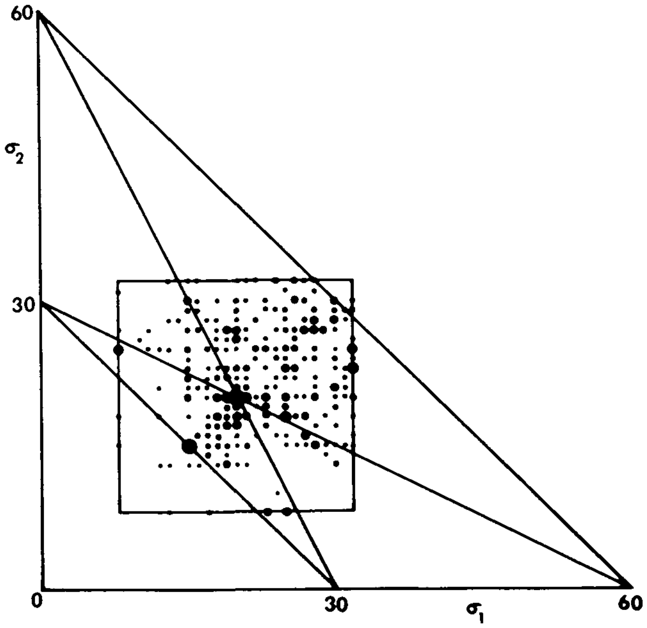

Fig. 1. Output choices made in the experiments of Fouraker and Siegel in the case of two player competition with complete information.

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In all games a total of 25 transactions took place, the first 3 being trials not resulting in a real monetary profit. In each of the two versions of the duopoly game there were 16 pairs of players, each carrying out the 25 transactions. In each of the two versions of the triopoly game there were 11 trios, also each carrying out 25 transactions. No person was a player in more than one game. The large amount of data resulting from these replications of the four different experiments provides the raw material for quite a rigorous test of any theory about bargaining behaviour in such games.

As an example of the results, Figure 1 gives the quantity choices of all 16 pairs in all 25 transactions of the duopoly game with complete information (here the size of dot indicates the number of times the pair occurs). We have superimposed all this data because it reveals properties of the "average behaviour" of players which is not as clear from data on individual transactions or individual pairs. While our theory is generally applicable at an individual level, its effects may be more apparent in the aggregate.

2. Theories of bargaining behaviour for duopolists

Fouraker and Siegel [1] compare their results with 3 traditional game theory optima.

(i) The *Cournot optimum*. Here both players maximize their own profit on the assumption that their competitor holds his output fixed. This implies that

$$\frac{\partial J_i}{\partial \sigma_i} = 0 \text{ for } i = 1, 2, \tag{2.1}$$

which, from (1.1), gives

$$\sigma_1 = \sigma_2 = 20. \tag{2.2}$$

(ii) The *Pareto optimum*. In the present case this is just the set of output pairs which maximizes the joint profit of the 2 players. Thus

$$\frac{\partial}{\partial \sigma_i}(J_1 + J_2) = 0 \text{ for } i = 1, 2$$

which, from (1.1), gives

$$\sigma_1 + \sigma_2 = 30. \tag{2.3}$$

This optimum is intended to describe the behaviour of cooperating players since it results in mutual benefit. Clearly, it can also be described as a set of σ's such that any move away from the set results in disadvantage to at least one player. The manner in which the joint profit (36 cents) is shared remains unspecified in theory, but is determined in practice by the bargaining process.

(iii) The *competitive optimum*. Here at least one player chooses an output which reduces the profit of each player to zero. Thus

$$\sigma_1 + \sigma_2 = 60 \tag{2.4}$$

in this case, while individual outputs remain unspecified. This is sometimes called a *threat optimum* or *threat curve* [5] since it is envisaged that it would only be realized if imposed by a strong firm, with large assets perhaps, which failed to gain the compliance of a competitor.

In papers [2] and [3] we introduced several classes of new optima which happen to coincide for duopolies; more will be said about the different classes in the triopoly analysis. These optima take account of the possibility that a firm can be disciplined by its competitor if it tries to make a profit-increasing adjustment. More precisely, if firm 1 makes a small change ε in its output σ_1 thereby increasing its profit, $J_1(\sigma_1 + \varepsilon, \sigma_2) > J_1(\sigma_1, \sigma_2)$, then firm 2 can make a small change δ in its output σ_2 so as to restore its own profit, $J_2(\sigma_1 + \varepsilon, \sigma_2 + \delta) = J_2(\sigma_1, \sigma_2)$, and leave firm 1 with a net reduction in profit, $J_1(\sigma_1 + \varepsilon, \sigma_2 + \delta) < J_1(\sigma_1, \sigma_2)$. Firm 2 is said to have *disciplined* firm 1.

