DEDUCING THICKNESS CHANGES OF AN ICE SHEET FROM RADIO-ECHO AND OTHER MEASUREMENTS

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ABSTRACT. The displacement of the surface of an ice sheet and of markers set in its top layers can be measured geodetically, and also, it is expected, by radio-echo methods. The paper discusses how such measurements could be interpreted as showing long-term changes in the thickness of the ice sheet; in particular it discusses how one might design an experiment so as to avoid unwanted effects due to short-term changes in rate of accumulation. The analysis is similar to that of Federer and others (1970), but it corrects an error, so that when applied to their results for central Greenland it gives a different result for the lowering of the surface. Federer and others have already concluded that the average accumulation rates during the past 100 years have been below those needed to keep in balance with the velocity of the ice sheet as a whole. Using a particular model, it is found that this has resulted in the surface lowering at a mean rate of 0.050 m a^{-1} between 1871 and 1968, and a mean rate of 0.140 m a^{-1} between 1959 and 1968.

RÉSUMÉ. *Détermination de changement d'epaisseur d'une calotte glaciaire à partir des radio-échos et d'autres mesures.* Le déplacement de la surface d'une calotte glaciaire et des témoins installés dans les niveaux supérieurs de celle-ci peuvent être mesurés par les méthodes géodésiques et aussi, on prévoit, par les radio-échos. Cet article discute la manière d'interpréter de telles mesures pour mettre en évidence des changements à long terme dans l'épaisseur de la calotte; on discute en particulier comment il est possible de conduire l'expérience de manière à éviter les effets indésirables dûs à des changements à courte périodicité dans le taux d'accumulation. L'analyse est semblable à celle de Federer et autres (1970), mais elle corrige une erreur qui, appliquée aux résultats de ces auteurs pour le Groenland central, donne un résultat différent pour l'abaissement du niveau de la surface. Federer et autres ont déjà conclu que le taux moyen d'accumulation durant les dernières cent années avait été inférieur à celui nécessaire pour maintenir un équilibre avec la vitesse d'écoulement de l'ensemble de la calotte. En utilisant un modèle particulier, on trouve qu'il en est résulté un abaissement du niveau de la surface de 0,050 m par an entre 1871 et 1968 et de 0,140 m par an entre 1959 et 1968.

ZUSAMMENFASSUNG. *Die Herleitung von Dickenänderungen einer Eisdecke aus Radar-Echolotungen und anderen Messungen.* Die Verlagerungen der Oberfläche einer Eisdecke und von Pegeln, die in ihren obersten Schichten verankert sind, können mit geodätischen Verfahren, aber vermutlich auch aus Radar-Echolotungen bestimmt werden. In diesem Beitrag wird diskutiert, in wieweit derartige Messungen als Indikatoren für langfristige Änderungen in der Dicke der Eisdecke herangezogen werden können; insbesondere wird dargelegt, wie ein Experiment anzulegen wäre, das unerwünschte Einflüsse von kurzzeitigen Veränderungen der Akkumulationsrate ausschliesst. Die Analyse ähnelt der von Federer u.A. (1970), es wird jedoch ein Fehler berichtigt, so dass die Anwendung auf deren Resultate für Zentral-Grönland zu einem unterschiedlichen Ergebnis für das Absinken der Oberfläche führt. Federer u.A. haben bereits den Schluss gezogen, dass die durchschnittlichen Akkumulationsraten während der letzten 100 Jahre unter denen gelegen haben, die nötig sind, um ein Gleichgewicht zur Geschwindigkeit des Inlandeises als Ganzes aufrechtzuerhalten. Bei Annahme eines bestimmten Modells ergibt sich, dass dies ein mittleres Absinken der Oberfläche von 0,050 m pro Jahr zwischen 1871 und 1968 und von 0,140 m pro Jahr zwischen 1959 und 1968 zur Folge hatte.