In our new optima, called *extended optima*, each firm can discipline the other. Such a state $\boldsymbol{\sigma}$ is an optimum for sophisticated bargainers in the sense that each firm is reluctant to move through fear of being disciplined, and gains a sense of stability through the knowledge that it can discipline its competitor if necessary. For unsophisticated players, or players with inadequate information, it would seem to influence players over successive transactions through their being accidentally disciplined for unsuitable moves (a rigorous proof is given in [4] for a particular adjustment process).

The resulting extended optima comprise (see [3], equation (4.10)) a set O_I^W with two triangular regions, within $\sigma_1 \geqslant 0$, $\sigma_2 \geqslant 0$,

$$\{\sigma_1 + \sigma_2 \geqslant 30,\ \sigma_1 + 2\sigma_2 \leqslant 60,\ 2\sigma_1 + \sigma_2 \leqslant 60\} \tag{2.5}$$

and

$$\{\sigma_1 + \sigma_2 \leqslant 60,\ \sigma_1 + 2\sigma_2 \geqslant 60,\ 2\sigma_1 + \sigma_2 \geqslant 60.\} \tag{2.6}$$

These are shown in Figure 1. They contain all three traditional optima discussed above. Our optima therefore incorporate a variety of different bargaining behaviours, and provide a unification and extension of the traditional concepts of optimal behaviour. From a practical point of view this would seem to offer advantages, because in predicting gross market behaviour it is unlikely that one would be able to characterize the set of firms as cooperative, threatening or as independent maximizers. Different firms are likely to behave differently.

3. Comparisons with duopoly data

In the game with complete information (experiment 10 of [1]), Fouraker and Siegel show that there is no statistically significant preference for any of the 3 traditional optima in the data. Although they considered in detail only the 24th transaction, the same applies to any transaction, or to the whole set of transactions, as is fairly clear from Figure 1. It is apparent also from Figure 1 that, on a per area basis, there is a disproportionately large number of points $\boldsymbol{\sigma}$ in the triangles (2.5) and (2.6) defining our extended optima, and a distinct paucity of points in the remaining regions of the admitted production space, namely the square $8 \leqslant \sigma_1 \leqslant 32$, $8 \leqslant \sigma_2 \leqslant 32$.

To make this more precise, let $n(t)$ be the number of points $\boldsymbol{\sigma}$ *not* in (2.5) or (2.6), from all pairs of players, during transaction number t, where $t = 1, 2, \ldots, 25$. Thus a fraction $n(t)/16$ are *not* extended optima. Of the 625 possible points, 259 are not in (2.5) or (2.6). If the $\boldsymbol{\sigma}$'s at transaction t were purely random, then they should be uniformly distributed among the 625 points, implying that $n(t)/16$ would equal 259/625, and hence that

$$r(t) \equiv 625n(t)/4144 \tag{3.1}$$

would equal unity, at least within a calculable statistical tolerance. Table 1 shows that $r(t)$ becomes significantly less than 1 as t increases; the statistical assessment of this significance is discussed later in Section 6. To clarify the trend, the proportions

$$R(t) = \{r(1) + r(2) + \cdots + r(t)\}/t \tag{3.2}$$

of points from the first t transactions, which are not extended optima are also listed in Table 1. This shows a fairly smooth decline from $R(1) \simeq .75$ to $R(25) \simeq .36$. The value .75 is barely significantly less than 1, indicating that initial products could be considered as random. Thereafter the players apparently learn by experience the disadvantage of adopting a $\boldsymbol{\sigma}$ which is not an extended optimum. Roughly speaking, once the firms have chosen a $\boldsymbol{\sigma}$ which is an extended optimum, each firm is likely to be disciplined by its competitor if it tries to increase its profit. There is therefore an increasing tendency for each firm not to move from such a $\boldsymbol{\sigma}$.