INTRODUCTION

The proposed radio-echo method of measuring the movement of an ice sheet (Nye and others, 1972[b]; Walford, 1972; Nye and others, 1972[a]) works in essence by enabling one to set up a horizontal and vertical coordinate system, fixed relative to the rock bed, precise to a few centimetres, and independent of visible landmarks. It has been suggested that the displacement of features such as the surface of the snow, a particular firn layer, or a marker set in the snow, could be measured relative to this fixed frame. This paper discusses how such displacement measurements could be interpreted, and, in particular, to what extent they could be interpreted as showing long-term changes in the thickness of the ice sheet. Federer and others (1970) analyse a similar problem in relation to the measurements of horizontal and vertical displacement made by the repeated geodetic surveys of the Expedition Glaciologique Internationale au Groenland (EGIG 1 and 2) in central Greenland. The topic is not as straightforward as it may at first appear.

If an ice sheet is in a steady state, with a certain rate of accumulation, and if the accumulation rate is suddenly changed, the new steady state is approached asymptotically with a time constant of the order of 10³ to 10⁴ years (Nye, 1960; Haefeli, 1961). Thus, even with a rapidly

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varying accumulation rate, the velocity pattern of the ice sheet as a whole is expected to change rather slowly with time; this is in contrast to the accumulation rate itself, which is expected to show fluctuations on a wide variety of time scales, from single storms, through seasonal changes to long-term climatic changes.

The direct approach to the problem of discovering long-term changes in the thickness of an ice sheet would be simply to measure the change in height of the surface over a certain time interval. The difficulty with this method is that the result would obviously be very sensitive to the particular values taken by the accumulation rate during the interval; one does not want the experiment to be merely a complicated way of discovering that it has recently been snowing exceptionally hard. To distinguish the long-term trend from short-term "noise" of this sort one might try to measure the change in height, not of the horizon of age zero, which is the surface, but of the firn horizon which is, say, ten years old. Such a horizon of course moves relative to the firn; and so the method would involve identifying the ten-year-old horizon by stratigraphic studies at the beginning and at the end of the time interval. Even if the identification were correct, the measured height changes would still be far from steady, for they would still reflect short-term fluctuations in accumulation rate, such as single storms or single years with high accumulation. Thus a direct approach of this sort would only be successful if the measurement interval were very long.

Rather than trying to identify repeatedly a firn horizon of given age, which moves relative to the firn, it is much easier to use markers which are fixed relative to the firn. At the same time it will be noticed that the direct method just described does not make use of the extensive information on the history of the rate of accumulation which has been the main result of the many stratigraphic studies carried out in Greenland and Antarctica. But, although it provides a much better method, the use of markers fixed in the firn, together with information on accumulation rate, brings its own problems. For example, the downward velocity of markers set in the upper layers of the firn is only partly due to the velocity of the ice sheet as a whole; a large part is due to the compaction of the firn. The latter component is affected by fluctuations in accumulation rate, and this must be taken into account. Knowing the velocity of the markers, and with information from stratigraphy about the history of accumulation rate, we can try to answer the question: by how much is the velocity of the ice sheet as a whole out of balance with the current long-term accumulation rate? When the question is put in this form it is obvious that the precise answer must depend on the time period over which one chooses to average the accumulation rate. The direct method would measure the rate of increase of thickness of the ice sheet averaged simply over the period of observation; by contrast, the method which uses markers in the firn in conjunction with the accumulation rate effectively makes the averaging period equal to the length of the stratigraphic record; this will usually be much longer, and the result will be correspondingly better.

A full analysis of the motion of the markers would require a theory of the snow compaction process with a varying accumulation rate and temperature, which is a very difficult matter. The theory that follows avoids this by stating conclusions entirely in terms of steady accumulation rates, although a model involving a sudden change from one steady rate to another is also considered.

THEORY

If there were a perfectly steady state, snow would fall in a given location at a steady rate and the velocity of the ice sheet would be just sufficient to ensure that the surface remained fixed in space. The compaction of the snow into firn would result in a density : depth relation that was constant with time. But we may expect that, provided the accumulation rate and the atmospheric conditions remain steady, even if the velocity of the ice sheet is out of balance with the accumulation rate, the physics of the compaction process will still be essentially the

same; and therefore the density : depth relation will be constant, provided the depth is measured relative to the surface. This is Sorge's Law (Bader, 1960, 1963).

The first model we use is thus as follows. The accumulation rate is constant. The movement of the ice sheet as a whole is out of balance with this accumulation rate, and so the thickness h at the place in question changes. Over a time interval short compared with 10^3 to 10^4 years, $\partial h/\partial t$ will be constant, to a sufficient approximation. We wish to relate $\partial h/\partial t$ to the motion of a marker in the firn. P in Figure 1 is a marker put at a certain depth in the firn at time t. The age, at time t, of the firn layer in which the marker lies is T. At a later time $t+\Delta t$ the marker (assumed fixed in the medium) is at P' and is now in firn of age $T+\Delta t$. PQ is a line drawn parallel to the surface of the ice sheet and P'Q is perpendicular to it.
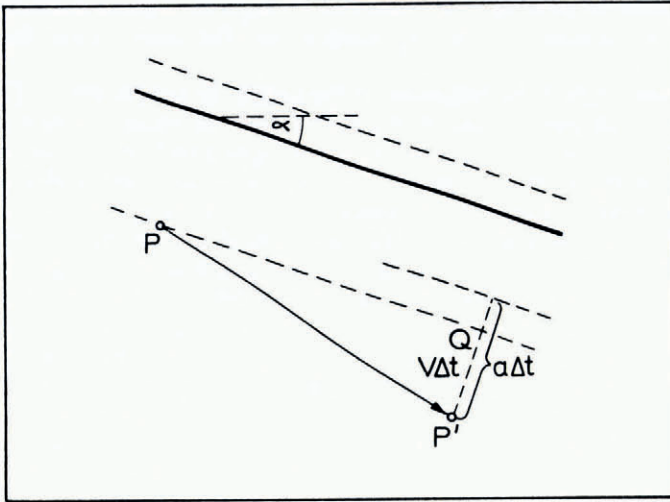

Fig. 1. P is a marker buried in the firn. In the time interval Δt it moves to P'. PQ is drawn parallel to the surface.

P'Q equals $V\Delta t$, where V is the downward velocity component of P normal to the surface. At time $t+\Delta t$ the firn layer of age T is above the marker by an amount $a\Delta t$, measured parallel to P'Q, where a is the average rate of accumulation measured normal to the surface as thickness of firn per unit time between the depths of P and P'. Let us assume that the water equivalent is fully preserved in a vertical column; there is no thinning of the layers by lateral flow. Then a is related to the rate of accumulation $a^\star$ measured normal to the surface as thickness of water equivalent by

$$a^\star \rho_w = a\rho,$$

where ρ_w is the density of water and ρ is the density at the depth concerned (the mean density between the depths of P and P'). Hence the horizon of age T has moved up parallel to its normal by a distance $a\Delta t - V\Delta t$. But, according to the model underlying Sorge's Law, the firn layer of age T is always to be found at the same depth below the surface. Hence the upward displacement of the horizon of age T is identical with the upward displacement of the surface, which, in turn, is equal to the increase Δh in thickness of the ice sheet measured normal to its surface. Hence

$$\Delta h = a\Delta t - V\Delta t, \qquad (1)$$

or, taking the limit,

$$\frac{\partial h}{\partial t} = a - V. \tag{2}$$

In this equation both a and V vary with depth z, but their difference is constant. In terms of a density : depth profile $\rho(z)$ we have

$$\frac{\partial h}{\partial t} = \frac{a^\star \rho_w}{\rho(z)} - V(z). \tag{3}$$

For an alternative derivation of Equation (3), consider an imaginary parallel-sided slab of constant thickness z whose top surface coincides with the moving top surface of the ice sheet. The slab is not fixed in space, neither is it fixed within the medium; it moves up along the normal to the surface with velocity $\partial h/\partial t$. The mass flux through the top surface is the rate of accumulation of mass $a^\star \rho_w$; the mass flux through the bottom surface is $(\partial h/\partial t + V)\rho$, where ρ is the density at depth z; and, by Sorge's Law, the mass within the slab is constant. Equation (3) follows.