The figures offer clear support for the applicability of the extended optimum, at least when the alternative is the rather implausible hypothesis of uniform random choice of $\boldsymbol{\sigma}$ at each transaction. Are there any more plausible alternative explanations of the data? Looking at Figure 1 we see that the most obvious paucity of points is in the 2 triangles

$$\{\sigma_1 \geqslant 8, \sigma_2 \geqslant 8, \sigma_1 + \sigma_2 < 30\} \tag{3.3}$$

TABLE 1

Experiment 10: duopoly, complete information

t	$r(t)$	$R(t)$	$s(t)$	$S(t)$
1	.75	.75	.76	.76
2	.60	.68	.89	.83
3	.60	.65	.71	.79
4	.60	.64	.89	.81
5	.15	.54	.22	.69
6	.75	.58	.94	.73
7	.30	.54	.44	.69
8	.30	.51	.44	.66
9	.15	.47	.22	.61
10	.60	.48	.71	.62
11	.15	.45	.22	.58
12	.00	.41	.00	.53
13	.00	.38	.00	.49
14	.45	.39	.66	.50
15	.15	.37	.22	.48
16	.15	.36	.22	.47
17	.00	.34	.00	.44
18	.60	.35	.71	.45
19	.45	.36	.25	.44
20	.45	.36	.47	.44
21	.15	.35	.22	.43
22	.45	.36	.47	.43
23	.30	.35	.44	.44
24	.45	.36	.47	.44

and

$$\{\sigma_1 \leqslant 32, \sigma_2 \leqslant 32, \sigma_1 + \sigma_2 > 60\}. \tag{3.4}$$

One might then entertain the hypothesis, H1, that the only distinct preference is for the region $30 \leqslant \sigma_1 + \sigma_2 \leqslant 60$ between the Pareto optima $\sigma_1 + \sigma_2 = 30$ and the competitive optima $\sigma_1 + \sigma_2 = 60$, and that within this region the $\boldsymbol{\sigma}$'s are purely random.

To test this hypothesis, let $m(t)$ be the number of points $\boldsymbol{\sigma}$ which fall in this region during transaction t. There are 510 possible $\boldsymbol{\sigma}$'s in the region, and 144 of these are not in the triangles (2.5) and (2.6). If $\mu(t)$ is the number of $\boldsymbol{\sigma}$'s from transaction t which lie in $30 < \sigma_1 + \sigma_2 < 60$ but not in (2.5) or (2.6) then the hypothesis implies that the proportion

$$s(t) = 510\mu(t)/\{144m(t)\} \tag{3.5}$$

of $\boldsymbol{\sigma}$'s, on a per area basis, which are not extended optima should be insignificantly different from unity. Table 1 shows that, in fact, the $s(t)$ are very significantly less than unity, and that the accumulated proportions

$$S(t) = 510\sum_1^t \mu(r)/\left\{144\sum_1^t m(r)\right\} \tag{3.6}$$

up to transaction t, decline from early values of about .8 to a final value of .44. Hypothesis H1, therefore, receives significantly less support from the data than do the extended optima.

A second hypothesis, H2, is that the Cournot solution alone is preferred, producing an excess concentration of $\boldsymbol{\sigma}$'s near $\sigma_1 = \sigma_2 = 20$ and a resulting paucity of $\boldsymbol{\sigma}$'s near the corners of the square in Figure 1. To test this hypothesis, one can look at points $\boldsymbol{\sigma}$ contained in discs of various radii centred at $\sigma_1 = \sigma_2 = 20$. The proportions per area from all transactions, R_i say, that are in such a disc of radius i but *not* in the triangles (2.5) and (2.6) are as follows:

i	=	2	3	4	5	6	7	8	9	10	11	12
R_i	=	0	.23	.54	.58	.62	.69	.55	.61	.57	.53	.47

Since all R_i are very significantly less than 1, H2 also receives significantly less support from the data than do the extended optima.

Note, however, that a concentration of points $\boldsymbol{\sigma}$ at the Cournot optimum $\sigma_1 = \sigma_2 = 20$ or on the lines $\sigma_1 = 20$ and $\sigma_2 = 20$ (corresponding to at least one player choosing his production at the Cournot value) would favour the extended optima since such points are contained in the triangles (2.5) and (2.6). Thus suggests the hypothesis, H3, that at least one player is adopting the Cournot optimum. To test this one can look at the proportions R_i' defined as the R_i above except that points on the lines $\sigma_1 = 20$ and $\sigma_2 = 20$ are omitted. The results are:

i	=	2	3	4	5	6	7	8	9	10	11	12
R_i'	=	0	.57	.97	1.02	.90	.98	.65	.71	.67	.62	.59

These figures are less clear-cut than previously. Once the Pareto optimum point (15, 15) has entered the disc, at $i = 8$, the ratios do remain significantly less than unity. The midrange ratios, however, are compatible with H3, suggesting that if one or both players are not producing at the Cournot value 20 they are relatively indifferent as to what $\boldsymbol{\sigma}$ is, provided they are reasonable close to the Cournot optimum. We suggest one reason why this result should be regarded with caution in Section 6.