These derivations neglect the thinning of the layers by lateral flow. This effect gives a correction term in Equation (3) which, for the Greenland measurements referred to later, amounts to about 2% of the individual terms on the right-hand side. So as not to obscure the main issue we shall neglect this term in what follows. It is discussed in the Appendix.

The radio-echo method essentially enables one to relocate the point P at the later time $t + \Delta t$, when the marker has reached P′. Hence it gives the vector PP′, from which the component P′Q, equal to $V\Delta t$, may be deduced. This is emphasized because from one point of view the method measures horizontal and vertical displacements separately—horizontal displacement by observing the fading pattern of the envelope of the echo, and vertical displacement by observing the phase of the echo. This would make it seem as if the results should be referred to a coordinate system parallel and perpendicular to the bed of the ice sheet, with the difficulty that the slope of the bed is not known with any accuracy. The difficulty is avoided when it is realized that the method simply relocates P; one can then measure the components of the vector PP′ in whatever coordinate system happens to be convenient.

Thus, to apply Equation (3), one measures V at a number of depths and one also needs to know ρ at these depths and the steady accumulation rate $a^\star$; $\partial h/\partial t$ is then deduced for each depth. If the model were correct, $\partial h/\partial t$ should be a constant. In fact we know that the accumulation rate fluctuates, and so $\partial h/\partial t$ as deduced in this way, using an average $a^\star$ and an average $\rho(z)$ curve, will fluctuate with depth. But we expect the irregularities to diminish with depth, for the following reason. Suppose there is one year with an unusually high accumulation. A marker set in the surface at the beginning of that year will show a larger value of V than usual because the consolidation of the firn beneath it is taking place under a load that is higher than usual. A marker set at depth at the beginning of the same year will also show a larger value of V than usual, but now the effect will be smaller, because the consolidation of the firn beneath this deeper marker is taking place under a load that is made up of several annual layers, and the unusually thick one at the top is only one of many, the rest of which are of the normal thickness. So the abnormality in V will diminish with depth. $a^\star$ and $\rho(z)$, being long-term averages, are only slightly altered by the single bad year, and therefore we expect that the values of $\partial h/\partial t$ deduced using Equation (3) from the deeper layers will be the most reliable; a plot of $\partial h/\partial t$ versus depth should approach a steady value. In this way the effect of short-term fluctuations of accumulation rate on the rate of consolidation should be smoothed out. One is left with the question of what long-term average to take for $a^\star$ in Equation (3), but, as we have already said, this problem is inherent in the original question: how much is the velocity of the ice sheet as a whole out of balance with the current long-term accumulation rate?

An advantage of the method described, using Equation (3), is that the measurements to be made are essentially simple. It is not necessary to find the age of particular firn layers; the only stratigraphy needed is that necessary to establish the long-term accumulation rate.

The measurements described give the change of thickness at the site; they do not of themselves disclose whether it is characteristic of the ice sheet as a whole, or whether, for example, it is cause by the horizontal movement through the measuring site of a bulge in thickness. These effects could only be separated by measuring at a number of different sites.

APPLICATION

Radio-echo observations to measure V, and hence $\partial h/\partial t$, are being planned at South Pole Station, Antarctica, but the result will probably not be known for some 5 to 10 years. However, closely related observations have already been analysed for Jarl-Joset station in central Greenland by Federer and others (1970); see also Quervain (1969, p. 135–41). The height of a point in an inclined shaft 40 m below the surface was surveyed on two occasions, in 1960 and 1968, and the vertical velocity was deduced. The horizontal eastward component of the velocity at the station was also measured. The authors deduce the resultant lowering of the surface, but the calculation contains an error, of a confusing kind. Apart from this their analysis is basically the same as ours. To relate the two approaches we divide Equation (2) by cos α, where α ($\approx$ 0.003 rad) is the slope of the surface, and express V in terms of u and v, the horizontal and vertical components of the total velocity, thus

$$V = v \cos \alpha - u \sin \alpha$$

to give

$$\frac{\partial h}{\partial t} \sec \alpha = a \sec \alpha - v + u \tan \alpha.$$

We use a subscript v for quantities measured vertically, rather than normal to the surface; thus

$$\frac{\partial h_\mathrm{v}}{\partial t} = \frac{\partial h}{\partial t} \sec \alpha, \qquad a_\mathrm{v} = a \sec \alpha, \qquad a_\mathrm{v}^\star = a^\star \sec \alpha$$

Then,

$$\frac{\partial h_\mathrm{v}}{\partial t} = a_\mathrm{v} - v + u \tan \alpha,$$

and hence, multiplying by ρ/ρ_w,

$$\frac{\rho}{\rho_\mathrm{w}} \frac{\partial h_\mathrm{v}}{\partial t} = a_\mathrm{v}^\star - V_r^\star, \qquad\qquad (4)$$

where $V_r^\star = (\rho/\rho_\mathrm{w})(v - u \tan \alpha)$ as defined by Federer and others (1970) in their Equations (1) and (2). Thus

$$V_r^\star = \frac{\rho}{\rho_\mathrm{w}} V \sec \alpha;$$

$V_r^\star$ is the downward velocity normal to the surface, V, converted to water equivalent and multiplied by sec α.

Federer and others (1970) find $V_r^\star = 0.29$ m a^{-1} in the period 1960 to 1968; they compare this with the corresponding 9-year mean accumulation rate $a_\mathrm{v}^\star$, equal to 0.19 m a^{-1}, to obtain a deficit of 0.10 m a^{-1} water equivalent over this period. To obtain the resultant lowering of the surface they multiply this water equivalent by ρ_w and divide by a mean density value of the material between the surface in 1968 and the layer of the year 1960, namely 480 kg m^{-3}. But Equation (4) shows that to obtain $\partial h_\mathrm{v}/\partial t$ the water equivalent ($a_\mathrm{v}^\star - V_r^\star$) should be

multiplied by ρ_w/ρ, where ρ is the density measured at the depth of the marker, 40 m below the surface, namely 690 kg m^{-3}. Thus

$$\frac{\partial h_v}{\partial t} = -\frac{1}{0.690}\times 0.10 \text{ m a}^{-1} = -0.145 \text{ m a}^{-1},$$

which gives a lowering of the surface between 1960 and 1968 of 1.16 m (rather than 1.67 m). The measured lowering of the surface was 1.74 m, which agrees less well than before. The reason for the difference must either be errors of measurement or that the model used for calculation, which assumes a constant rate of accumulation, is inadequate.

TABLE I. MEASURED RATES OF ACCUMULATION AT JARL-JOSET STATION, GREENLAND

Period	Interval years	$a_v^\star$ m a^{-1}		
1871–1944	73	0.265		} 0.262
1945–60	15	0.258	} 0.249	
1945–59	14	0.240		
1959–68	9	0.206	} 0.193	
1959–68	9	0.180		