Clearly, one can continue to construct variations on these hypotheses but they are likely to become progressively more arbitrary and data-inspired. Without fresh experiments to examine them, therefore, they must be treated extremely cautiously.

For the game with incomplete information studied in experiment 7 of Fouraker and Siegel [1], the same analyses give the results shown in Table 2. Again $R(t)$ declines from an early value of .75 to a final value of .40, which confirms a tendency towards the extended optimum. Similarly $S(t)$ declines to .44, which is significant evidence against hypothesis H1.

TABLE 2

Experiment 7: duopoly, incomplete information

t	$r(t)$	$R(t)$	$s(t)$	$S(t)$
1	.45	.45	.25	.25
2	.60	.53	.51	.38
3	1.21	.75	.97	.54
4	.30	.64	.24	.46
5	.30	.57	.24	.41
6	.60	.58	.71	.46
7	.60	.58	.51	.47
8	.90	.62	1.18	.56
9	.60	.62	.51	.56
10	.15	.57	.22	.52
11	.00	.52	.00	.47
12	.30	.50	.44	.47
13	.60	.51	.71	.48
14	.30	.50	.44	.48
15	.15	.47	.22	.46
16	.30	.46	.44	.46
17	.15	.44	.22	.45
18	.15	.43	.22	.43
19	.90	.45	1.33	.48
20	.15	.44	.22	.47
21	.15	.42	.22	.46
22	.30	.42	.24	.45
23	.15	.41	.22	.44
24	.30	.40	.44	.44
25	.30	.40	.44	.44

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Fouraker and Siegel [1] found that this experiment offers strong support for the Cournot optimum. There is a very evident concentration of points $\boldsymbol{\sigma}$ on and near $\sigma_1 = \sigma_2 = 20$ in the later transactions, but the following values of R_i:

i	=	2	3	4	5	6	7	8	9	10	11	12
R_i	=	.12	.31	.35	.38	.37	.57	.63	.52	.48	.45	.45

continue significantly less than 1, so that hypothesis H2 is inadequate to explain the data. The values of R_i':

i	=	2	3	4	5	6	7	8	9	10	11	12
R_i'	=	1.00	1.04	.98	.99	.75	1.08	1.11	.91	.85	.76	.76

lie rather close to one except for large i so that hypothesis H3 might be considered almost adequate. Inspection of the data (page 256 of [1]), however, reveals that the behaviour of player pair 14 is anomalous. If this pair is omitted the new R_i's are

i	=	2	3	4	5	6	7	8	9	10	11	12
R_i'	=	1.00	1.04	.98	1.00	.76	.97	.83	.67	.62	.57	.59

and these cast much more doubt on the adequacy of H3.

It is not surprising that the Cournot optimum is favoured in experiment 7 but not in experiment 10. In the former, the lack of information makes it fairly natural to follow the *Cournot strategy*, namely to maximize one's profit at each transaction on the assumption that one's competitor's output has its value at the preceding transaction. This leads precisely to the Cournot solution if followed exactly (Theocharis [6]). For the experiment 10 with complete information, the extra information may obscure the obvious strategy, or lead players to try more subtle cooperative or competitive strategies.

The Cournot strategy has the property that $\boldsymbol{\sigma}$'s initially in the set (2.5) and (2.6) of extended optima remain therein. Similarly $\boldsymbol{\sigma}$'s which are initially in the 2 triangles

$$\{\sigma_1 + 2\sigma_2 > 60, 2\sigma_1 + \sigma_2 < 60\} \tag{3.7}$$

and

$$\{\sigma_1 + 2\sigma_2 < 60, 2\sigma_1 + \sigma_2 > 60\} \tag{3.8}$$

also remain therein. Consequently, this strategy predicts that $S(t) = S(1)$ for all t. The values of $S(t)$ given previously therefore suggest that the Cournot strategy does not adequately explain the distribution of $\boldsymbol{\sigma}$'s; one point of uncertainty is the rounding which was done in the profit tables for all experiments and which would certainly induce perturbations on a pure Cournot strategy.

A further point of interest connected with the Cournot solution is that it predicts $\sigma_1 + \sigma_2 = 40$. Fouraker and Siegel [1, pages 131–132] stress the fact that, in experiment 7, 14 out of 16 totals for transaction 24 support the Cournot value; they regard the totals as 'more important to the economic theory than the individual σ_i', [1, page 135]. This 'Cournot line', however, lies entirely outside O_I^W, except for the point (20, 20), and we have seen that O_I^W is favoured by the data even when Cournot-based alternatives are considered. This highlights the inadequacy of the simple classification of points used in [1] and some of the conclusions derived therefrom.

A notable feature of the R_i and R_i' is that, broadly, for both experiments, they initially rise for small i, reach a maximum and then decline, the dependence on i being reasonably smooth. We have no convincing explanation of this effect, but suspect it is geometric in origin and of no real economic significance.

4. Triopoly theory

Fouraker and Siegel [1] again compare their results with the 3 traditional optima:

(i) the Cournot optimum

$$\sigma_1 = \sigma_2 = \sigma_3 = 15, \tag{4.1}$$

(ii) the Pareto optimum

$$\sigma_1 + \sigma_2 + \sigma_3 = 30, \tag{4.2}$$

and

(iii) the competitive optimum

$$\sigma_1 + \sigma_2 + \sigma_3 = 60. \tag{4.3}$$

Our new extended optima described in [2] and [3] are of 4 types, I, II, III and IV, and the corresponding sets of $\boldsymbol{\sigma}$'s are denoted by O_I^W to O_{IV}^W, respectively. For the weakest optimum, type I, at least one firm can be disciplined by one or both competitors, as described in [2] and [3]. The resulting set O_I^W *excludes* only those $\boldsymbol{\sigma}$'s strictly below $\sigma_T = 30$ or strictly above $\sigma_T = 60$, or within the 3 tetrahedra

$$\{2\sigma_T - \sigma_i < 60, \sigma_T + \sigma_i > 60\} \tag{4.4}$$

for $i = 1, 2, 3$.

The optimum set O_{III}^W consists of those $\boldsymbol{\sigma}$'s for which *every* firm can be disciplined by one or both competitors. Besides the regions excluded by O_I^W it excludes the 6 regions T_1, T_2, T_3, U_1, U_2 and U_3, where

$$T_i = \{2\sigma_T - \sigma_i > 60, 2\sigma_T - \sigma_j < 60 \text{ for } j \neq i\} \tag{4.5}$$

and

$$U_i = \{\sigma_i + \sigma_T < 60, \sigma_j + \sigma_T > 60 \text{ for } j \neq i\}, \tag{4.6}$$

(see Table 1 of [3]). Clearly O_{III}^W is a subset of O_I^W.

The other two optima do not seem to play a significant role in the experiments, so they are not enlarged upon here.

It is worth noting here that Fouraker and Siegel appear to use the terms 'competitive' and 'rivalistic' almost interchangeably in the text and data analysis, despite having made a clear theoretical distinction between them [1, page 97]. While the two concepts coincide for duopolies they are different in the triopoly case. In fact, they seem always to mean 'competitive' in discussions of the experiments.

5. Comparison of theory with triopoly results

For their experiment 9, wikth complete information, Fouraker and Siegel [1] show that the results indicate no significant preference for any single one of the 3 traditional optima: Cournot, Pareto or competitive. To test the new optimum O_I^W, we follow the procedure of Section 3. Let $n_I(t)$ be the number of trios out of the 11 whose σ does *not* lie in O_I^W during transaction t. Of the 15625 possible σ in the cube $\{8 \leqslant \sigma_i \leqslant 32\}$ from which the players choose, 7703 are not in O_I^W. If the value of the proportion

$$r_I(t) \equiv 15625 n_I(t) / (11 \times 7703) \tag{5.1}$$

of points not in O_I^W on a per volume basis is significantly less than one, the new optimum O_I^W is favoured. The values of $r_I(t)$ in Table 3 clearly support our optimum against the hypothesis of uniform randomness for the σ's. The accumulated proportions

$$R_I(t) = \{r_I(1) + \cdots + r_I(t)\} \tag{5.2}$$

up to transaction t decrease from an initial value of 0.74 to a final value of 0.24, showing the optimum gradually makes its impression on the players, presumably through the disciplining effects of unfavourable output choices, as in the duopoly case.

To test O_I^W more stringently we omit the σ below $\sigma_T = 30$ and above $\sigma_T = 60$ from consideration in accordance with hypothesis H1 of Section 3. Of the remaining 7991 possible σ's only 69 are not in O_I^W. We find that none of these 69 is ever realized by any of the 11 trios in any of the 25 transactions. This is the best possible support for O_I^W and provides moderately significant evidence against H1 and in favour of O_I^W.