Table I of Federer and others (1970) shows measured rates of accumulation for different periods and is reproduced, in part, in Table I. Weighting the measured values in column 3 of Table I according to the time intervals, we see that, while the average accumulation rate for 1871–1959 was 0.262 m a^{-1}, during 1959–68 it was only 0.193 m a^{-1}. This suggests a model in which the accumulation rate is a step function falling suddenly from 0.262 to 0.193 m a^{-1} in 1959. We can no longer use Sorge's Law, and a complete analysis would involve considering the details of the snow compaction process. However, as an approximation we might proceed as follows. Suppose first that the accumulation rate had continued steadily at 0.262 m a^{-1}. The expected lowering of the surface from 1960 to 1968 can then be estimated from Equation (4) by using the values of $V_r^\star$ and ρ measured at 40 m depth (the actual values $V_r^\star$ and ρ in this hypothetical situation would be slightly different). The lowering would have been 0.32 m. In fact the accumulation rate from 1959–68 was 0.069 m a^{-1} lower than the value just assumed, a deficit of 0.621 m of water. Suppose that the only effect of this is that a corresponding layer of firn of density 480 kg m^{-3} is missing; we are assuming that, despite the thinner-than-usual top strata, the compaction below proceeded unaltered. It is then as if a layer 1.29 m thick had been sliced off. The total lowering would be 0.32 m + 1.29 m = 1.61 m. This is evidently an overestimate because in fact the compaction rate would be slowed down. Nevertheless it is still less than the observed value of 1.74 m (but an improvement on the value of 1.16 m obtained by assuming a steady accumulation rate of 0.19 m a^{-1}). By making the step occur in 1958 instead of 1959 the total calculated lowering would be 1.76 m, virtually as observed. If the step occurred in 1957 the calculated lowering would be 1.90 m. Again, these are both overestimates.

Let us return to the original question: by how much is the velocity of the ice sheet as a whole out of balance with the current long-term accumulation rate? From Equation (4), using values from the 40 m depth level, we find that, if the "current long-term accumulation rate" is interpreted as meaning the mean value for the 97 year period 1871–1968 ($a_v^\star = 0.255$ m of water per year), the rate of fall of the surface is 0.050 m a^{-1}. Strictly interpreted, this is the rate at which the surface would have fallen if the accumulation rate had been steady at its measured average value during the period, and if, under these hypothetical

conditions, ρ and $V_r{}^\star$ at 40 m depth had taken their measured values. Table II shows the results if the "current long-term accumulation rate" is taken instead as the mean for 1871–1959 or for 1959–68.

The final column in Table II gives $1 - (a_v{}^\star/V_r{}^\star)$ as a percentage, the amount by which the accumulation rate is below the balance value.

<div align="center">TABLE II</div>

Period	$a_v{}^\star$ m a^{-1}	$\partial h_v/\partial t$ m a^{-1}	Deficit %
1871–1968	0.255	−0.050	12
1871–1959	0.262	−0.040	10
1959–68	0.193	−0.140	33

REFERENCES

Bader, H. 1960. Theory of densification of dry snow on high polar glaciers. *U.S. Snow, Ice and Permafrost Research Establishment. Research Report* 69.

Bader, H. 1963. Theory of densification of dry snow on high polar glaciers. II. (*In* Kingery, W. D., *ed. Ice and snow; properties, processes, and applications: proceedings of a conference held at the Massachusetts Institute of Technology, February 12–16, 1962.* Cambridge, Mass., M.I.T. Press, p. 351–76.)

Federer, B., *and others.* 1970. Outflow and accumulation of ice in Jarl-Joset station, Greenland, by B. Federer, H.[-V.] v[on] Sury, K. Philberth and M. [R.] de Quervain. *Journal of Geophysical Research,* Vol. 75, No. 24, p. 4567–69.

Haefeli, R. 1961. Contribution to the movement and the form of ice sheets in the Arctic and Antarctic. *Journal of Glaciology,* Vol. 3, No. 30, p. 1133–51.

Nye, J. F. 1960. The response of glaciers and ice-sheets to seasonal and climatic changes. *Proceedings of the Royal Society of London,* Ser. A, Vol. 256, No. 1287, p. 559–84.