TABLE 3

Experiment 9: triopoly, complete information

t	$r_I(t)$	$R_I(t)$	$r_I'(t)$	$R_I'(t)$	$q_{III}(t)$	$Q_{III}(t)$
1	.74	.74	.80	.80	1.16	1.16
2	.74	.74	.80	.80	.00	.58
3	.74	.74	.57	.73	.00	.39
4	.00	.55	.00	.55	1.48	.76
5	.00	.44	.00	.44	.00	.57
6	.00	.37	.00	.37	.00	.45
7	.00	.32	.00	.31	.00	.38
8	.37	.32	.40	.32	1.81	.55
9	.18	.31	.00	.29	.00	.48
10	.37	.31	.20	.29	.00	.44
11	.00	.29	.00	.25	1.48	.55
12	.00	.26	.00	.23	2.22	.70
13	.18	.26	.00	.21	1.63	.78
14	.37	.26	.20	.21	.00	.73
15	.18	.26	.00	.20	.81	.73
16	.00	.24	.00	.19	.00	.68
17	.00	.23	.00	.18	.00	.64
18	.00	.22	.00	.17	1.48	.69
19	.18	.21	.20	.17	.00	.65
20	.37	.22	.20	.17	.90	.66
21	.18	.22	.00	.16	.00	.63
22	.18	.22	.20	.16	.00	.60
23	.37	.22	.40	.17	.90	.62
24	.55	.24	.20	.17	1.02	.63
25	.18	.24	.20	.18	.81	.64

The hypotheses H2 and H3 of Section 3, that a preference for the Cournot optimum is the only effect, can be tested by omitting σ's on the 3 surfaces $\sigma_1 = 15$, $\sigma_2 = 15$ and $\sigma_3 = 15$; note that we are not considering points in concentric spheres on this occasion. The proportions corresponding to $r_I(t)$ and $R_I(t)$ with such σ omitted are denoted by $r_I'(t)$ and $R_I'(t)$ and listed in Table 3. Again there is a clear and very significant fall in $R_I'(t)$, down to 0.16, which provides very strong evidence against these hypotheses.

Since states in O^W_{III} correspond to *every* firm being disciplinable, one might expect that such states would be favoured among the O^W_I states. There are 7019

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TABLE 4

Experiment 8: triopoly, incomplete information

t	$r_I(t)$	$R_I(t)$	$r'_I(t)$	$R'_I(t)$	$q_{III}(t)$	$Q_{III}(t)$
1	.55	.55	.60	.60	.96	.96
2	.92	.74	.80	.70	.51	.77
3	.55	.68	.60	.66	.58	.70
4	.00	.51	.00	.50	.98	.79
5	.18	.44	.20	.44	.62	.75
6	.00	.37	.00	.36	.98	.80
7	.37	.37	.40	.37	.51	.76
8	.37	.37	.40	.37	.86	.77
9	.37	.37	.40	.38	.51	.74
10	.00	.33	.00	.34	.84	.75
11	.55	.35	.40	.34	.77	.76
12	.18	.34	.20	.33	1.08	.79
13	.00	.31	.00	.31	1.12	.82
14	.18	.30	.20	.30	.77	.81
15	.18	.30	.20	.29	.77	.81
16	.18	.29	.20	.29	1.23	.84
17	.37	.29	.20	.28	1.03	.85
18	.18	.29	.00	.27	1.39	.88
19	.00	.27	.00	.25	.70	.87
20	.00	.26	.00	.24	.70	.86
21	.00	.25	.00	.23	.98	.87
22	.00	.23	.00	.22	.70	.86
23	.18	.23	.20	.22	1.39	.88
24	.00	.22	.00	.21	1.12	.89
25	.00	.21	.00	.20	1.12	.91

points in O_{III}^W (described in Section 4) all contained within the set of 7922 points of O_I^W. Let $m_{III}(t)$ be the number of σ's, from the 11 generated in transaction t, which lie among the 903 which belong to O_I^W but *not* to O_{III}^W. Then

$$q_{III}(t) = 7922\, m_{III}(t) / \{903(11 - n_I(t))\} \tag{5.3}$$

is the proportion of σ, on a per volume basis, which is in $O_I^W - O_{III}^W$ during transaction t.

The values of $q_{III}(t)$ and the corresponding accumulated proportions

$$Q_{III}(t) = 7922 \sum_{\tau=1}^{t} m_{III}(\tau) \Big/ \left\{ 903 \sum_{\tau=1}^{t} (11 - n_I(\tau)) \right\} \tag{5.4}$$

up to transaction t are also given in Table 3. The individual q_{III}'s are highly variable, as one would expect with such samples, though as pointed out in Section 6 they do provide some evidence in favour of the type III optimum. The cumulative proportions decline to a value well below one, thus supporting the suggestion of preference for $\mathrm{O}_{\mathrm{III}}^{\mathrm{W}}$. It would appear that, from about the 12th transaction, the players have learned through the experience of being disciplined for unfavourable moves. There is no clear preference for the optima of types II and IV described in [2] and [3].

In the experiment 8, with incomplete information, Fouraker and Siegel found some preference for the Cournot optimum compared with the Pareto and competitive optima, especially when total productions σ_T are examined. We analyse the experiment in the same fashion as experiment 9 with the results shown in Table 4. Again, none of the 69 points in the 3 tetrahedra (4.4) were ever realized.

The results in the first four columns once again provide very convincing evidence against all our alternatives (uniform randomness and H1, H2, H3), and in favour of the type I optima. It is especially interesting that, despite Fouraker and Siegel's analysis and the conclusions we drew from both duopoly experiments, hypotheses H3, of the preference of at least one player for a Cournot production, has no support here. This perhaps suggests that the added complexity of a third player obscures the simple maximizing strategy. Of course, the problems due to rounding in the profit tables mentioned in Sections 3 and 6 also occur here. Another possiblity is that the data reflect the oscillatory nature of the Cournot strategy for triopolies (Theocharis [6]).

The figures in the final columns of Table 4 show there is no real evidence of preference for $\mathrm{O}_{\mathrm{III}}^{\mathrm{W}}$ within $\mathrm{O}_{\mathrm{I}}^{\mathrm{W}}$ now. Perhaps the lack of information in experiment 8 means that a player is less able to undertake, or less likely to notice, disciplining moves.

In the analysis of experiment 7, we mentioned that Fouraker and Siegel's claim of support for the 'Cournot line' $\sigma_1 + \sigma_2 = 40$ was inconsistent with our finding of preference for $\mathrm{O}_{\mathrm{I}}^{\mathrm{W}}$. For triopolies, the 'Cournot plane' is $\sigma_1 + \sigma_2 + \sigma_3 = 45$ which does lie to a considerable extent within $\mathrm{O}_{\mathrm{I}}^{\mathrm{W}}$ (this is perhaps most easily seen by noting it is equivalent to $\Sigma_1^3 y_i = 3$, in the notation of [3]). Thus to make any similar comparison would take much more analysis, which we shall not perform.

6. Statistical analysis

At various points in the preceding sections we have made qualitative reference to the statistical significance of ratios and cumulative ratios calculated from the experimental data as measures of evidence for a range of hypotheses. Unlike Fouraker and Siegel, we shall not formally derive significance levels for our statistics, for the following reasons.

(i) We have a very large number of ratios, and it would be meaningless to produce an equivalent number of significance levels–for example, 1 in 20 of the levels around 5% significance will be misleading and there may be a considerable number of these.

(ii) Individual ratios for different transactions (Tables 1–4), or for concentric circles of different radii (see Section 3) are likely to be highly correlated, making the interpretation of the corresponding significance levels very difficult. The cumulative ratios are certainly correlated.

(iii) If we pick single ratios in each sequence, to overcome the objections in (i) and (ii), there seems to be no clear objective way of selecting them.

(iv) At least one alternative hypothesis to the extended optima, that of the σ_i being independent and uniform random variables, is not really plausible and is only introduced as a natural base to help concentrate the mind. Formal significance levels are unlikely to aid this process.

If, however, one does wish to make formal tests, the procedures are quite straightforward. Each individual ratio can be tested by referring the numerator observation to a binomial distribution whose index and parameter are clear from context; the necessary assumptions of independence and homogeneity of players seem to be true here. For example, from (3.1) and Table 1, the significance level of $r(24)$ is calculated from a binomial (16, 259/625) distribution, the probability of an observation from this distribution being less than or equal to $n(24)$'s observed value of 3 being .052. This observation, then, provides strong evidence against the uniformity hypothesis.

Two comments on this follow. First, in some cases the binomial index will also depend on the data, in (3.5) for instance. Our tests are then performed conditionally on the observed value of the index, on the assumption that this value does not contain any information about the hypothesis under test (technically, it is an ancillary statistic). Secondly, in general we would employ one-sided tests since we have reason to believe that ratio values significantly greater than one will occur; the test is also more stringent. A possible exception to the one-sided rule is the problem described in the penultimate paragraph of Section 3.