Nye, J. F., *and others.* 1972[a]. Measuring the change in thickness of the Antarctic ice sheet, by J. F. Nye, M. V. Berry and M. E. R. Walford. *Nature, Physical Science,* Vol. 240, No. 97, p. 7–9.

Nye, J. F., *and others.* 1972[b]. Proposal for measuring the movement of a large ice sheet by observing radio echoes, by J. F. Nye, R. G. Kyte and D. C. Threlfall. *Journal of Glaciology,* Vol. 11, No. 63, p. 319–25.

Quervain, M. R. de. 1969. Schneekundliche Arbeiten der Internationalen Glaziologischen Grönlandexpedition (Nivologie). *Meddelelser om Grønland,* Bd. 177, Nr. 4.

Walford, M. E. R. 1972. Glacier movement measured with a radio echo technique. *Nature,* Vol. 239, No. 5367, p. 93–95.

APPENDIX

THE EFFECT OF THINNING OF THE LAYERS BY LATERAL FLOW

The lateral spreading of the ice sheet causes a thinning of the annual layers which is in addition to the effect of compaction; at sufficient depth, where the density has become nearly constant, this is the dominant effect. The lateral spreading rate will be almost uniform with depth in the upper layers, but for the sake of generality we will use a model in which it varies with depth. To derive the appropriate correction term to Equation (3) we first examine Sorge's Law a little more carefully.

Consider a model in which the accumulation rate is steady in time but variable in space. The spreading rate likewise varies in space, both vertically and horizontally, but measured relative to the surface of the ice sheet it is steady in time; that is to say, it depends on horizontal position and on depth. The spreading rate is out of balance with the accumulation rate and so the thickness of the ice sheet changes steadily with time, but non-uniformly in space. Such a condition obviously could not persist for all time (because the thickness cannot be negative) but we could imagine it persisting for a period of, say, 10^3 years. So we adopt this model, but with the proviso that we must not draw conclusions from it about the state of layers older than 10^3 years, because such layers experienced an unspecified spreading before our steady spreading rate started. With this proviso and provided atmospheric conditions are steady, we assert that the density : depth curve will be constant in time.

Now we repeat the mass flux argument which follows Equation (3) in the main part of the paper, but paying attention to the lateral flux. Take as the control volume a parallel-sided slab of constant thickness z whose moving top surface, of area S_0, coincides with the moving top surface of the ice sheet. The sides of the slab are normal to the top and bottom faces and are fixed in space (that is, they only move perpendicular to the surface). The lateral spreading rate is measured by specifying the proportional rate of change $e(z)$ of a test area S fixed in the firn and parallel to the surface, namely

$$\frac{1}{S}\frac{dS}{dt} = e(z).$$

If there was no change of density, e would also be the compressive strain-rate normal to the surface. Thus e may be thought of as the compressive strain-rate normal to the surface that acts in addition to compaction. The mass flux through the sides of the slab is then $\bar{\rho e}zS_0$, where the bar denotes the average value between the surface and depth z. By the generalization of Sorge's Law asserted above, the mass within the slab remains constant. Hence

$$a^\star\rho_w S_0 = \left(\frac{\partial h}{\partial t}+V\right)\rho S_0+\bar{\rho e}zS_0,$$

and thus

$$\frac{\partial h}{\partial t} = \frac{a^\star\rho_w}{\rho} - V - \frac{\bar{\rho e}z}{\rho}, \tag{3a}$$

where ρ, V and $\bar{\rho e}$ are all functions of z. The last term is the required correction to Equation (3). To estimate its magnitude, neglect all density variations and put $\bar{e} \approx V/h$. The term is then $V(z/h)$, which for the Greenland example is 1.6% of V.

The correction can also be expressed in terms of the age of the layer at depth z, but the form of Equation (3a) has the advantage of involving quantities which can be measured without stratigraphy.