Cumulative ratios, such as $R(t)$ in (3.2), will also have a binomial distribution, if one ignores the probable dependence of summands mentioned in (ii). For moderate t, a normal approximation will be quite adequate, except perhaps when the hypothesized proportion parameter is very near zero.

On occasions a rather more *ad hoc* analysis may be appropriate. One such case is the $q_{III}(t)$ sequence of Table 3, which we noted at the time was highly variable, and for which the proportion of transactions with no O_I^W points in $O_I^W - O_{III}^W$ is perhaps a more meaningful quantity. For each transaction, the probability of the event is $(7019/7922)^{11-n_t}$ under the uniform hypothesis, ranging from about .26 to .42 on this data. The observed proportion of the event is .52 which is suggestive evidence against no preference for type III optima (though because of the dependence between transactions a formal test is dubious).

Finally, we have mentioned before that the hypotheses chosen in this paper as possible alternatives to the extended optima all have the weakness that, apart perhaps from specifying certain 'target' sets like the Cournot optimum, they assume individual productions are randomly and uniformly distributed in particular regions. In the absence of any really plausible notion to the contrary, this assumption seems the most useful as a basis for comparison and the lack of support for it from data on any specific occasion should not be regarded as a fatal flaw in the hypothesis. However, one might argue that the rounding introduced in the profit tables in [1] to simplify the numbers could plausibly induce a degree of uniform dispersion about targets such as the maximum profit at a fixed production level of the competitors. This effect can cut both ways; the apparent uniformity suggested by the R_i' statistics in experiment 10 is queried anew while the potential discrepancy of our analysis and that of Fouraker and Siegel in experiment 8 is partially resolved. In other words, these random perturbations due to rounding may mask more precise forms of bargaining behaviour and make the interpretation of formal and informal statistical analysis more uncertain.

7. Discussion of results

At several places in the preceding sections we have discussed the results of our analysis. In this final section we draw the threads together and add some new points.

The analysis of Fouraker and Siegel [1] concentrates on the three traditional optima of game theory, examines the relative support for these in the different experiments and attempts to explain any apparent differences in terms of variables such as the amount of information available and the number of players and their psychological characteristics. Our analysis revolves around the weak type I optima introduced in [2] and [3] and investigates to what extent this set of σ's is preferred by players during the course of a game.

We have already criticized several aspects of the analysis in [1]. However, in some ways the two approaches look at different parts of the general problem. Fouraker and Siegel are exclusively concerned with the ultimate targets of the players, these being of course very small (zero measure) subsets of the production space. How the players behave en route is not very relevant for them. In contrast, our theory describes the broad pattern of the sequence of bargaining moves, *via* the concept of discipline, and although the traditional goals are included in our 'allowed' regions they are treated only as a part of the whole process. It is this intermediate structure which is perhaps the most novel feature of our work.

A few other comments on our theory are:

(1) Although the adjustment process inherent in the description of discipline may appear sequential (player 2 moves after player 1), this is not necessary. Rather, with knowledge of the market each player knows in theory what *can* happen if he tries to move unilaterally in certain directions, so he will be discouraged from so doing.

(2) It is possible to relate discipline to the psychological states discussed in [1]. Roughly, this is because the lower triangle of O_I^W, (2.5), is involved with discouraging adjustments of a non-cooperative nature, while in the upper one, (2.6), the disciplining moves have more of the nature of a threat. Thus adjustments may be more convincingly related to discipline than to signalling (see [1, pages 151–154]). We should note, though, that a 'rivalist' will, almost tautologically, be neither concerned with, nor influenced by, disciplining.

(3) Although our theory does not specifically mention cooperative behaviour it certainly does not eliminate it; on the contrary, it is in a sense encouraged because non-cooperative moves are disciplinable, hence not attractive.

While we would not claim that the extended optima are unequivocally supported by the data, if only because the experimental conditions are not completely suitable (unrestricted production changes cause the theory some problems), we feel that there is a considerable amount of evidence in favour of our new optima. We have mentioned earlier that our alternative hypotheses (H1, H2, H3) considered are not always completely plausible, and clearly it is possible to keep on constructing ever more involved hypotheses that try to explain the observations, but these rapidly become complex and arbitrary. *We have not found any simple, natural alternatives to the extended optima that convincingly explain the whole pattern of the data.*

In [1, pages 150–151] we find: "It seems clear that bargaining by oligopolists under complete information may lead to results falling all along the distribution of possible outcomes. [Our] theory......predicts dispersion as a function of the combination of bargaining types and signals....". The extended optima provide a clear and rational explanation of this dispersion.

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